Unraveling in Matching Markets

By Hao Li and Sherwin Rosen*

We use a two-period matching model with initial uncertainty about productivities of participants to analyze incentives for early contracting or unraveling. Unraveling provides insurance in the absence of complete markets, but causes inefficient assignments. Unraveling is more likely, the smaller the applicant pool, the smaller the proportion of more promising applicants, and the greater the heterogeneity in the pool. Banning early contracts hurts firms and benefits less promising applicants; the effects on more promising applicants depend on how the gains from early contracts are shared. Ex post *buyouts eliminate inefficient assignments, and more promising applicants always unravel.* (*JEL* D40, J44, C78)

* Li: School of Economics and Finance, University of Hong Kong, Pokfulam Road, Hong Kong (haoli@econ.hku.hk); Rosen: Department of Economics, University of Chicago, 1126 East 57th Street, Chicago, IL 60637, USA (rosen@cicero.spc.uchicago.edu). The authors are grateful for the comments and suggestions made by Preston McAfee, Alvin Roth, Wing Suen and anonymous referees.

The timing of transactions is important in markets where buyers and sellers need to be matched. There are many examples. Elite colleges have both regular and early admission programs. Most graduate and undergraduate admission procedures agree not to inform applicants prior to a common date. New entrants into many professional sports choose when to expose themselves to the draft. Much attention has been focused on the trend toward recruiting younger and less experienced players in recent National Basketball Association drafts. This year the National Basketball Association (NBA) draft set a record in that the first 7 picks and 17 among 29 first-round selections were not college seniors. In the past few years some highly ranked players have skipped college altogether. In spite of the posturing by NBA executives urging players to stay in school and finish their education, this year's outcome is along the creeping trend toward earlier entrance into the NBA.

Difficulties in controlling the timing of recruiting new entrants of highly trained professionals have been pointed out in a set of interesting papers by Roth (1984,1991), and Mongell and Roth (1991). Roth and Xing (1994) observe that these difficulties appear in many other markets, including the medieval weekly markets for ordinary commodities, medical interns and residents, and postseason college football bowl games.

Two related aspects of timing of market transactions must be distinguished. In one, labeled "jumping the gun" by Roth and Xing, participants compete for a limited supply of the best-qualified candidates or best positions in strategically timing their offers. For instance, firms sometimes make exploding offers that quickly expire, while job candidates often try to delay making a choice from available offers in the hope of receiving better ones. Another aspect of timing is unraveling of the appointment date. In some entry-level professional labor markets, employment begins only after attainment and certification of professional qualifications, but the appointment date sometimes unravels to a few years before that. Unraveling is defined as future employment contracts that are signed long before employment is to begin. Examples include the placement of medical interns and residents before the 1950's, summer job programs for law students prior to offers of longer term positions upon graduation, and early commitment by judges to clerks. Often there are natural dates when the potential of candidates can be best assessed. Unraveling of the appointment date is not so much about competing through strategically timing proposals and acceptances. It is more about *ex post* inefficiencies caused by making early contracts with incomplete information.

The two aspects are related. Sometimes jumping the gun can be the principal reason for unraveling of the appointment date. Still, isolating the insurance aspect of "futures" contracts from the strategic aspect helps us understand how market imperfections affect timing problems. It also allows the social costs and benefits of early contracting to be assessed. This paper analyzes price competition and market unraveling based on incomplete information about agents' future productive characteristics. Agents have to deal with both individual uncertainty about their traits, as well as aggregate uncertainty about the market value of those traits. Aggregate uncertainty arises in these market because assignments are indivisible. If appointments are made after individual uncertainty is resolved, *ex post* efficiency of assignments is guaranteed. But agents are exposed to payoff risks because that they can be on the long or short side of the market. Complete Arrow-Debreu securities markets would eliminate payoff risks to agents and guarantee *ex post* efficient job assignments. In the absence of such markets, unraveling (early contracting) can bring about limited insurance at the expense of *ex post* inefficient assignments. Unraveling is a manifestation of risk-aversion under incomplete insurance markets. It relieves some of the anxiety about availability of jobs for applicants and of qualified candidates for firms.

We analyze markets where individual uncertainty applies only to job applicants, and consider two contractual situations, one where firms are bound by *ex post* unsuccessful early contracts, and another where buyouts of inefficient matches by firms are allowed. Without buyouts, unraveling need not occur. It is more likely, the smaller the proportion of more promising candidates in the applicant pool, and the smaller the total applicant pool relative to the number of positions. In our model, payoff risks to firms are the main source of insurance gains from early contracts. These gains are large when the applicant pool or proportion of promising applicants are small, because firms are less likely to fill their positions in the spot market. An increase in the degree of heterogeneity increases the likelihood that more promising applicants unravel. In this model, unraveling always reduces the probability that productive applicants will be short and firms will be long in the *ex post* spot market. Therefore a ban on early contracts unambiguously decreases the *ex ante* welfare of firms. Since less promising applicants never receive any rents from early contracts, the ban unambiguously improves their welfare. Whether the ban increases or decreases welfare of more promising applicants depends on the gains they receive.

When buyouts of *ex post* unsuccessful early contracts are allowed, unraveling is more prevalent than when early contracts are binding. Buyouts amount to an up-front contingency clause in early contracts specifying the terms under which contracts are terminated *ex post*. Buyout provisions benefit participants as a whole because they eliminate *ex post* inefficient assignments and increase total gains from trade. Buyouts always increase the welfare of those less promising candidates who would choose to wait when *ex ante* contracts are binding. Whether buyouts increase or decrease the welfare of firms and more promising candidates depends on how the gains they receive with buyouts compare with the gains they receive without them.

The next section introduces a two-period assignment model, defines individual and aggregate uncertainties that are central to our analysis of unraveling, and identifies the market imperfections that cause unraveling to occur. Section II analyzes unraveling in a competitive market equilibrium without buyouts, and considers how relative size, composition and the degree of heterogeneity in the applicant pool affect unraveling. Section III considers unraveling when buyouts are allowed. The last section contains a brief summary of this paper and some comments on our findings.

I. The Assignment Market: Background

There are two models of matching: the assignment market (Koopmans and Beckmann, 1953, and Shapley and Shubik, 1972), and the marriage problem (Gale and Shapley, 1962). These exchanges differ from others in that objects of trade are indivisible. Typically, each participant has only one unit to buy or sell. In an assignment market, money is transferable among all participants and matching is made through market prices. In a marriage problem, only rank orderings of preferences matter and prices play no allocating role.[1] We use the assignment market and competitive mechanism as the framework of our analysis.

A. Competitive equilibrium in the assignment market

In an assignment market, there are two groups of agents, workers and firms. A production function describes a non-negative joint output of an assignment of a worker to a firm. The joint output of assigning either two workers or two firms to each other is zero. The output of an unassigned agent is normalized to zero. The largest total output of any number of agents is the maximal sum of outputs that can be produced by pair-wise assignments of these agents.

