# Optimal Delay in Committees*

Ettore Damiano
University of Toronto

Li, Hao
University of British Columbia

Wing Suen
University of Hong Kong

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*Abstract.* Delay after disagreement in collective decision making may foster information aggregation but is costly ex post. Limited commitment entails an upper bound on the expected delay cost that can credibly be imposed. We consider the design of dynamic delay mechanisms, and show that an optimal mechanism does not impose a constant delay that reaches the upper bound throughout successive rounds of the decision making process. Instead, it induces in equilibrium start-and-stop cycles where players alternate between making the maximum concession to avoid disagreement and making no concession at all. The start-and-stop feature is robust to modeling delay cost by discounting instead of money-burning, and the optimal mechanism is shown to be "redesign-proof" when the committee can only commit to a small number of rounds of delay.

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## 1. Introduction

The committee problem is a prime example of strategic information aggregation. The committee decision is public, affecting the payoff of each committee member; the information for the decision is dispersed in the committee and is private to committee members; and committee members have conflicting interests in some states and common interests in others.[1] As a mechanism design problem, the absence of side transfers in the committee problem means that costly delay naturally emerges as a tool to provide incentives to elicit private information from committee members, much as in the implementation literature where a universally bad outcome or a sufficiently large penalty can be useful toward implementing desirable social choice rules when agents have complete information about one another's preferences (Moore and Repullo 1990; Dutta and Sen 1991). However, it may not be credible to impose a lengthy delay which is costly ex post, unless there is strong commitment power.

This paper takes a limited commitment approach to mechanism design in committee problems.[2] Specifically we assume that the committee can commit to self-imposing a delay penalty and not renegotiating it away immediately upon a disagreement, but there is an upper bound on the expected length of delay that the committee can commit to. That is, the committee can commit to "wasting" every member a small amount of value or time, but not too much, and the upper bound on the expected delay reflects the commitment power. A sufficiently tight bound on delay implies that the Pareto efficient decision cannot be reached immediately, and that delay will occur in equilibrium. In this case, because private information is not fully revealed, there is room for further information aggregation after committee members have incurred the delay cost. This gives rise to dynamic delay mechanisms in which committee members can make the collective decision in a number of rounds, punctuated by a sequence of delays between successive rounds, and with each delay uniformly bounded from above due to the limited commitment problem. We ask: Should there be a sequence of maximum delays between successive rounds? Does delay work better if it is front-loaded or back-loaded? Do deadlines for agreements arise endogenously as an optimal arrangement?

We present the underlying committee decision problem in this paper in the context of a symmetric two-member recruiting committee problem for an academic department. The two members have different research fields, and must choose between two candi-

[1] See Li, Rosen and Suen (2001) for an example and Li and Suen (2009) for a literature review.

[2] See Bester and Strausz (2001), Skreta (2006), and Kolotilin, Li and Li (2013) for other models of limited commitment.

dates, each specializing in one of the two research fields. In a "common-interest" state, with one high quality candidate and one low quality candidate, both committee members prefer the high quality one, regardless of their own research fields or the candidates'. In a "conflict" state, with both candidates having the same high quality or both low quality, each committee member naturally prefers the candidate of his own research field. Complicating the recruiting decision, however, is that due to their own research expertise, each member can ascertain the quality of the candidate of his own field (his "own" candidate), but not that of the "other" candidate. In particular, a committee member whose own candidate is of low quality will not go along with the other candidate unless he is optimistic that the latter has high quality. Indeed, if his belief about the quality of the other candidate is sufficiently pessimistic, there is no incentive compatible mechanism that Pareto-dominates choosing between the candidates by a coin flip. This is a stark illustration of the difficulties of efficient information aggregation—the members would have agreed to make the same choice in a common interest state had they been able to share their information. Although highly stylized, our model of recruiting committees thus captures the difficulties of reaching a mutually preferred collective decision when preference-driven disagreement is confounded with information-driven disagreement.[3]

Introducing a collective punishment for disagreements may improve the committee decision, when otherwise the best is a random decision. In the absence of side transfers, naturally this punishment is delay in making the decision, modeled as an additive disutility to each committee member that is proportional to its magnitude or length. The threat of delay motivates a committee member whose own candidate has a low quality to "concede" even though he is not optimistic about the quality of the other candidate. Understanding how to use delay to improve the committee decision requires us to impose more structure on both the preferences and information of the two types of committee members, the "high type" who knows his own candidate has high quality, and the "low type" who has a low-quality candidate. We assume that the high type has a stronger relative preference for his own candidate (a greater payoff difference) than the low type for the same belief about the quality of the other candidate, and that the high type is initially less optimistic about the quality of the other candidate (a higher initial belief that the other candidate's quality is low) than the low type. These two assumptions together constitute a single crossing property in our model, ensuring that the high type has a stronger incentive to "persist" with his own candidate than the low type does in any delay mechanism.

[3]Such difficulties are also present when competing firms choose to adopt a common industry standard, or when separated spouses decide on child custody.

In Section 2 we formally model a "delay mechanism" as an extensive-form game of potentially infinitely many rounds of simultaneous voting for one of the two candidates. In each round the decision is made according to the agreement if both members vote for the same candidate or through a coin flip if both concede, or postponed to the next round after delay if both persist. There may or may not be a terminal round, and if there is one, the decision is made through a coin flip after the last delay. The objective of this paper is to provide the complete characterization of the "optimal" delay mechanism, which maximizes the ex ante symmetric payoff of the two members (before they know their type), subject to a limited commitment constraint. We take a reduced-form approach to modeling this constraint and assume that delay in each round of the mechanism must not exceed an exogenous upper bound.

The dynamic game induced by a delay mechanism resembles a war of attrition with incomplete information and interdependent values.[4] Thanks to the single crossing property, in any equilibrium of this game, the high type always persists with his own candidate, so long as the low type weakly prefers persisting to conceding. As the game continues with the low type randomizing between persisting and conceding, both the low type and the high type become increasingly optimistic about the quality of the other candidate, while the latter stays less optimistic than the former. This characterization means that finding an equilibrium involves jointly solving for the sequences of actions, beliefs and payoff values of the low type only. For an arbitrary sequence of delays, however, such an approach is not manageable and does not yield any particular insights. In this paper, we introduce a "localized variations method" to study the design of an optimal delay mechanism. Consider changing the delay at some round $t$. We study its effect by adjusting the delay in round $t-1$, through the introduction an extra round if necessary, in such a way that keeps the equilibrium payoff of the low type for round $t-1$ fixed, and simultaneously adjusting the delay in round $t+1$, also through the introduction of an extra round if necessary, in such a way that keeps the continuation payoff of the low type after round $t+1$ constant. Since the effects of these variations are confined to a narrow window, there is no need to compute the entire sequences of actions, beliefs, and payoffs. It turns out that just by employing this localized variations method, we can arrive at an essentially complete characterization of optimal delay mechanisms.