A pair-wise assignment is feasible if each agent is matched to at most one other agent. Associated with a feasible assignment is a feasible outcome, a vector of non-negative payoffs such that the sum of the payoffs to each matched pair is no greater than the joint output of the pair and the payoff to each unmatched agent is zero. An assignment is efficient if among all feasible assignments it yields the largest sum of joint outputs. A feasible outcome is stable if the sum of the payoffs to any pair of agents is greater than or equal to the joint output they can produce. Thus, in a stable outcome, the sum of payoffs equals the joint output for each matched pair. For any pair not matched to each other, the sum of payoffs for the two agents is greater than the joint output they can produce, so that the pair does not form a "blocking" coalition that improves the payoff for both of them.

The main theorem of the assignment market is that if a feasible outcome is stable then the associated assignment is efficient, and if a feasible assignment is efficient then there exists a feasible outcome associated with it that is stable. This is analogous to the two welfare theorems in a decentralized economy. Koopmans and Beckmann (1953) formulate the efficient assignment as the solution to a linear programming problem and give a competitive market equilibrium interpretation to the associated outcome. Shapley and Shubik (1972) formulate stable outcomes as equilibria of a cooperative game and show that side-payments do not occur in equilibrium. In this paper, we refer to each stable outcome as a competitive equilibrium.

B. Payoff risks

Parallels between the assignment theory and competitive markets are best revealed by an example with two distinct types of identical agents in each group. Suppose that workers and firms are either productive or unproductive, and that assigning a productive worker to a productive firm produces unit output, while all other kinds of assignments produce nothing.

To put the assignment market in a competitive framework, it is useful to think of one group of agents, say the firms, as residual income recipients. Unproductive firms cannot bid positive prices for workers and receive a payoff of zero in the market. Similarly, unproductive workers cannot ask positive prices of firms and receive zero payoff. Only productive workers and productive firms potentially have positive payoffs. Their actual payoffs in a competitive market depend on their relative numbers. If there are equal numbers of productive workers and firms, all are matched one-to-one. Any division of the pair-wise output that gives all agents in each group the same payoff is a competitive equilibrium. Suppose, however, that there are more productive workers than firms. Now the supply of productive workers exceeds demand and their payoff is driven down to zero. Productive firms get all the return because they are the scarce factor. In the opposite case where there are more productive firms than workers, the workers receive 1 in the competitive equilibrium. In our simple example of two types of each group, uncertainty about the relative size of the two productive types translates into payoff risks before they enter the market. Avoiding this kind of payoff risks is a main motivation for individual agents to contract early and "unravel" the market.[2]

Discontinuities in equilibrium prices or payoffs with respect to supply and demand are inherent in the indivisibility of assignments (pair-wise here) and in the discreteness in the types. Indivisibility implies that payoffs are sensitive only to the sign of excess demand or supply, not to its magnitude. Whether productive workers outnumber productive firms by a few or by many does not change the equilibrium payoffs of zero for productive workers and 1 for productive firms. The discontinuities do not disappear as the size of the market gets large. They are more sensitive to the discreteness of types. For example, suppose the joint output between an unproductive agent and a productive agent of the other group is $\mu \in (0, 1/2)$, instead of 0. Then the consequences of being on the long or short side of the market are less severe. If productive workers outnumber productive firms but there are more firms than productive workers, so that each productive worker is matched to some firm, their equilibrium payoff is μ instead of zero because they add value μ to assignments with unproductive firms. Productive firms receive $1 - \mu$ instead of 1. In the limit of a continuum of types, the discontinuities disappear.

C. An example of unraveling

The essential aspects of market unraveling are illustrated by a two-period model with both individual uncertainty and aggregate uncertainty. Individual uncertainty arises because there is incomplete information in the first period by all agents on how productive a particular agent will be in the second period. Individual uncertainty creates randomness in supplies of different types of agents, and results in aggregate uncertainty due to discontinuities in equilibrium payoffs. Unraveling (first period contracting) eliminates aggregate uncertainty but causes *ex post* inefficient assignments due to individual uncertainty. Contracting after individual uncertainty is resolved ensures *ex post* efficient assignments, but exposes agents to payoff risks due to aggregate uncertainty.

These points are illustrated by extending the example. In the first period productivities of agents are unknown. All agents on each side of the market are identical with probability λ of becoming productive in the second period. All are risk-averse with utility function u. For simplicity, we assume that there are equal numbers of workers and firms.

The second period competitive equilibrium gives agents who turn out to be productive a payoff of 1 or 0 depending on excess demand or supply.[3] The uncertainty of second period payoffs may motivate agents to contract with each other in the first period, before their productivity is known. Imagine the following game. In the first period, there are pairwise meetings between workers and firms. Each pair of agents chooses whether or not to sign the following contract: the worker receives a payoff r and the firm receives $1 - r$ in the second period if both turn out to be productive; otherwise, both receive 0. The market is said to unravel if there exists some $r \in [0, 1]$ such that some pairs of agents choose to sign the first period contract rather than to wait. From the individual point of view, the first period contract eliminates risks that the two agents may be on the long or short side of the market, should they be productive. But the contracts create *ex post* assignment inefficiencies because information necessary to achieve efficient assignments is available only in the second period after the contracts have been signed. The cost of these inefficiencies to the two agents is reflected in the fact that the payoff to the two agents depends on the joint probability that both are productive.

When λ is sufficiently large the insurance gains from contracting early and eliminating aggregate uncertainty outweigh the loss due to *ex post* inefficient assignments. The market unravels. The argument runs as follows. In the second period, the payoffs to productive agents are 1 or 0 depending on excess demand or supply. The expected utility to each agent from waiting is $\lambda[u(1)/2 + u(0)/2] + (1 - \lambda)u(0)$. If two agents sign the first period contract the expected utility to the worker is $\lambda^2 u(r) + (1 - \lambda^2)u(0)$ and the expected utility to the firm is $\lambda^2 u(1 - r) + (1 - \lambda^2)u(0)$. Let r^w be the minimum payoff required for the worker to sign the contract and r^f be the maximum payoff that the firm is willing to pay the worker to sign the contract. Equating the expressions above we have that the ask price r^w and the bid price r^f are implicitly defined by the condition

$$(1) \qquad u(r^w) = u(1 - r^f) = u(0) + [u(1) - u(0)]/(2\lambda).$$

If λ is large enough then the ask price r^w falls below the bid price r^f and unraveling occurs. There is not enough structure in this example to determine the market equilibrium price uniquely, but it must be between the bid and ask. When λ is sufficiently small, $r^f < r^w$. The market does not unravel. The value of insurance from a first period contract is small because the probability of exposure to aggregate uncertainty in the second period is small. The loss due to *ex post* inefficient assignments dominates the insurance gains.

D. Unraveling and market incompleteness

Before we analyze richer assignment problems, it is important to understand that unraveling is really caused by market incompleteness. In the above example, the optimal insurance arrangement is for all agents to sign a first period contract in which they all receive the same share of maximum total output of all participants after waiting till the second period to achieve *ex post* efficient assignments. A decentralized Arrow-Debreu securities market in the first period together with a second period spot market achieves this.

Imagine that in the first period each agent sells 100 perfectly divisible shares of the claim to the agent's payoff from the second period spot market. For example, holding 50 shares of one claim entitles the buyer to 1/2 of the seller's equilibrium payoff from the spot market. A competitive equilibrium in the securities market is the set of share prices, such that given these prices, each agent chooses the numbers of shares to maximize expected utility subject to the budget constraint, and the market clears for shares of each claim. Since all agents are indistinguishable in the first period the competitive equilibrium has equal share prices for all claims and each agent holds an equal number of shares of each claim, including the agent's own claim. This equilibrium is *ex post* efficient, because the second period spot market produces the efficient assignments that maximize total output. It is also *ex ante* optimal, because each agent spreads risks as much as possible.

If markets were complete and agents could legally sell claims to their future payoffs, unraveling would not be observed. Unraveling or early contracting is a manifestation of market failure. For instance, it is never said to be observed in markets for agricultural commodities where futures contracts are common. Rather, it is restricted to those labor markets where indivisible assignments are important and where complete markets of the kind described above are vulnerable to moral hazard problems. These problems are especially severe in labor markets because *ex post* productivities of job applicants depend critically on the efforts they make after signing early contracts. Insurance provided by complete markets may lead to shirking and disinvestment in human capital by the applicants. In what follows we take it for granted that complete markets do not exist, so that unraveling may occur.

II. Unraveling without Reentry

In this section, we consider a richer version of the example above. There agents are indistinguishable in the first period and unraveling affects all market participants in the same way. Here we allow for two kinds of heterogeneity among participants. First, the two sides in the assignment market differ in the individual uncertainty they face. In markets for entry-level professionals, individual uncertainty about future productivity is a substantial issue only for job applicants. Firms operate each year and have established reputations. Second, there is heterogeneity among the workers. Some have more promise to become productive later than others. Unraveling in this situation can affect more promising workers differently than less promising ones. The competitive equilibrium in the first period market is characterized next, assuming that agents who sign first period contracts do not enter the spot market in the second period. The case of unraveling with buyouts and reentry is considered later.

A. The model

As before, in the second period workers are either productive or unproductive, and the joint output is 1 between a productive worker and a productive firm and 0 otherwise. In the first period, there are n types of workers, whose prospects differ. Type i ($i = 1, \ldots, n$) workers, of size n_i, become productive in the second period with probability λ_i. Type 1 workers are the most promising. Less promising types (greater i) have smaller λ_i. To capture the idea that firms face little uncertainty about themselves, we assume that all firms are identical and all are productive. All workers have the same concave utility function u over payoffs, and all firms have the same utility function v. Without loss of generality, assume that $u(0) = v(0) = 0$.

The equilibrium offers written in the first period contracts and the number of each type of agents who sign them are endogenous. For agents who choose to wait, the second period spot market is described as before: productive agents receive a payoff of 1 if they are on the short side of the market. All other agents receive zero. Since there is no reentry, the first period contract between a type i worker and a firm takes the following form: any type i worker who turns out to be productive receives r_i and the firm receives $1 - r_i$. Otherwise both receive zero. We refer to r_i as the "price" of the first period contract with type i workers.[4]

We proceed by defining bid and ask prices for the workers. Different types of workers may have different bid and ask prices. These prices depend on the number y_i of type i workers who sign first period contracts because aggregate uncertainty—the discontinuous payoffs in the second period spot market conditional on excess demand or supply—depends on y_i. For any $y_1, \ldots, y_n$, let $\pi(y_1, \ldots, y_n)$ be the probability that there are more firms in the second period spot market than productive workers. That is, $\pi(y_1, \ldots, y_n)$ is the probability that *ex post* productive workers who have not signed early contracts receive 1 in the second period spot market.[5] It represents aggregate uncertainty. To shorten notation, we write π instead of $\pi(y_1, \ldots, y_n)$ whenever the meaning is clear.

B. Bid and ask prices

Given $\pi \in [0, 1]$, type i workers are indifferent between signing a first period contract at price r_i and waiting for the second period spot market, if

$$(1 - \lambda_i)u(0) + \lambda_i u(r_i) = (1 - \lambda_i)u(0) + \lambda_i[\pi u(1) + (1 - \pi)u(0)]. \tag{2}$$

This simplifies to

$$u(r_i) = \pi u(1). \tag{3}$$

Therefore, the ask price of all types of workers is the same. Let $r^w(\pi)$ denote the common ask price as a function of π. In order for firms to be indifferent between signing a first period contract with a type i worker at price r_i and waiting for the second period spot market, we must have

$$\lambda_i v(1 - r_i) = (1 - \pi)v(1). \tag{4}$$

Let $r_i^f(\pi)$ denote the bid price for type i workers. For any types $i < j$, $r_i^f(\pi) \geq r_j^f(\pi)$ for all π, with equality if and only if $\pi = 1$.

Taking derivatives with respect to π, we have

$$\frac{dr^w(\pi)}{d\pi} = \frac{u(1)}{u'(r^w(\pi))}; \tag{5}$$

$$\frac{dr_i^f(\pi)}{d\pi} = \frac{v(1)}{\lambda_i v'(1 - r_i^f(\pi))}. \tag{6}$$

Thus, $r^w(\pi)$ is increasing and convex and $r_i^f(\pi)$ is increasing and concave. One can easily verify that $r^w(0) = 0$, $r_i^f(0) < 0$, and $r^w(1) = r_i^f(1) = 1$ for each i.

[Insert Figure 1 Here]

Figure 1 depicts the relative positions of the ask price r^w and bid prices r_i^f and r_j^f for $i < j$. In the diagram, r^w intersects r_i^f at $\pi_i \in (0, 1)$ and it intersects r_j^f at $\pi_j \in (\pi_i, 1)$. $r^w(\pi) > r_i^f(\pi) > r_j^f(\pi)$ for $\pi \in [0, \pi_i)$, $r_i^f(\pi) > r^w(\pi) > r_j^f(\pi)$ for $\pi \in (\pi_i, \pi_j)$, and $r_i^f(\pi) > r_j^f(\pi) > r^w(\pi)$ for $\pi \in (\pi_j, 1)$. Therefore, when $\pi > \pi_j$, insurance gains exist from first period contracts with both type i and type j workers. Between π_i and π_j insurance gains exist for type i workers but not for type j. Below π_i the ask price exceeds both bid prices so there is no gain from insurance for either type. That insurance gains exist only when π is high is due to the absence of individual uncertainty for firms. When π is very low, the ask price of both types of workers is close to zero, but because of the individual uncertainty of workers, the bid price for each type is negative: firms prefer to wait for the spot market rather than to sign first period contracts with either type of worker at any price. In this sense we can say that payoff risks to firms are the main source of insurance gains in our model.

For a given type i, the ask price function $r^w(\pi)$ may lie entirely above the bid price $r_i^f(\pi)$ in Figure 1 for all $\pi < 1$. Then type i workers never participate in unraveling. In this case, we define the threshold π_i to be 1. Whether π_i is equal to 1 or strictly less than 1 depends on the curvatures of r^w and r_i^f, and on the probability λ_i if u or v is concave. The curvatures of r^w and r^f in turn depend on risk-aversion. If both u and v are linear (risk-neutral), $dr^w/d\pi = 1$ and $dr_i^f/d\pi = 1/\lambda_i$: there are no gains from first period contracts regardless of λ_i. The more risk-averse the agents, the more likely gains from insurance. If u or v are concave, the bid price for type i workers is increasing in λ_i, and insurance gains are more likely to exist if λ_i is greater.