The main result of this paper is a characterization of all delay mechanisms that have a symmetric perfect Bayesian equilibrium with the maximum ex ante expected payoff

[4]See also Hendricks, Weiss and Wilson (1988), Cramton (1992), Abreu and Gul (2000), and Deneckere and Liang (2006).

to each member. Such optimal delay mechanisms have interesting properties that we highlight in Section 3.2 and establish separately in Section 4. First, any optimal delay mechanism is effectively finite. If the low types make some concession in an infinite number of rounds, the gain from information aggregation would be too small relative to the delay cost after sufficiently many rounds of concession. If the low types exit (by conceding with probability one) and the high types remain for infinitely many rounds, the high types would be playing a pure war of attrition, which is worse than a coin flip. Second, we show that an equilibrium of an optimal delay mechanism necessarily induces the low type to concede with probability one at the deadline. This implies that the decision reached by the committee is always Pareto efficient in equilibrium as the high type always persists with his candidate with probability one. Moreover, in any optimal mechanism in which the low type makes concessions in at least two rounds, he enters the deadline round with a belief that is just sufficient to induce him to concede with probability one. In other words, it is not optimal to have the low type make more concessions than is necessary for reaching the Pareto efficient decision at the deadline. Third, we show that an optimal delay mechanism induces a "start-and-stop" pattern of making concessions by the low type. At the first round, the low type starts by adopting a mixed strategy with the maximum feasible probability of conceding to his fellow member; thus, the incentives for concessions are front-loaded. If the committee fails to reach an agreement, the low type would make no concession in the next round or next few rounds. After one or more rounds of no concession, the low type starts making the maximum feasible concession again, and would stop making any concession for one or more rounds upon failure to reach an agreement. Thus the equilibrium play under the optimal delay mechanism alternates between maximum concession and no concession.[5] To achieve this "start-and-stop" pattern of equilibrium play, the length of delay between successive rounds cannot be constant throughout. Instead, the expected delay cost is equal to the limited commitment bound in rounds when members are making maximum concessions, and is strictly lower than the bound in rounds when they are not making concessions.

In an earlier paper (Damiano, Li and Suen 2012), we show that introducing delay in committee decision-making can result in efficient information aggregation and ex ante welfare gain among committee members. That paper adopts a continuous time model with a simpler payoff structure and only one conflict state (there is no state with two

[5] A round of no concession following each round of maximal concession may be interpreted as temporary "cooling off" in a negotiation process. For negotiation practitioners, such cooling off is often seen as necessary to keep disruptive emotions in check and avoid break-downs, and sometimes as a useful negotiation tactic (see, for example, Adler, Rosen and Silverstein, 1998) . Our characterization of the start-and-stop feature of optimal delay mechanism provides an alternative explanation.

high types). We show in Section 5.3 that as the uniform upper bound on delay goes to zero, optimal delay mechanisms characterized here converge to the optimal deadline in the continuous-delay model of Damiano, Li and Suen (2012). While our previous paper explicitly solves a class of war of attrition games and performs comparative statics of the ex ante welfare with respect to the deadline, the present paper achieves a more ambitious goal of characterizing the optimal dynamic mechanisms.

The model of costly delay adopted in this paper is akin to "money-burning." The analytical advantage of the model is that the cost of postponing a decision by a fixed length of time is independent of the decision and the underlying state. Section 6.1 briefly addresses the robustness issue of modeling costly delay through discounting. The assumption of limited commitment is translated into a lower bound on the discount factor between two consecutive rounds. With additional assumptions on the preferences, we are able to show that the start-and-stop dynamics of optimal delay mechanisms remains intact under discounting.

Our analysis implicitly assumes that a mechanism can commit to an arbitrarily long sequence of delays, as long as each delay in this sequence is within the upper bound on expected delay. Section 6.2 explores the implications of mechanisms which can credibly commit to delay sequences of up to $\Omega$ rounds (where $\Omega$ is fixed and the delay at each round is still required to be within the cap). We introduce the concept of "$\Omega$-redesign proof" mechanisms. Although an optimal mechanism we identify in our main result can involve $T$ rounds, we show that the equilibrium ex ante payoff can be achieved by a redesign proof mechanism as long as the length of delay sequences that can be committed to is at least $\Omega$, where $\Omega$ can be considerably smaller than $T$. This result follows because inducing the start-and-stop cycles of an optimal dynamic mechanism requires only the ability to commit to delay sequences of length 2.

## 2. Model

### 2.1. A simple committee problem

Consider the following symmetric joint decision problem. There are two members, and two alternatives. Each member has a different "favorite" alternative.

Each member is initially either a low type, $\theta = L$, or a high type, $\theta = H$. The type information is private and unverifiable. Denote the initial belief that his opponent is low type as $\gamma_1$ for a low type member, and as $\mu_1$ for a high type member.

**Assumption 1.** (NEGATIVE CORRELATION) $\gamma_1 < \mu_1$.

The implied common prior beliefs for each of the four possible states are given by the following table:

| | $L$ | $H$ |
|---|---|---|
| $L$ | $\frac{\gamma_1\mu_1}{1-\gamma_1+\mu_1}$ | $\frac{(1-\gamma_1)\mu_1}{1-\gamma_1+\mu_1}$ |
| $H$ | $\frac{(1-\gamma_1)\mu_1}{1-\gamma_1+\mu_1}$ | $\frac{(1-\gamma_1)(1-\mu_1)}{1-\gamma_1+\mu_1}$ |

Assumption 1 amounts to a strictly negative correlation between the types of the two members. In the recruiting example, if committee members know only that at most one of the two candidates is of high quality, then a high type knows that his opponent must be a low type. More generally, as long as the two alternatives are drawn independently without replacement from a finite pool of alternatives (in which some are of high quality and some are of low quality), this will induce a negative correlation that we need.[6]

Denote as $\overline{\pi}_{\theta\theta'}$ the payoff from choosing his favorite alternative to the member of type $\theta$ when his opponent's type is $\theta'$, and $\underline{\pi}_{\theta\theta'}$ as his payoff from choosing his opponent's favorite. We assume that: (i) each member strictly prefers his opponent's favorite alternative when his own type's low and his opponent's type is high, and otherwise prefers his own favorite (strictly except when both types are high); and (ii) each member has a stronger incentive to choose his favorite alternative when his own type is high than when it is low, for the same type of his opponent (strictly when his own type is $H$ and weakly when it is $L$). These assumptions can be combined as follows:

**Assumption 2.** (CONFLICTS AND COMMON INTERESTS) $\overline{\pi}_{HH} - \underline{\pi}_{HH} \geq 0 > \overline{\pi}_{LH} - \underline{\pi}_{LH}$, *and* $\overline{\pi}_{HL} - \underline{\pi}_{HL} \geq \overline{\pi}_{LL} - \underline{\pi}_{LL} > 0$.

The states $LL$ and $HH$ are conflict states, where each member prefers his own favorite alternative (strictly in the former case and weakly in the latter); while the states $LH$ and $HL$ are common-interest states, where both members strictly prefer the favorite alternative of the high type member. To simplify notation, for all $\theta, \theta' = L, H$, we define

$$\phi_{\theta\theta'} = \frac{1}{2}(\overline{\pi}_{\theta\theta'} + \underline{\pi}_{\theta\theta'}); \qquad \lambda_{\theta\theta'} = \frac{1}{2}(\overline{\pi}_{\theta\theta'} - \underline{\pi}_{\theta\theta'}).$$

When the opponent's type is $\theta'$, the payoff for type $\theta$ from a coin flip is $\phi_{\theta\theta'}$, and the payoff difference for type $\theta$ between getting his favorite alternative and a coin flip is $\lambda_{\theta\theta'}$.

[6] If instead types are independent, the delay mechanisms characterized by our main result remain optimal, but there are optimal delay mechanisms that do not have all the characterized features. Specifically, under independence all optimal delay mechanisms satisfy Propositions 1, 2 (if we strengthen Assumption (2) by assuming that $\lambda_{HL} > \lambda_{LL}$) and 3, but necessarily Propositions 4 and 5.

The above payoff assumptions are natural if we interpret a high type as one whose favorite alternative is of high quality and a low type likewise low quality, and the payoff to a member from a given alternative is the sum of its quality and a private benefit when it is his own favorite. Then, as in the recruiting committee example in the introduction, a common-interest state represents a situation where the quality difference is sufficient to overcome the private benefit so the low type member is willing to go along with his opponent's favorite alternative, and a conflict state is such that there is no quality difference so each member prefers his own favorite.

Assumptions 1 and 2 play two roles in our analysis. First, they are used to establish an equilibrium property that type $H$ has stronger incentives than type $L$ to persist with his own favorite alternative. Second, they are used to prove an optimality property that type $H$ benefits whenever type $L$ is induced by a greater delay to make concessions.

The unique symmetric *first-best* outcome in this problem is to choose the favorite alternative of the member if his type is high and his opponent's type is low, and otherwise flip a coin. In the absence of side transfers, there is a mechanism that achieves this outcome as an equilibrium if

$$\gamma_1 \leq \gamma_* \equiv \frac{-\lambda_{LH}}{\lambda_{LL} - \lambda_{LH}}.$$

Consider a voting game where each member chooses between the two alternatives, with the agreed alternative implemented immediately and any disagreement leading to a fair coin flip between the two alternatives. We say that a member *persists* if he votes for his favorite alternative, and that he *concedes* if he votes for his opponent's favorite alternative. It is a dominant strategy for type $H$ to vote for his favorite alternative, regardless of his belief $\mu_1$ about the type of his opponent. We let $x_1$ represent the probability that a low type persists. For any $x_1$ of the opponent low type, it is optimal for type $L$ to concede, as

$$\gamma_1\left(x_1\underline{\pi}_{LL} + (1-x_1)\phi_{LL}\right) + (1-\gamma_1)\underline{\pi}_{LH} \geq \gamma_1\left(x_1\phi_{LL} + (1-x_1)\overline{\pi}_{LL}\right) + (1-\gamma_1)\phi_{LH},$$

with strict inequality if $\gamma_1 < \gamma_*$.

In contrast, if $\gamma_1 > \gamma_*$, the unique equilibrium in the above voting game has both type $L$ and type $H$ voting for their favorite alternatives. The decision is always made by a coin flip in equilibrium, despite the presence of a mutually preferred alternative in the two common interest states. In fact, there is no symmetric, incentive-compatible mechanism that Pareto-dominates flipping a coin.[7] Our model provides a stark environment that

[7] Consider any symmetric mechanism where each alternative is implemented with probability $\frac{1}{2}$ in states $LL$ and $HH$, and let $q$ be the common probability that type $H$'s favorite alternative is implemented in states

illustrates the severe restrictions on efficient information aggregation in committees when side transfers are not allowed. Throughout the paper, we assume that $\gamma_1 > \gamma_*$.

### 2.2. Second-best delay mechanism

As suggested in our previous work (Damiano, Li and Suen 2012), delay in making decisions can improve information aggregation and ex ante welfare in the absence of side transfers. We model delay by an additive payoff loss to the members. Properly employed by a mechanism designer, delay helps improve information aggregation by "punishing" type $L$ when he acts like the high type.

Imagine we modify the voting game in Section 2.1 by adding delay: when both members vote for their favorite alternatives, a delay $\delta_1$ is imposed on the members before the decision is made by flipping a coin. We refer to this game as a *one-round delay mechanism*, and claim that the uniquely optimal one-round delay mechanism is $\delta_1$ such that type $L$ concedes with probability one in equilibrium, and is indifferent between conceding and persisting.

For any given $\gamma_1 > \gamma_*$, assuming that type $H$ persists with probability one and type $L$ persists with some probability $x_1 \in [0,1]$, we have the following indifference condition of type $L$ between persisting and conceding:

$$\begin{aligned} U_1 &= \gamma_1 \left(x_1 \left(-\delta_1 + \phi_{LL}\right) + (1-x_1)\overline{\pi}_{LL}\right) + (1-\gamma_1)\left(-\delta_1 + \phi_{LH}\right) \\ &= \gamma_1 \left(x_1 \underline{\pi}_{LL} + (1-x_1)\phi_{LL}\right) + (1-\gamma_1)\underline{\pi}_{LH}. \end{aligned}$$

By Assumptions 1 and 2, the indifference condition of type $L$ implies that type $H$ strictly prefers persisting to conceding:

$$\begin{aligned} V_1 &= \mu_1 \left(x_1 \left(-\delta_1 + \phi_{HL}\right) + (1-x_1)\overline{\pi}_{HL}\right) + (1-\mu_1)\left(-\delta_1 + \phi_{HH}\right) \\ &> \mu_1 \left(x_1 \underline{\pi}_{HL} + (1-x_1)\phi_{HL}\right) + (1-\mu_1)\underline{\pi}_{HH}, \end{aligned}$$

confirming that it is an equilibrium for type $H$ to persist with probability one and type $L$

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$LH$ and $HL$. The incentive condition for type $L$ to truthfully reveal his type is

$$\gamma_1\phi_{LL} + (1-\gamma_1)(q\underline{\pi}_{LH} + (1-q)\overline{\pi}_{LH}) \geq \gamma_1(q\overline{\pi}_{LL} + (1-q)\underline{\pi}_{LL}) + (1-\gamma_1)\phi_{LH}.$$

Since $\gamma_1 > \gamma_*$, the above condition requires $q \leq \frac{1}{2}$. Thus, the best that can be achieved by a symmetric, incentive compatible mechanism when $\gamma_1 > \gamma_*$ is setting $q = \frac{1}{2}$, which is the same as flipping a coin.

to persist with probability $x_1$. The average payoff to each member is

$$W_1 \equiv \frac{\mu_1}{1-\gamma_1+\mu_1} U_1 + \frac{1-\gamma_1}{1-\gamma_1+\mu_1} V_1. \tag{1}$$

As $\delta_1$ increases, the payoff from persisting for the low type shifts down so the indifference condition implies a smaller value of $x_1$. A smaller $x_1$ raises $U_1$ because $\lambda_{LL} > 0$. For the high type, it is straightforward to verify (see Lemma 2 below) that

$$\begin{aligned} \gamma_1 V_1 = \mu_1 U_1 + \gamma_1 \mu_1 \left(\overline{\pi}_{HL} - \overline{\pi}_{LL} - x_1(\lambda_{HL} - \lambda_{LL})\right) + (\mu_1 - \gamma_1)\delta_1 \\ + \gamma_1(1-\mu_1)\phi_{HH} - (1-\gamma_1)\mu_1\phi_{LH}. \end{aligned}$$

Because a higher $\delta_1$ lowers $x_1$ and raises $U_1$, the above equation implies that a higher $\delta_1$ increases $V_1$. We conclude that the lowest $\delta_1$ such that $x_1 = 0$ maximizes $W_1$: if we increase the delay $\delta_1$ further, type $L$ would strictly prefer to concede, but this would raise the payoff loss to type $H$ without changing the decision. The equilibrium outcome of this optimal one-round delay mechanism is that the Pareto efficient decision is reached with minimum delay. Both type $L$ and type $H$ obtain their respective highest possible symmetric decision payoffs, while type $H$ incurs the necessary payoff loss when encountering another type $H$. We refer to this outcome as the *second-best*.