C. Ordering property and monotonicity property

It is useful to think of the first period market as an assignment market. If a worker and a firm sign a first period contract, they are said to be assigned to each other. If a worker or a firm chooses to wait for the second period market, the agent is said to be unassigned. In an assignment problem without uncertainty, the payoff to an unassigned worker is exogenous. Here it depends both on the type of the worker and on y_i, because these numbers affect aggregate uncertainty in the second period spot market. Furthermore, the numbers are endogenous variables, determined in the first period market equilibrium.

Unraveling is signing a first period contract. Type i workers unravel if the bid price exceeds the ask price. The market equilibrium price r_i lies between the bid and ask. Unraveling of type i workers is complete if all type i workers sign first period contracts: $y_i = n_i$. Unraveling of type i is incomplete if $y_i < n_i$. In this case all type i workers are indifferent between signing and waiting for the second period spot market. If firms sign first period contracts with more than one type of worker, contract prices must be such that firms are indifferent between all these types. If some firms sign while others wait, we say there is incomplete unraveling of firms, and firms are indifferent between signing and waiting. We assume that the total number of n types of workers exceeds the number of firms.[6]

Two properties of the model greatly simplify the characterization of market equilibrium. First, unraveling is ordered: unraveling of type i workers "precedes" that of type $j > i$ workers. This ordering property limits the number of corner solutions that must be considered. Second, the probability $\pi(y_1, \ldots, y_n)$ is monotonically decreasing in each of its arguments. This monotonicity property further simplifies characterization of equilibrium.

Consider two types of workers, i and $j > i$. The ask price for the two types is the same. Since type i has a greater probability of becoming productive than type j, the bid price for i is greater than that for j for any $\pi < 1$. Thus, unraveling of type j implies complete unraveling of type i. The reason is as follows. Since there is unraveling of type j, the price r_j paid by firms for contracts with type j is higher than the ask price $r^w(\pi)$. If there was incomplete unraveling for type i, the price r_i for the contract with type i would equal the ask price $r^w(\pi)$, implying $r_i \le r_j$. But then $\lambda_i v(1 - r_i) > \lambda_j v(1 - r_j)$ because $\lambda_i > \lambda_j$, and firms strictly prefer signing first period contracts with type i to signing with type j, contradicting the requirement that firms be indifferent between two types of workers if they sign with both.

The absence of individual uncertainty for firms implies that $\pi(y_1, \ldots, y_n)$ is a decreasing function in each of its arguments. If an additional pair of a type i worker and a firm sign a first period contract, the number of firms remaining in the second period spot market falls by one. Individual uncertainty for type i workers means that the expected number of productive workers participating in the second period market falls by less than one.

Therefore, the probability that productive workers will be short in the second period spot market decreases with y_i, the number of type i workers who sign first period contracts. The formal argument is straightforward and omitted.

D. Market equilibria

Comparisons between aggregate uncertainty π and the thresholds π_i determine the unraveling equilibrium in the first period market. Recall from Figure 1 that π_i is defined as the intersection between the bid and ask functions of type i workers, and that less promising types have a greater π_i. Gains from insurance for type i exist only if π exceeds π_i. Aggregate uncertainty depends on the number y_i of each type i workers who sign first period contracts and on the size n_i of type i. The ordering property implies that in any equilibrium there is a cutoff, least promising type i_* of workers that participate in unraveling. Recall that $\pi(0,\ldots,0)$ is the probability that productive workers are short in the second period spot market when no one signs a first period contract. To find i_*, we first compare $\pi(0,\ldots,0)$ to π_1. If $\pi(0,\ldots,0) \leq \pi_1$, set $i_* = 0$. Otherwise, compare $\pi(n_1,0,\ldots,0)$ to π_2. If $\pi(n_1,0,\ldots,0) \leq \pi_2$, set $i_* = 1$. Otherwise, compare $\pi(n_1,n_2,0,\ldots,0)$ to π_3. If $\pi(n_1,n_2,0,\ldots,0) \leq \pi_3$, set $i_* = 2$. Otherwise, compare $\pi(n_1,n_2,n_3,0,\ldots,0)$ to π_4. And so on. This process stops before we reach the $(n+1)$-th step, because the total number of workers exceeds the number of firms.

If the process stops in the first step and $i_* = 0$, there is no unraveling of any type. At and below the initial condition $\pi(0,\ldots,0) \leq \pi_1$, there is no insurance gain for type 1, hence no insurance gain for any other types. Unraveling of type 1 would only cause the aggregate uncertainty π to decrease further. If $i_* \geq 1$, by construction it satisfies

$$\pi(n_1,\ldots,n_{i_*-1},0,\ldots,0) > \pi_{i_*}; \tag{7}$$

$$\pi(n_1,\ldots,n_{i_*},0,\ldots,0) \leq \pi_{i_*+1}. \tag{8}$$

The first inequality implies positive insurance gain for type i_* workers after all more promising types have unraveled; the second shows that insurance gains for type i_*+1 do not exist after type i_* and all the more promising types have unraveled. By the ordering property, type i_* is the least promising type of workers who unravel.

The equilibrium prices and π depend on whether unraveling of the cutoff type i_* workers is complete or incomplete. If $\pi(n_1, \dots, n_{i_*}, 0, \dots, 0) \geq \pi_{i_*}$, unraveling of type i_* is complete. Insurance gains for type i_* workers still exist at the point where all of them have signed early contracts, but no more of them are available. The equilibrium π is $\pi(n_1, \dots, n_{i_*}, 0, \dots, 0)$. There is incomplete unraveling of firms, who are indifferent between signing first period contracts with types $i = 1, \dots, i_*$ and waiting for the second period spot market.[7] The equilibrium price of the first period contract for type i workers ($i = 1, \dots, i_*$) is $r_i = r_i^f(\pi(n_1, \dots, n_{i_*}, 0, \dots, 0))$, the bid price for type i evaluated at the equilibrium π. Workers who unravel receive all the rent from first period contracts.

If $\pi(n_1, \dots, n_{i_*}, 0, \dots, 0) < \pi_{i_*}$, unraveling of type i_* is incomplete. Unraveling continues until the equilibrium π is driven down to π_{i_*}.[8] All the insurance gains from first period contracting are exhausted at this point. Again, the equilibrium price r_i of the first period contract for type i workers ($i = 1, \dots, i_*$) is given by $r_i = r_i^f(\pi_{i_*})$, on the bid function for type i, and workers more promising than type i_* receive all the rent. Both type i_* workers and firms are indifferent between signing and waiting, but more promising types of workers strictly prefer signing to waiting.

E. Unraveling with Full Insurance

In the first period contracts considered above, payments received by applicants depend on their *ex post* productivity. This kind of contract has the advantage of preserving workers' incentives when *ex post* productivity of applicants depends critically on the efforts they make after signing early contracts. Firms do not fully insure applicants in this case, even if they are risk-neutral. Nonetheless, it is interesting to analyze first period contracts that provide full insurance. For example, if investments by applicants after first period contracts are not firm-specific, then the applicants pursue their own long term interests by making the investment. In these situations, risk-neutral firms may be willing to offer full insurance to applicants in early contracts.