### 2.3. Limited commitment

Using delay to improve information aggregation in committees is both natural and, as a mechanism, simple to implement. However, as a form of collective punishment, some degree of commitment is needed. For initial beliefs $\gamma_1$ just above $\gamma_*$, the required delay $\delta_1$ to achieve the second-best is small. As $\gamma_1$ increases, however, achieving the second best outcome requires an ever larger delay; as $\gamma_1$ approaches 1, the required $\delta_1$ would have to be arbitrarily large, which presents a serious credibility issue. We model limited commitment by assuming there is an upper bound $\Delta > 0$ on the delay $\delta$ that can be credibly imposed on the committee when both committee members vote for their favorite alternatives.

**Assumption 3.** (BOUNDED EXPECTED DELAY) $d \leq \Delta$.

The above assumption is weak. Since committee members are risk-neutral, all we require is that the delay committed ex ante in any mechanism is not always enforced ex post. The idea is that longer delays are more costly, so the committee members have more incentive to renegotiate it away. We sidestep the issue of modeling renegotiation explicitly by taking the maximum expected delay $\Delta$ as a primitive. This is admittedly a crude way of modeling the constraint on commitment power, but it nonetheless captures the essential idea that destruction of value is unlikely to be credible unless the amount involved is small relative to the decision at stake.

### 2.4. Dynamic delay mechanisms

Given an upper-bound $\Delta$ on the expected cost of delay, the second-best outcome becomes unachievable for initial beliefs $\gamma_1$ of type $L$ that are too high. From the indifference condition, we find that the second-best outcome is achievable if and only if

$$\gamma_1 \leq \gamma^* \equiv \frac{-\lambda_{LH} + \Delta}{\lambda_{LL} - \lambda_{LH} + \Delta}.$$

Clearly, we have $\gamma^* > \gamma_*$. Throughout the main part of our analysis, we assume $\gamma_1 > \gamma^*$ so that the second best outcome is not achievable in a one-round delay mechanism. For $\gamma_1$ just above $\gamma^*$, the optimal one-round mechanism is to set the expected delay $\delta_1$ to the upper-bound $\Delta$, so as to maximize concession by the low type.[8] As $\gamma_1$ increases, however, type $L$ votes for his favorite alternative with a greater probability, which leads to a greater payoff loss due to delay, and for sufficiently high $\gamma_1$, the benefit of inducing type $L$ to vote for the opponent's favorite alternative is outweighed by the payoff loss due to delay. The best one-round mechanism is the trivial one with $\delta_1 = 0$, equivalent to a coin flip.

Can the committee do better by committing ex ante to repeated delays when they disagree? Imagine that we modify the one-round mechanism by replacing coin flip after delay with a continuation one-round mechanism with some delay $\delta_2 \leq \Delta$. Suppose that in this *two-round delay mechanism* we can choose $\delta_2$ such that type $L$ obtains a continuation payoff in the second round which is exactly equal to the coin-flip payoff, but is obtained through equilibrium randomization with probability $x_2 < 1$ of voting for his ex ante favorite. Then, it remains an equilibrium for type $L$ to vote for his favorite alternative in the first round with the same probability $x_1$ as in the original one-round mechanism. As both $x_1$ and the continuation payoff remain unchanged in the modified mechanism, the equilibrium payoff to type $L$ is the same as in the one-round mechanism. However, because a smaller $x_2$ benefits type $H$ more than it benefits the low type, whenever type $L$ is indifferent between a continuation round with $x_2 < 1$ and $\delta_2 > 0$ and a coin flip with $x_2 = 1$ and $\delta_2 = 0$, type $H$ is strictly better off with the former than with the latter. Thus, this two-round mechanism delivers the same payoff to type $L$ as in the original one-round

[8] This is formally proved in Section 5.1.

mechanism but improves the payoff of the high type.

That a two-round mechanism can improve a one-round mechanism motivates us to consider general dynamic delay mechanisms. Formally, a *delay mechanism* is a multi-round voting game where in each round $t \leq T$, with $T$ infinite or finite, conditional on the game not having ended, each member chooses between voting for his favorite alternative (persisting) and voting against it (conceding). If the two votes agree, the agreed alternative is implemented immediately and the game ends. If both members concede (we call this a *reverse disagreement*), the decision is made by a coin flip without delay. If both persist (*regular disagreement*), the expected delay cost $\delta_t \in (0, \Delta]$ is imposed; the game moves on to the next round if $t < T$, or ends with a coin flip if $t = T$. Since the game has no discounting, for the game to be well-defined in the case of $T = \infty$, we need to specify the payoff if both members always vote for their own favorite alternative; for simplicity we assume that the payoff is strictly lower than the minimum of implementing any decision across states. We often represent a delay mechanism by the corresponding sequence of delays, $(\delta_1, \delta_2, \ldots, \delta_T)$.

Given the initial beliefs $\gamma_1$ and $\mu_1$, and given the upper-bound on delay $\Delta$, we say that a delay mechanism, together with a symmetric perfect Bayesian equilibrium in the extensive-form game defined by the mechanism, is *optimal* if there is no delay mechanism with a symmetric perfect Bayesian equilibrium that gives a strictly higher ex ante payoff (1) to each member. This definition of optimality allows for multiple symmetric perfect Bayesian equilibria in a given delay mechanism.[9]

# 3. Results

## 3.1. Preliminary analysis

Fix a delay mechanism $(\delta_1, \ldots, \delta_T)$, where $T$ can be finite or infinite. Denote as $x_t$ the equilibrium probability that type $L$ persists in round $t$, and as $y_t$ the equilibrium probability that type $H$ persists. Let $\gamma_t$ be the equilibrium belief of type $L$, and $\mu_t$ be the belief of type $H$, that his opponent is of type $L$ at the beginning of round $t$. Given the initial belief $\gamma_1$ and $\mu_1$, whenever applicable the beliefs $\gamma_t$ and $\mu_t$ in subsequent rounds are derived from Bayes' rule:

$$\gamma_{t+1} = \frac{\gamma_t x_t}{\gamma_t x_t + (1-\gamma_t) y_t}; \qquad \mu_{t+1} = \frac{\mu_t x_t}{\mu_t x_t + (1-\mu_t) y_t}.$$

[9]The main restriction we impose is symmetry. Generally there are asymmetric equilibria in which only one type $H$ member concedes with a positive probability. Our approach is to impose symmetry and establish (in Section 4.1) that type $H$ persists with probability one in any equilibrium of an optimal mechanism. This is a more natural approach given the underlying committee problem.

Next, we denote as $U_t$ and respectively $V_t$ the equilibrium expected payoff of type $L$ and type $H$ at the beginning of round $t$. We have

$$U_t = \gamma_t U_{t,L} + (1-\gamma_t)U_{t,H}, \qquad V_t = \mu_t V_{t,L} + (1-\mu_t)V_{t,H},$$

where $U_{t,\theta}$ is the equilibrium payoff of type $L$ against a type $\theta = L, H$ opponent, with analogous expressions of $V_{t,\theta}$. When $T$ is finite, we have $U_{T+1,\theta} = \phi_{L\theta}$ and $V_{T+1,\theta} = \phi_{H\theta}$ for $\theta = L, H$. In equilibrium, $U_t$ is given by the following recursive formula

$$\begin{cases} \gamma_t \left(x_t(-\delta_t + U_{t+1,L}) + (1-x_t)\overline{\pi}_{LL}\right) + (1-\gamma_t)\left(y_t(-\delta_t + U_{t+1,H}) + (1-y_t)\overline{\pi}_{LH}\right) & \text{if } x_t > 0, \\ \gamma_t \left(x_t \underline{\pi}_{LL} + (1-x_t)\phi_{LL}\right) + (1-\gamma_t)\left(y_t \underline{\pi}_{LH} + (1-y_t)\phi_{LH}\right) & \text{if } x_t < 1. \end{cases}$$

In the above, the top expression is the expected payoff from persisting and the bottom expression is the expected payoff from conceding, with the two equated if $x_t \in (0,1)$. Similarly, $V_t$ is given by

$$\begin{cases} \mu_t \left(x_t(-\delta_t + V_{t+1,L}) + (1-x_t)\overline{\pi}_{HL}\right) + (1-\mu_t)\left(y_t(-\delta_t + V_{t+1,H}) + (1-y_t)\overline{\pi}_{HH}\right) & \text{if } y_t > 0, \\ \mu_t \left(x_t \underline{\pi}_{HL} + (1-x_t)\phi_{HL}\right) + (1-\mu_t)\left(y_t \underline{\pi}_{LH} + (1-y_t)\phi_{HH}\right) & \text{if } y_t < 1. \end{cases}$$

Finally, the ex ante payoff $W_1$ of each member, before they learn their types, is given by (1). An optimal delay mechanism maximizes $W_1$.

Given a delay mechanism $(\delta_1, \ldots, \delta_T)$, an equilibrium of the induced game can be characterized by a sequence $\{(\gamma_t, x_t, U_t), (\mu_t, y_t, V_t)\}_{t=1}^{T}$ that satisfies the evolutions of the beliefs and the recursive conditions of the equilibrium values. The "boundary conditions" are provided by the initial beliefs $\gamma_1$ and $\mu_1$ and, if $T$ is finite, by the payoffs from coin flips in the event that both members persist in the last round $T$. Although it is possible to solve for the equilibrium for some particular delay mechanism (such as one with constant delay), characterizing all equilibria for any given mechanism is neither feasible nor insightful. We introduce a "localized variations method" to derive necessary conditions for an equilibrium induced by an optimal delay mechanism. This method is most useful in our context because the following screening result shows that in any equilibrium type $H$ strictly prefers persisting to conceding so long as type $L$ weakly does so. The key is that type $H$ could always mimic type $L$'s equilibrium strategy in the continuation after persisting in round $t$.

**Lemma 1.** (SCREENING) *Suppose that $\gamma_t < \mu_t$. Then, $x_t > 0$ implies $y_t = 1$.*

*Proof.* For a fixed type $\theta = H, L$ of his opponent, let $\hat{V}_{t+1,\theta}$ be the continuation payoff of type $H$ from mimicking type $L$'s equilibrium strategy after persisting in round $t$. Since $x_t > 0$, type $L$ weakly prefers persisting:

$$\gamma_t \left(x_t(-\delta_t + U_{t+1,L} - \underline{\pi}_{LL}) + (1-x_t)\lambda_{LL}\right) \geq (1-\gamma_t)\left(y_t(\delta_t + \underline{\pi}_{LH} - U_{t+1,H}) - (1-y_t)\lambda_{LH}\right).$$

If $y_t = 1$, we are already done; otherwise, the right-hand-side of the above expression is strictly positive because $U_{t+1,H} \leq \underline{\pi}_{LH}$ and $\lambda_{LH} < 0$. Since $\gamma_t < \mu_t$, we can replace $\gamma_t$ in the above inequality with $\mu_t$ to get

$$\mu_t \left(x_t(-\delta_t + U_{t+1,L} - \underline{\pi}_{LL}) + (1-x_t)\lambda_{LL}\right) > (1-\mu_t)\left(y_t(\delta_t + \underline{\pi}_{LH} - U_{t+1,H}) - (1-y_t)\lambda_{LH}\right).$$

Against each type $\theta$, by construction type $L$ and type $H$ have the same expected payoff loss from delay and the same total probability that their favorite alternative is implemented. It then follows from Assumption 2 that $U_{t+1,\theta} - \underline{\pi}_{L\theta} \leq \hat{V}_{t+1,\theta} - \underline{\pi}_{H\theta}$ for $\theta = H, L$. Since $\lambda_{H\theta} \geq \lambda_{L\theta}$ for each $\theta$ by Assumption 2, we have

$$\mu_t \left(x_t(-\delta_t + \hat{V}_{t+1,L} - \underline{\pi}_{HL}) + (1-x_t)\lambda_{HL}\right) > (1-\mu_t)\left(y_t(\delta_t + \underline{\pi}_{HH} - \hat{V}_{t+1,H}) - (1-y_t)\lambda_{HH}\right),$$

implying that type $H$ strictly prefers persisting to conceding, and thus $y_t = 1$. ∎

The above result means that we can focus our equilibrium analysis on the incentives of type $L$ alone, at least until type $L$ concedes. Given Assumption 1, if $x_1 > 0$, then by the above lemma we have $y_1 = 1$. Bayes' rule then implies that $\gamma_2 < \mu_2$, so now $y_2 = 1$ if $x_2 > 0$. This continues until the game ends in some round $n$ for type $L$ when he concedes with probability one. Thereafter we have $\gamma_t = \mu_t = 0$ for all $t \geq n+1$.

Our localized variations method requires us to study the changes to the payoffs to both type $L$ and type $H$ when we vary a delay mechanism locally. The following lemma provides a link between the equilibrium payoffs of the two types.

**Lemma 2.** (LINKAGE) *Suppose that $x_\tau > 0$ for $\tau = t, \ldots, t'$. Then,*

$$\gamma_t V_t = \mu_t U_t + \gamma_t \mu_t \left(\prod_{\tau=t}^{t'} x_\tau (V_{t'+1,L} - U_{t'+1,L}) + \left(1 - \prod_{\tau=t}^{t'} x_\tau\right)(\overline{\pi}_{HL} - \overline{\pi}_{LL})\right)$$
$$+ (\mu_t - \gamma_t)\sum_{\tau=t}^{t'} \delta_\tau + \gamma_t(1-\mu_t)V_{t'+1,H} - (1-\gamma_t)\mu_t U_{t'+1,H}. \quad (2)$$

*Proof.* Since $x_\tau > 0$ for $\tau = t, \dots, t'$, we can write $U_t$ as the payoff from persisting with probability one from $t$ through to $t'$, followed by the equilibrium strategy from $t'+1$ onwards. By the Screening Lemma, we have $y_\tau = 1$ for $\tau = t, \dots, t'$, and thus $U_t$ is

$$\gamma_t \left( \prod_{\tau=t}^{t'} x_\tau U_{t'+1,L} + \left(1 - \prod_{\tau=t}^{t'} x_\tau\right) \overline{\pi}_{LL} - \sum_{\tau=t}^{t'} \prod_{i=t}^{\tau} x_i \delta_\tau \right) + (1-\gamma_t) \left( - \sum_{\tau=t}^{t'} \delta_\tau + U_{t'+1,H} \right).$$

Similarly, $V_t$ is given by

$$\mu_t \left( \prod_{\tau=t}^{t'} x_\tau V_{t'+1,L} + \left(1 - \prod_{\tau=t}^{t'} x_\tau\right) \overline{\pi}_{HL} - \sum_{\tau=t}^{t'} \prod_{i=t}^{\tau} x_\tau \delta_\tau \right) + (1-\mu_t) \left( - \sum_{\tau=t}^{t'} \delta_\tau + V_{t'+1,H} \right).$$

Multiplying $V_t$ by $\gamma_t$ and $U_t$ by $\mu_t$, and taking the difference, we obtain (2). ■