When risk-neutral firms offer full insurance to workers, the first period contract with a type i ($i = 1, \dots, n$) worker specifies a transfer payment r_i to the worker regardless of his *ex post* productivity. For any given aggregate uncertainty $\pi \in [0,1]$, the ask price $r_i^w(\pi)$ of type i is the solution to

$$u(r) = \lambda_i \pi u(1). \tag{9}$$

The bid price for type i is given by

$$r_i^f(\pi) = \lambda_i - (1-\pi). \tag{10}$$

Ask and bid functions have the following properties: (i) both are increasing in π; (ii) ask price is zero and bid price is negative at $\pi = 0$, and bid price exceeds ask price at $\pi = 1$; and (iii) ask and bid functions have a unique intersection π_i.[9] See Figure 2.

[Insert Figure 2 Here]

The ask and bid functions can be compared to their counterparts in Figure 1 when firms are risk-neutral (v is linear) but first period contracts do not provide full insurance. We observe that there is more to gain when firms offer full insurance, even though both ask and bid functions shift down (with full insurance workers ask less and firms bid less). Without full insurance, gains from insurance may not exist for any π if λ_i is too small (ask exceeds bid for all $\pi < 1$), but insurance gains always exist with full insurance for any $\lambda_i < 1$ if π is great enough. See Figure 2. Moreover, for any λ_i, the critical aggregate uncertainty π_i, below which insurance gains do not exist, can be shown to be lower with full insurance than without full insurance.

As before, insurance gains are greater for more promising types: the critical aggregate uncertainty π_i is smaller for more promising types. Therefore, the ordering property is preserved under full insurance. Unraveling of any type i implies complete unraveling of all types more promising than i. To see this, note that unraveling for firms cannot be complete because there are no insurance gains for any type when aggregate uncertainty π equals 0. Thus, in equilibrium firms are indifferent between all types of workers with whom they sign early contracts and waiting for the second period market. This implies that equilibrium first period contract prices for these types are given by their bid prices

$r_i^f(\pi)$ evaluated at the equilibrium aggregate uncertainty. If some type j workers unravel, then $r_j^f(\pi) \geq r_j^w(\pi)$, implying $\pi \geq \pi_j$. Since $\pi_i < \pi_j$ for any type i more promising than type j, $\pi > \pi_i$, and so $r_i^f(\pi) > r_j^w(\pi)$. Therefore, unraveling of type i is complete.

Since the monotonicity property of aggregate uncertainty holds as before, we can find the equilibrium π, the cutoff type, and equilibrium prices in exactly the same way as in the previous section. The discussions about comparative statics analyzed next apply to both early contracts without full insurance and with full insurance.

F. Comparative statics

Using the characterization of unraveling equilibrium established above, we now address how changes in relative numbers of different kinds of agents, the average quality of workers, and the degree of heterogeneity affect the likelihood of unraveling of different types, equilibrium prices and *ex ante* welfare in terms of first period expected utility.

From the ordering and monotonicity properties, the necessary and sufficient condition for unraveling of type i is $\pi(n_1, \ldots, n_{i-1}, 0, \ldots, 0) > \pi_i$. In an assignment market without uncertainty, equilibrium payoffs are determined by relative numbers of different types of market participants up to a boundary condition that those remaining unassigned receive an exogenous payoff. Here, the "effective" numbers of market participants $\pi(n_1, \ldots, n_{i-1}, 0, \ldots, 0)$ (relative to π_i) replace the sizes n_i as the determinants of equilibrium. The thresholds π_i do not depend on relative numbers of market participants, but $\pi(n_1, \ldots, n_{i-1}, 0, \ldots, 0)$ decreases as the number n_i of any type i workers increases or as the number of firms decreases. Therefore, unraveling of any type i workers becomes less likely as the relative supply of the n types of workers increases. Furthermore, given the total number of workers, unraveling of more promising types is less likely the greater the proportion of more promising types of workers in the distribution of types, because a greater proportion of more promising types on balance decreases $\pi(n_1, \ldots, n_{i-1}, 0, \ldots, 0)$ for any small i.

The effects of numbers of market participants on equilibrium prices and on *ex ante* welfare of market participants depend on whether unraveling of the cutoff type i_* is complete or not. Relative numbers do not affect bid and ask functions, so if unraveling of type i_* is incomplete, changes in the numbers of different types of market participants on the margin have no effects on prices and hence no effects on the expected utility of any type of agents who enter early contracts. However, if unraveling of type i_* is complete, changes in the numbers of market participants can affect the equilibrium prices through $\pi(n_1, \ldots, n_{i_*}, 0, \ldots, 0)$. Increasing the number of any type i of workers and decreasing the number of firms have the same effect of reducing the equilibrium prices r_i for types $i \leq i_*$ because they reduce $\pi(n_1, \ldots, n_{i_*}, 0, \ldots, 0)$. The *ex ante* welfare of all types of workers who unravel is decreased while welfare of firms is increased. Types that in equilibrium do not unravel are also worse off, because the equilibrium π, the probability of *ex post* productive workers receiving 1 in the second period spot market, falls.

Across-the-board changes in the prospects λ_i of applicants have ambiguous effects on the likelihood of unraveling. With an increase in all λ_i, the bid function for each type i shifts up while the ask function stays the same, so π_i decreases and insurance gains increase for type i workers. See Figure 1 above. This tends to increase the likelihood of unraveling for type i workers. But an improvement in the prospects of all applicants also leads to a decrease in the probabilities $\pi(n_1, \ldots, n_{i-1}, 0, \ldots, 0)$ for all i. This tends to decrease the likelihood of unraveling for all types. Thus, changes in the likelihood of unraveling are ambiguous.

Similarly, changes in the degree of heterogeneity among workers affect both the average quality of workers available in the second period spot market through changes in $\pi(n_1, \ldots, n_{i-1}, 0, \ldots, 0)$, and the insurance gains for individual types through changes in π_i. A "compensated" change in heterogeneity is one that isolates the two effects. For example, suppose initially $\pi(0, \ldots, 0) < \pi_1$ so that there is no unraveling. Consider an increase in λ_1 and a decrease in λ_n that increases heterogeneity among workers but keeps $\pi(0, \ldots, 0)$ unchanged. This decreases π_1 and increases the insurance gains for type 1. If the changes are sufficiently large, type 1 unravels. Moreover, $\pi(n_1, \ldots, n_{i-1}, 0, \ldots, 0)$ increases for all i. Therefore, the likelihood of unraveling for all types (except perhaps for type n) increases as a result of the increase in heterogeneity.

We conclude with the welfare effects of banning first period contracting. Workers who do not unravel in equilibrium benefit from such a ban, because under the ban the probability that *ex post* productive workers receive 1 equals $\pi(0,\ldots,0)$, which is greater than the equilibrium probability $\pi(n_1,\ldots,n_{i_*},0,\ldots,0)$ or π_{i_*}. For the same reason, the ban hurts firms unambiguously. The welfare effects of the ban on workers who unravel can be ambiguous. These workers receive rents from signing first period contracts rather than waiting. Whether they are made better off or worse off by the ban depends on the rent they receive and the impact the ban has on the difference between the equilibrium value of π and $\pi(0,\ldots,0)$. More promising types are more likely to be hurt by the ban because they receive bigger rents.