To illustrate how the Linkage Lemma can be fruitfully used, consider a delay mechanism, $(\delta_1, \dots, \delta_T)$, with an equilibrium of the induced game given by the sequence, $\{(\gamma_\tau, x_\tau, U_\tau), (\mu_\tau, y_\tau, V_\tau)\}_{t=1}^{T}$. Suppose we change this delay mechanism in such a way that $\delta_\tau$ remains the same as in the original mechanism for $\tau < t$ and $\tau > t'$. Further, suppose we choose a new subsequence of delays $(\delta_t, \dots, \delta_{t'})$ (by inserting extra rounds if necessary) in such a way that "total persistence" between rounds $t$ and $t'$, $\prod_{\tau=t}^{t'} x_\tau$, is the same as in the original mechanism and $U_t$ is also the same as in the original mechanism. Then the equilibrium play of the low type prior to round $t$ is unchanged by this *localized variation* of the original mechanism because $U_t$ is held fixed. The equilibrium play of the low type after round $t'$ is also unchanged, because $\gamma_{t'+1}$ is determined by $\gamma_t$ and total persistence $\prod_{\tau=t}^{t'} x_\tau$, which are not affected the this localized variation. As a result, the only way such a localized variation can affect $V_t$ is through the term $\sum_{\tau=t}^{t'} \delta_\tau$ in equation (2). If "total delay" between rounds $t$ and $t'$, $\sum_{\tau=t}^{t'} \delta_\tau$, under such a localized variation is higher than that in the original mechanism, then the value of $V_t$ will also be higher under the localized variation. Because the equilibrium play prior to round $t$ is not affected by the localized variation, this implies that $V_1$ would increase while $U_1$ would remain unchanged. In the ensuing analysis, we employ this and other types of localized variations to avoid constructing the entire equilibrium sequence, and use the Linkage Lemma to study the effect of these localized variations on $V_1$ and $U_1$.

The third and last ingredient in our localized variations method is a tight upper bound on how much concession in equilibrium in a given round $t$ that type $L$ with belief $\gamma_t$ can make. The following lemma requires $\gamma_t > \Delta/(\lambda_{LL} + \Delta)$, so that the bound we derive on $x_t$ is strictly positive. This condition is satisfied if $\gamma_t > \gamma^*$.

**Lemma 3.** (MAXIMAL CONCESSION) *Suppose that* $\gamma_t > \Delta/(\lambda_{LL} + \Delta)$ *and* $y_t = y_{t+1} = 1$ *for some* $t < T$*. Then,*

$$x_t \geq \chi(\gamma_t) \equiv \frac{\gamma_t \lambda_{LL} - (1-\gamma_t)\Delta}{\gamma_t(\lambda_{LL} + \Delta)}; \qquad \gamma_{t+1} \geq \Gamma(\gamma_t) \equiv \frac{\gamma_t \lambda_{LL} - (1-\gamma_t)\Delta}{\lambda_{LL}}.$$

*Proof.* Given $y_t = 1$, type $L$ weakly prefers conceding to persisting if

$$\gamma_t \left( x_t \underline{\pi}_{LL} - (1-x_t)\lambda_{LL} \right) + (1-\gamma_t)\underline{\pi}_{LH} \geq (\gamma_t x_t + 1 - \gamma_t)(-\delta_t + U_{t+1}).$$

In round $t+1$, type $L$ can always concede. Given that $y_{t+1} = 1$, we thus have

$$U_{t+1} \geq \gamma_{t+1} \left( \underline{\pi}_{LL} + (1-x_{t+1})\lambda_{LL} \right) + (1-\gamma_{t+1})\underline{\pi}_{LH}.$$

The above reaches the minimum when $x_{t+1} = 1$. With this bound on $U_{t+1}$ and the bound on $\delta_t$, applying Bayes' rule shows that type $L$ weakly prefers conceding to persisting if

$$(\gamma_t x_t + 1 - \gamma_t)\Delta \geq \gamma_t(1-x_t)\lambda_{LL}.$$

From the above we then obtain $x_t \geq \chi(\gamma_t)$, which is positive if $\gamma_t > \Delta/(\lambda_{LL} + \Delta)$. Since $\gamma_{t+1}$ is increasing in $x_t$, using Bayes' rule with $x_t = \chi(\gamma_t)$ gives $\gamma_{t+1} \geq \Gamma(\gamma_t)$. ■

We say that round $t < T$ is *active* if $x_t \in (0,1)$. The above lemma leads to the following useful definition: we say that there is *no slack* in an active round $t$ if $\delta_t = \Delta$ and $U_{t+1} = \gamma_{t+1}\underline{\pi}_{LL} + (1-\gamma_{t+1})\underline{\pi}_{LH}$. When there is no slack, the "static" incentive in round $t$ for truth-telling for type $L$ is maximized, as regular disagreement is punished by the maximum credible expected delay cost $\Delta$. Furthermore, the "dynamic" incentive in round $t$ is also maximized, as the continuation payoff $U_{t+1}$ is minimized. The latter occurs in equilibrium if, after regular disagreement in round $t$, type $L$ persists with probability one in round $t+1$ but is indifferent between conceding and persisting. Therefore minimizing the continuation payoff for type $L$ requires no concession in the following round. A dynamic delay mechanism with maximal concession in some round $t$ necessarily results in this kind of "start-and-stop" behavior.

Maximal concession, or equivalently no slack, is suggestive of how to increase concession in a localized variation of an equilibrium of a given delay mechanism. In an active round $t$, if $\delta_t < \Delta$, we can raise $\delta_t$ to induce another equilibrium with a lower equilibrium $x_t$. If $\delta_t = \Delta$ but $U_{t+1} > \gamma_{t+1}\underline{\pi}_{LL} + (1-\gamma_{t+1})\underline{\pi}_{LH}$, we can achieve the same outcome by inserting a round $s$ with delay $\delta_s$ between $t$ and $t+1$. For $\delta_s$ small, it is an equilibrium for type $L$ to persist with probability one, with $x_s = 1$. But this will make it more costly for type $L$ to persist in round $t$ and can thus induce another equilibrium with a lower $x_t$. We then ensure that this new equilibrium differs from the original one only locally around $t$ with additional adjustments to the mechanism, so that we can apply the Linkage Lemma to evaluate the effects on the payoffs.

When there is maximal concession, the evolution of the belief of type $L$ is pinned down by the Maximal Concession Lemma. For any fixed $\gamma_1 \in (\gamma^*, 1)$, denote as $n^*$ the smallest integer $n$ satisfying

$$\left(\frac{\lambda_{LL} + \Delta}{\lambda_{LL}}\right)^n \geq \frac{1-\gamma^*}{1-\gamma_1}. \tag{3}$$

Then, $n^*$ is the least number of active rounds for the belief to reach from $\gamma_1$ to $\gamma^*$ or below. A tighter commitment bound $\Delta$ requires more active rounds for the initial degree of conflict $\gamma_1$ to reach the level $\gamma^*$, when the second-best can be achieved. Define the "residue" $\eta$ such that

$$\eta \left(\frac{\lambda_{LL} + \Delta}{\lambda_{LL}}\right)^{n^*-1} = \frac{1-\gamma^*}{1-\gamma_1}. \tag{4}$$

By definition, $\eta \in (1, (\lambda_{LL} + \Delta)/\lambda_{LL}]$, and (3) holds as an equality if $\eta = (\lambda_{LL} + \Delta)/\lambda_{LL}$.

The Maximal Concession Lemma implies that an active round with no slack is necessarily followed by an *inactive* round, where type $L$ persists with probability one. Since $x_t = 1$ in an inactive round $t$, it is irrelevant how the total delay in the inactive rounds between two consecutive active rounds are divided into rounds.[10] Although how we number the rounds in any dynamic delay mechanism has a degree of arbitrariness, we denote the active rounds in a mechanism consecutively as $(1) < \ldots < (i) < \ldots$; so $t = (i)$ is the $i$-th active round in the mechanism. If the number of active rounds is finite in a delay mechanism, say some $n$, then following the last active round $(n)$ and possibly inactive rounds, there must be either a *deadline round* after which the game is ended with a coin flip, or an *exiting round* at which type $L$ concedes with probability one before the game ends.[11] We denote the deadline round or the exiting round as round $[n+1]$. Finally, for convenience, for any active round $(i)$, we denote as $\sigma_{(i)}$ the sum of delay $\delta_{(i)}$ in round $(i)$ and the total delay in all subsequent inactive rounds before the next active round $(i+1)$ or the exiting or deadline round $[i+1]$, and refer to it as the *effective delay* of round $(i)$.

[10]In particular, any positive total delay can be divided into infinitely many inactive rounds with a geometric series. As a result, an equilibrium in an infinite mechanism with $T = \infty$ can be outcome-equivalent to another equilibrium in a finite mechanism.

[11]Otherwise type $H$ also persists with probability one after round $(n)$ by the Screening Lemma. This cannot be an equilibrium because by assumption the payoff to either type from not implementing any decision is strictly lower than the payoff from making any decision immediately.

### 3.2. Main result

The main result is the following theorem on optimal delay mechanisms. It is restated and proved in Section 5 after the characterization results of Propositions 1 to 5 in Section 4.

**Theorem.** *For any $\mu_1 \in [\gamma^*, 1]$, there exist two boundary functions $\underline{g}(\mu_1)$ and $\overline{g}(\mu_1)$ satisfying $\gamma^* \leq \underline{g}(\mu_1) \leq \overline{g}(\mu_1) \leq \mu_1$, such that*

*(a) for $\gamma_1 \in (\gamma^*, \underline{g}(\mu_1))$, the optimal delay mechanism is one round, where $\delta_{(1)} = \Delta$;*

*(b) for $\gamma_1 \in [\underline{g}(\mu_1), \min\{\overline{g}(\mu_1), \Gamma^{-1}(\gamma^*)\})$, any optimal delay mechanism has a single active round (1) with no slack and a deadline round [2], in which $\sigma_{(1)} = \gamma_1(\lambda_{LL} + \Delta)$, and $\delta_{[2]} = \min\{\Gamma(\gamma_1)\lambda_{LL}/(1 - \Gamma(\gamma_1)) + \lambda_{LH}, 0\}$;*

*(c) for $\gamma_1 \in [\Gamma^{-1}(\gamma^*), \overline{g}(\mu_1))$, any optimal delay mechanism has $n^* - 1$ active rounds with no slack, one active round $(j)$ with slack for some $j$ such that $2 \leq j \leq n^*$, and a deadline round $[n^* + 1]$, and is payoff-equivalent to one where $j = n^*$, with $\sigma_{(i)} = \Delta + \lambda_{LL}\Delta/(\Delta + \lambda_{LL})$ for each $i = 1, \ldots, n^* - 2$, $\sigma_{(n^*-1)} = \Delta + (\eta - 1)\lambda_{LL}/\eta$, $\sigma_{(n^*)} = (\eta - 1 + \gamma^*)\lambda_{LL}$, and $\delta_{[n^*+1]} = \Delta$;*

*(d) for $\gamma_1 \in [\overline{g}(\mu_1), \mu_1)$, the optimal delay mechanism is a coin flip, where $\delta_{(1)} = 0$.*

Figure 1 depicts the four regions for which each of the four cases in Theorem applies, assuming that $\Delta$ is sufficiently small.[12] The two boundary functions $\underline{g}(\mu_1)$ and $\overline{g}(\mu_1)$ are shown in the figure in red and blue respectively, and their exact characterizations are given in Propositions 6 and 7 in Section 5. One-round delay mechanism with maximum expected delay $\Delta$ is optimal when $\gamma_1$ is close to $\gamma^*$ (Case (a) in Theorem), while at the other end, a coin flip without delay when $\gamma_1$ is close to 1 (Case (d)). A dynamic delay mechanism (Case (b) and Case (c)) is optimal so long as the initial degree of conflict $\gamma_1$ is intermediate, that is, between $\underline{g}(\mu_1)$ and $\overline{g}(\mu_1)$. Case (b) has a single active round before the deadline round, while Case (c) has at least two active rounds; which case applies depends on whether it takes one or more rounds of maximum concession by the low type for the belief to reach $\gamma^*$ starting from the initial belief $\gamma_1$, that is, whether $\gamma_1$ is below or above $\Gamma^{-1}(\gamma^*)$.

In either Case (b) or Case (c), optimal dynamic delay mechanisms induce intuitive properties of equilibrium play, which highlight the logic of using delays dynamically to facilitate strategic information aggregation in our committee problem. The most interesting properties are:

[12] When $\Delta$ is sufficiently large, $\overline{g}(\mu_1)$ lies entirely to the left of $\gamma_1 = \Gamma^{-1}(\gamma^*)$ so Case (c) does not apply; in the intermediate case, $\overline{g}(\mu_1)$ crosses $\gamma_1 = \Gamma^{-1}(\gamma^*)$ from the left. See Proposition 7 for the details.

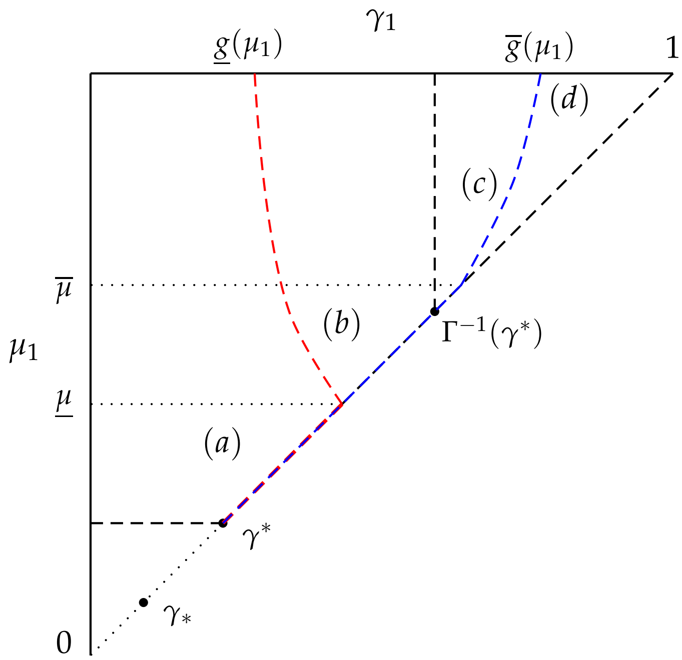


***Figure 1.*** *Regions (a), (b), (c) and (d) refer to the four respective cases in the space of initial beliefs for which Theorem applies.*

(i) Any optimal dynamic delay mechanism has a finite number of active rounds, with a *deadline round*.

(ii) Any optimal dynamic delay mechanism induces *efficient deadline concession*, with type $L$ conceding and type $H$ persisting with probability one, and if there are at least two active rounds, the belief of type $L$ in the deadline round is equal to $\gamma^*$.

(iii) Any optimal dynamic delay mechanism induces *start-and-stop* by type $L$ in that, except for a single round which cannot be the first one, in all active rounds type $L$ makes maximal concession and then no concessions in following inactive rounds.