III. Unraveling with Reentry

The contracts analyzed above are inefficient in both the first period and the second period. Pair-wise first period contracts cannot replicate the gains from risk sharing implied by complete contingent contracts. In the second period, *ex post* inefficiencies are caused by the restriction on recontracting for those *ex post* productive agents whose first period contracts proved unsuccessful because their partners turned out to be unproductive. In this section, *ex post* productive agents are allowed to enter the second period spot market. This amounts to allowing firms to "buy out" of their *ex post* unsuccessful first period contracts.[10] Inefficient risk-sharing remains in the first period, but *ex post* inefficiencies are eliminated because all *ex post* unsuccessful first period assignments are dissolved.

A. Buyout provisions

We need to analyze the terms of buyouts. Consider what happens when a type i worker ($i = 1,\ldots,n$) in a first period contract with a firm turns out to be productive. If the spot market happens to be long on productive workers and they receive a spot payoff zero, the firm must pay the first period contract price of r_i to the type i worker anyway, so there is no advantage to buying out. Similarly, if the spot price for productive workers is 1, there is no advantage to buying out for the worker. Thus, the terms set by the initial contract cannot be advantageously renegotiated if the early contract turns out to be successful. Next, consider what happens when a first period contract is successful. If the spot price for productive workers is 1, the firm and the worker gain nothing from buying out. Competition among firms implies that some of them will buy out of their contracts paying their *ex post* unproductive partners 0, and rematch to the *ex post* productive workers who have not signed first period contracts paying them 1. But for the worker and the firm who have signed a first period contract, it is as if the buyout had not taken place. Therefore, for these agents, meaningful buyouts occur only when the contract is unsuccessful and the spot price of productive workers is 0.

Because agents participating in a first period contract anticipate the possibility of buyout, renegotiation amounts to an up-front contingency clause that specifies the terms and the market circumstances of a buyout. The firm pays the *ex post* unproductive type i worker r' to buy out of the contract when productive workers are available in the second period spot market at the price of 0. Since the joint gain from buyout is 1, the two agents in the first period contract are in the same position as they would have been had the worker been productive. An optimal first period contract should share risks in the same way. Therefore, $r' = r_i$.[11]

B. Market equilibria

The analysis of equilibrium unraveling with reentry proceeds as before. In fact, the analysis becomes easier. With buyouts all *ex post* inefficient assignments are eliminated and the key probability π that productive workers are short in the second period market is independent of the number of workers y_i who sign first period contracts. Thus π is no longer endogenous.

As in the previous section, we first characterize the bid and ask functions. For any fixed $\pi \in [0,1]$ and $i = 1, \ldots, n$, let $r_i^w(\pi)$ be the solution to the equation

$$[1-(1-\lambda_i)\pi]u(r) = \lambda_i \pi u(1), \tag{11}$$

and let $r_i^f(\pi)$ be the solution to the equation

$$[1-(1-\lambda_i)\pi]v(1-r) = (1-\pi)v(1). \tag{12}$$

With reentry, each type of workers has distinct ask and bid functions. It is straightforward to establish that for each type i of worker, (i) ask and bid price functions are increasing in π; (ii) ask and bid prices equal zero at $\pi = 0$ and equal 1 at $\pi = 1$; and (iii) bid price is strictly greater than ask price for all π between 0 and 1. See Figure 3. Unlike the previous section, insurance gains exist for all values of π for all types of workers.

[Insert Figure 3 Here]

Without reentry, ask prices are the same for all types of workers and bid price is greater for more promising types. The value of insurance is greater for more promising types and unraveling of workers is ordered by type. Here, both the bid and ask prices are greater for type i workers than for type $j > i$ at each value of π. It can be shown that although type j workers are willing to accept a lower price for first period contracts, the fact that $\lambda_i > \lambda_j$ still implies that at any π firms prefer to sign with type i workers at their ask price $r_i^w(\pi)$ rather than with type j at $r_j^w(\pi)$. With this result, it can also be shown that any unraveling of type j workers implies complete unraveling of type i.[12] The order is preserved when buyout and reentry are permitted.

With reentry of firms whose first period contracts proved unsuccessful, the equilibrium probability that *ex post* productive workers who have not signed first period contracts receive a payoff 1 in the second period spot market, equals $\pi(0, \ldots, 0)$, just as if no agents had signed first period contracts. Finding the first period market equilibrium can be thought of as a pure assignment problem because aggregate uncertainty π is no longer endogenous. Since the ordering property is preserved, the least promising type of workers who participate in unraveling is type i_* such that the total number of firms is between $\sum_{i=1}^{i_*-1} n_i$ and $\sum_{i=1}^{i_*} n_i$. There is incomplete unraveling of the cutoff type i_*, and the equilibrium price r_{i_*} of first period contracts with the cutoff type workers equals their ask price $r_{i_*}^w(\pi(0, \ldots, 0))$. The equilibrium price r_i of contracts with any type i more promising than type i_* is determined by the indifference condition of firms between type i and type i_*:

$$(13) \quad [1 - (1 - \lambda_i)\pi(0, \ldots, 0)]v(1 - r_i) = [1 - (1 - \lambda_{i_*})\pi(0, \ldots, 0)]v(1 - r_{i_*}^w(\pi(0, \ldots, 0)).$$

Since at any π firms prefer to sign with type i workers at their ask price $r_i^w(\pi)$ rather than with the cutoff type i_* at $r_{i_*}^w(\pi)$, the equilibrium price r_i for type i workers is greater than their ask price $r_i^w(\pi(0,\ldots,0))$. Moreover, since bid price exceeds ask price for type i_* at any π, and since firms are indifferent between the cutoff type i_* and any more promising type i, the equilibrium price r_i of type i workers is below their bid price $r_i^f(\pi(0,\ldots,0))$. Thus, the rent from first period contracts is shared between workers and firms who sign them.

C. Welfare comparisons

Unraveling with recontracting provides some insurance in the first period without sacrificing the *ex post* efficiency of assignments. Therefore, a ban on first period contracting with reentry hurts market participants as a whole. In particular, firms and workers that are more promising than the cutoff type i_* are strictly worse off under the ban. Welfare of those less promising than the cutoff type is unaffected.

It is also of interest to compare welfare with and without reentry. Reentry shifts bid price functions up because firms are willing to pay more for first period contracts with any type i of worker at any π. It shifts ask price functions down because type i workers are willing to accept less to sign first period contracts. More workers and firms sign first period contracts when reentry is allowed. In general, buyouts must improve welfare, but there is a question of how the gains are distributed among various types of agents. It is possible that some agents become worse off with reentry.

Reentry unambiguously improves welfare of those workers who choose to wait in the equilibrium without reentry. With reentry, the more promising of these workers sign with firms in the first period, and they prefer signing to waiting at $\pi = \pi(0,\ldots,0)$. The rest still wait for the second period spot market. Both these two groups are better off, because with reentry $\pi(0,\ldots,0)$ is greater than the equilibrium π without reentry, and so waiting yields a greater expected utility when reentry is allowed. Welfare comparisons for firms and the types of workers who unravel in the equilibrium without reentry are more complicated. Whether allowing reentry makes firms better off depends on the rent they receive from first period contracts with reentry. Without reentry, firms are indifferent between signing and waiting at an equilibrium π that is smaller than $\pi(0,\ldots,0)$. With reentry, firms prefer signing to waiting at $\pi(0,\ldots,0)$. Allowing reentry makes firms worse off if the equilibrium

π without reentry is sufficiently below $\pi(0, \ldots, 0)$ that firms are able to sign first period contracts at very low prices. This occurs when workers are so risk-averse for low values of π that they will accept unfavorable terms in first period contracts without reentry. Similarly, the welfare effects of allowing reentry on the types of workers who participate in unraveling without reentry depend on how reentry changes the rent they receive from first period contracts.

IV. Concluding Remarks

In labor markets for young professionals and other situations where information is imperfect but matching people to positions is important, uncertainty produces anxiety over how participants will make out and to whom they will be matched. Every young person in the marriage market has worried about whether there exists a mate out there meant just for them. Applicants for jobs and for admission to schools are concerned that their preferred firm or school might not want them, and that there may be keen competition for positions. They fear being passed over and not "getting in."

The problem is not confined to applicants. Firms worry about filling available positions. Colleges that fail to attract their targeted freshmen class size leave serious money on the table: the marginal cost of students below capacity is much smaller than average cost per student. Hospitals that fail to find qualified interns and residents, and law firms unsuccessful in recruiting promising associates face higher costs because other people's time must be diverted to these tasks. Firms behave as if they were risk averse under such circumstances.

The model developed here is the first to analyze the competitive market and pricing aspects of these kinds of problems. Initial uncertainty about each candidate's future productivity is resolved only with time. Everyone, both workers and firms alike, share common knowledge of these prospects along the way. The tendency toward early contracting—unraveling—noted by Roth and others is a manifestation of attempts to eliminate these kinds of anxieties when, for a variety of reasons, insurance is incomplete. Early contracts provide limited guarantees of positions to parties who accept them. They partially insure workers against being shut out of the market should they prove productive but too many qualified applicants become available later. They partially insure firms against the risks of being shut out of the *ex post* spot market should qualified persons be in short supply. But there are costs of early contracts. They are made with imperfect information. Some of them are bound to be inefficient *ex post*. Nonetheless, the *ex ante* benefits from insurance to participants exceed the *ex post* inefficient matching costs when unraveling occurs. Unraveling is Pareto optimal for those who participate when market-wide insurance is impossible to arrange. In this sense, like all externalities, it is a manifestation or a "missing" market.

A main result of our analysis is that incentives for early contracts are ordered on the productivity potential of applicants. If observed at all, unraveling occurs among candidates who appear to be most promising a priori, before full information is revealed. This result contrasts with models based on private information, where the ordering goes in the opposite direction. For instance, Bergstrom and Bagnoli (1993) show that in marriage markets where participants possess private information about their prospects, more promising candidates marry later. Time signals their private information to others. Less desirable persons don't benefit from waiting and contract early.

There are virtues in both formulations, depending on the application. In real marriage markets, waiting and showing proof of one's superior qualities tends to attract superior mates. Isn't this the traditional economic reason why husbands tend to be older than wives in most societies? But in those labor and other markets where unraveling is observed, superior prospects typically are observed to sign early. This is implied by our model. Less attractive candidates are the ones who wait. In these situations, credentials, test scores, grades in school, and observations of teachers and more experienced hands are common knowledge. The data are rather more impersonal and more easily communicated directly to outsiders than are traits relevant to familial marriage. They are not the exclusive personal property of the candidates themselves. In fact there are serious questions of whether candidates can even form accurate and unbiased estimates of themselves in these situations (e.g., see Rosen, 1986).

Unraveling is not a common feature of entry level labor markets. The imperfect information aspects of matching that provoke it are not nearly so important as other features of employment in many labor market settings. Yet unraveling seems more prevalent today than in an earlier era, when information, standardized tests, and other credentials were far less routinized. Limitations on the technology of assessing prospects' potential productivity based on common criteria make it more costly to order prospects among participants. When there are more differences of opinion, fears of being shut out *ex post* are of less concern. Standardized tests and other improvements in the market's ability to classify applicants' future potential have encouraged the rush toward early contracts in many situations and likely will continue to do so.

Another important implication of our model is that unraveling does not always occur in a competitive matching market. For unraveling to arise, participants must anticipate that the most productive workers are likely to be on the short side of the spot market should everyone else choose to wait. This feature also accords with observation.

Unraveling in the medical interns and residents market first occurred after World War II, when there was an extreme shortage of qualified doctors and medical schools and hospitals were making great efforts to improve their quality. Early competition for clerks among American judges is a very recent phenomena unquestionably associated with the huge increase in demand for judicial litigation and increased court case loads. These demands impose much greater burdens on judges today. Judges caught short or without capable clerks have much more work to do, without a compensating increase in pay, than those who manage to attract talented aides. Similarly, early contracts via summer-intern network connections for young law associates are also a recent phenomenon. It has been caused by the rapid growth of large, general purpose law firms, in conjunction with the tendency for the size and importance of legal claims to be concentrated on the better firms (Spurr, 1987), and the high concentration of student credentials among a relatively small number of top law schools. Finally, the tendency for young basketball players to skip college or leave early for the pros has been associated with a general increase in demand for the game that increased the premium for outstanding talents and the inability or unwillingness of "amateur" teams to compete in the marketplace. Each of these examples lends credence and broad empirical support to our way of looking at the problem.

Given that unraveling is caused by a kind of market failure, it is easy for us as economists to think of improvements. One obvious alternative is for firms to buy out of early contracts that prove unsuccessful *ex post*. Provisions of this kind, however, are seldom observed. Perhaps additional moral hazard considerations are involved. It may be difficult for parties to agree on enforceable tests of what constitutes success. The abolition of mandatory retirement in the U.S. has made such problems familiar to academics. Or it may be largely distributional elements that suppress it, similar to the group interests that support many inefficient economic policies. The distributional consequences of shutting down the recontract market in our model are less than compelling on that score.

In fact the phenomena of early contracting and unraveling often provoke efforts to prohibit it, not unlike disarmament treaties. The medical interns and residents matching program that first motivated economic thinking on these matters is such a case. It arose out of the concerted attempts by hospitals to eliminate early contracting and the mismatching that resulted from it. In our model unraveling, whenever it occurs, unambiguously benefits firms *ex ante* and harms less promising candidates. Moreover, no compelling case can be made that it harms the better applicants —it is likely to help many of them—so this kind of "collusive" thinking is hard to apply in this context.

Before conceding much on this point, we hasten to point that our analysis is limited to price mechanisms. An important feature of the interns and residents matching program is that it greatly limits the role of prices. While wages and working conditions vary substantially from hospital to hospital, wages and other terms are posted in advance of the match and applied uniformly to all who are accepted. An analogy would be a situation where each graduate department posted identical fellowship terms that applied to all applicants who are admitted and accepted the offer. The point is that prices are not allowed to clear the assignment market at the actual contract point. Similar restrictions have been observed in allegedly collusive agreements among colleges to limit price cutting in the form of scholarships for freshmen enrollments. While they are outside the scope of our market-clearing model, it certainly seems possible that restrictions on price competition would be preferred by firms.

Finally, aggregate uncertainty about the value of productive characteristics plays a crucial role in our model. There is an important sense in which it causes uncertainty about the availability of positions that motivates the rush to sign early. We believe that these considerations are important for understanding assignment markets for discontinuous types of agents. However, it can be shown (Li and Suen, 1997) that aggregate uncertainty is not necessary for unraveling to occur. Individual uncertainty alone can produce situations where the gain from insurance in early contracts outweighs the loss of *ex post* efficiency from mismatches.

References

Bergstrom, Theodore and Bagnoli, Mark. "Courtship as a Waiting Game." *Journal of Political Economy*, February 1993, *101*(1), pp. 185-200.

Crawford, Vincent."Comparative Statics in Matching Markets." *Journal of Economic Theory*, August 1991, *54*(2), pp. 389-400.

Gale, David, and Shapley, Lloyd. "College Admissions and the Stability of Marriage." *American Mathematical Monthly*, January 1962, *69*(1), pp. 9-15.

Koopmans, Tjalling and Beckmann, Martin."Assignment Problems and the Location of Economic Activities." *Econometrica*, January 1953, *25*(1), pp. 53-76.

Li, Hao and Suen, Wing. "Risk-sharing, Sorting, and Early Contracting." Working Paper, University of Hong Kong, 1997.

Mongell, Susan and Roth, Alvin. "Sorority Rush as a Two-sided Matching Mechanism." *American Economic Review*, June 1991, *81*(3), pp. 441-464.

Rosen, Sherwin. "Prizes and Incentives in Elimination Tournaments." *American Economic Review*, September 1986, *76*(4), pp. 701-715.

Roth, Alvin."The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory." *Journal of Political Economy*, December 1984, *92*(6), pp. 991-1016.

Roth, Alvin."A Natural Experiment in the Organization of Entry-level Labor Markets: Regional Markets for New Physicians and Surgeons in the United Kingdom." *American Economic Review*, June 1984, *81*(3), pp. 415-440.

Roth, Alvin and Xing, Xiaolin."Jumping the Gun: Imperfections and Institutions Related to the Timing of Market Transactions." *American Economic Review*, September 1994, *84*(4), pp. 992-1044.

Shapley, Lloyd and Shubik, Martin."The Assignment Game I: The Core." *International Journal of Game Theory*, 1971, *1*(1), pp. 111-130.

Spurr, Stephen. "How the Market Solves an Assignment Problem: The Matching of Lawyers with Legal Claims." *Journal of Labor Economics*, October 1987, *5*(4), pp. 502-532.

Footnotes

1. For a discussion of modeling issues in the two problems, see Crawford [1991].

2. Roth and Xing [1994] identify two other causes of unraveling. One is instability: if the assignment in the market is unstable, then there may be incentives for agents to reach agreements early. However, this cannot happen in a competitive assignment market. They identify market competition as another cause of unraveling and give an example where despite the stability of assignment in the market, some agents have incentives to make early offers to their next best candidates by committing to not competing for these agents in the market in case they reject the offers.

3. The equilibrium payoffs are indeterminate if there are equal numbers of productive agents. In this case, we assume that the equilibrium payoff to all productive workers is 0 or 1 with probability 1/2. This assumption is non-consequential for what follows.

4. In general an optimal first period insurance contract requires non-zero transfer between the two parties when the worker turns out to be unproductive. Our assumption that firms do not pay *ex post* unproductive workers can be justified on the ground that the worker's *ex post* productivity depends on some effort he makes between the two periods. It also makes it easy to characterize the first period contracts. Section E relaxes this assumption.

5. For simplicity, indeterminacy of the spot market equilibrium is resolved by our assumption that when there are equal numbers of productive workers and firms, the equilibrium is the same as when there are fewer productive workers than firms.

6. If there are more firms than workers of all types, there is no insurance gain from early contracts because $\pi = 1$ and workers will definitely be short in the spot market. (Insurance gains still exist if firms offer full insurance to workers; see section E.) However, it is misleading to say that excess supply of workers is critical for unraveling to occur. What is more important in the early contracting market is the expected supply of productive workers. Indeed, Figure 1 shows that expected shortage of productive workers in the sense of relatively big π, is necessary for unraveling.

7. Unraveling for firms cannot be complete because there are no insurance gains when aggregate uncertainty π equals 0.

8. We ignore the integer problem and assume that y_i's can take real values. More rigorously we can assume agents use mixed strategies in the sense that the probability of signing a first period contract can be any number between 0 and 1.

9. We use the same notation to ease the burden.

10. The reason why firms buy out unsuccessful contracts here is that they have no individual uncertainty: *ex post* productive workers who sign first period contracts are always efficiently assigned to productive firms. Buyout will be two-sided if there is individual uncertainty for firms as well as for workers. See Li and Suen (1997). Also, note that when firms are risk-neutral and offer full insurance to workers, first period contracts are necessarily binding because there is no room for renegotiation.

11. When the cost of reentry is a number $k \in (0, 1)$, instead of 0 in this section and 1 in the previous section, equilibrium first period contracts specify the division of the output 1 when the type i worker turns out to be productive and of the output $1 - k$ when he is not productive but productive workers are on the long side in the second period market. The payoff risks in these two contingencies must be optimally shared between the two parties. The analysis of unraveling with costly reentry is similar to that in this section. The only change is that insurance gains may not exist when π is small because reentry is likely and the cost of reentry reduces the value of first period insurance contracts.

12. The argument is as follows. Unraveling of type j workers implies that there is a price $r_j \geq r_j^w(\pi)$. Since firms prefer first period contracts with type i workers at $r_i^w(\pi)$ to contracts with type j at $r_j^w(\pi)$, there exists a price $r_i > r_i^w(\pi)$ such that firms prefer contracting with type i to contracting with type j. Since type i workers also prefer first period contracts at such r_i to waiting, there is unraveling of type i if there unraveling for type j. Moreover, unraveling of type i is complete, because incomplete unraveling would imply that $r_i = r_i^w(\pi)$ and firms are indifferent between type i at $r_i^w(\pi)$ and type j at some $r_j \geq r_j^w(\pi)$, which is impossible.

Figure 1

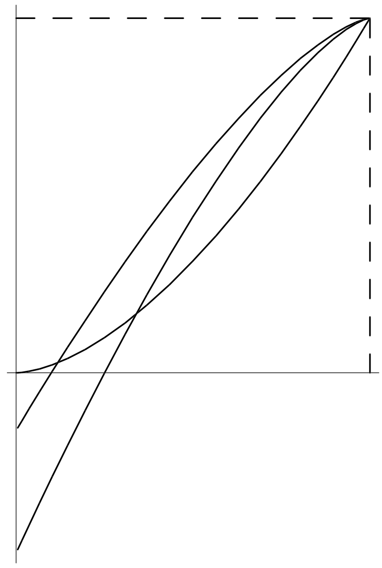

Figure 2

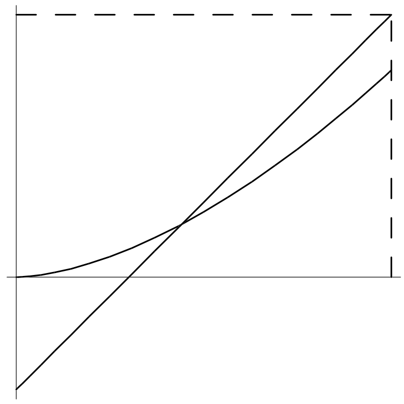

Figure 3