Property (i) gives the sense that an optimal mechanism must have a finite deadline. We show in Section 4.1 that not only there is a finite number of active rounds, but also the last active round is followed by a deadline round, instead of an exiting round in which the low type exits while the high type stays. It is not optimal to have two type $H$ members play a "pure" war-of-attrition, where an immediate coin flip is Pareto efficient after type $L$ has already exited the game. The mechanism is thus also finite for type $H$.

Property (ii) implies that type $L$ makes an efficient deadline concession. Since type $H$ persists throughout the game, the choice between the two alternatives is always Pareto efficient. We show in Section 4.2 that any optimal delay mechanism induces a belief of type $L$ in the deadline round that is less than or equal to $\gamma^*$. Intuitively dynamic delay mechanisms work by driving down type $L$'s belief that the state is a conflict state. Inducing a deadline belief greater than $\gamma^*$ would imply that type $L$ does not concede with probability one in the deadline round. As a result the Pareto efficient decision could not be achieved at the end, which cannot be optimal because adding more active rounds for type $L$ to have an opportunity to concede would improve the payoff of type $H$. This is the logic we have hinted at in Section 2.2: when $\gamma_1$ is sufficiently above $\gamma^*$, a dynamic delay mechanism with efficient deadline concession (Case (b) of Theorem) dominates a one-round mechanism with maximum feasible expected delay $\Delta$ (Case (a)). But driving down type $L$'s belief through delay is costly. It does not pay to induce a deadline belief too much below $\gamma^*$. When $n^* \geq 2$ (Case (c)), we show in Section 4.4 that the deadline belief must be exactly equal to $\gamma^*$. A lower deadline belief would imply that type $L$ would concede in the deadline round even if in the deadline round the limited commitment bound is slack, and so the delays before the deadline can be reduced while still guaranteeing the Pareto efficient decision at the end.

Property (iii) is perhaps the most interesting insight of this paper. It holds for any optimal dynamic delay mechanism, but start-and-stop cycles appear only in Case (c) of Theorem, as there need to be at least two active rounds. This property will be established in Section 4.5 below. We show that type $L$ needs to make concessions in a way so that the belief reaches $\gamma^*$ as quickly as possible, that is, with the least number of active rounds; otherwise, it is possible at some point of the mechanism to increase the total delay locally, and use the Linkage Lemma to increase the payoff to type $H$ without affecting the payoff to type $L$. Thus, except for a single active round where there is slack to prevent the belief of type $L$ from going down below $\gamma^*$, in each active round type $L$ must make maximum concession. As we have explained in the Maximum Concession Lemma, this requires simultaneously maximizing the immediate delay in an active round and minimizing the continuation payoff after a disagreement. The latter yields the start-and-stop property that, except for a single round, all active rounds in which type $L$ concedes with positive probability are followed by inactive rounds in which type $L$ concedes with zero probability. We show in Section 4.3 that the single active round with slack must not be the first round; that is, dynamic incentives to make maximum concessions are front-loaded.

Figure 2 gives an illustration of the start-and-stop feature in Case (c) of Theorem, where $n^* = 5$. As shown in Section 4.5, it is payoff-irrelevant where we put the single active round with slack, except that it cannot be the first one, so we have made it the last ac-

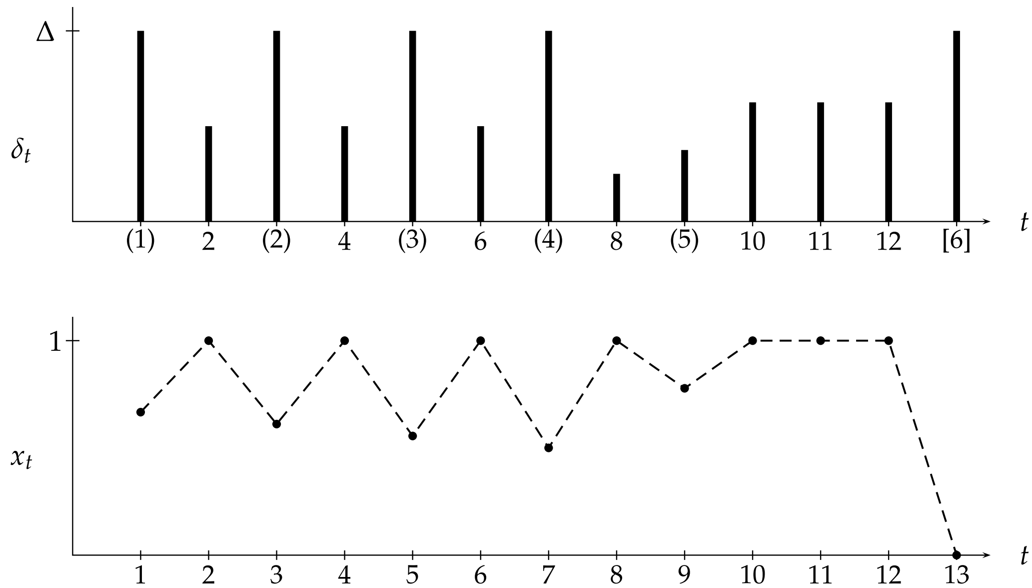


***Figure 2.*** *An example of an optimal delay mechanism with start-and-stop cycles in rounds 1 to 9, followed by a deadline round at round 13.*

tive round (round 9 in the figure) before the deadline round. Each of the first three active rounds has no slack, and as stated in Case (c), the effective delay is $\Delta + \lambda_{LL}\Delta/(\lambda_{LL} + \Delta)$. Note that no slack does not mean that $\delta$ is set equal to the upper bound in each round during this phase. Instead, we have $\delta_{(i)} = \Delta$, followed by one or more inactive rounds $t$ between successive active rounds with $\delta_t < \Delta$. The fourth active round (round 7 in the figure) also has no slack. However, because it is followed by an active round with slack (round 9), the effective delay associated with the fourth active round is lower than that for the earlier three active rounds. Finally, for the active round with slack, the effective delay given in Case (c) is $(\eta - 1 + \gamma^*)\lambda_{LL}$, which may or may not exceed $\Delta$, but since $(\eta - 1)\lambda_{LL} \leq \Delta$, we can always set the immediate delay to $(\eta - 1)\lambda_{LL}$ and allocate the rest of the effective delay evenly across additional inactive rounds subject to the bound $\Delta$. The resulting sequence of thirteen rounds of delays is shown in the top panel of Figure 2, where the first eight rounds consist of four cycles of the maximal delay equal to the limited commitment bound and a delay strictly within the bound, followed by the single active round with slack and three inactive rounds and finally the deadline round. The corresponding sequence of $x_t$ for $t = 1, \ldots, 13$, is shown in the bottom panel of Figure 2. There are four start-and-stop cycles, with the probability of persistence decreasing over the "start" round of the cycles.

# 4. Characterization

## 4.1. Finite deadline

Any delay mechanism has a finite number of active rounds. This means that any mechanism is effectively finite for type $L$, as the last active round is followed by either a deadline round or an exiting round. The reason is that, if a mechanism has an infinite number of active rounds, then by the Screening Lemma, $y_t = 1$ for all $t$. Since by assumption never implementing any alternative gives a payoff that is strictly lower than implementing any alternative to any member in any state, this cannot be an equilibrium.

If a mechanism has a finite number, say $n$, of active rounds followed by an exiting round in which type $L$ fully concedes before the game ends, then two members of type $H$ play a "pure" war of attrition after type $L$ has exited. This follows because $x_{[n+1]} = 0$ implies that $y_{[n+1]} > 0$.[13] We show that this cannot be optimal, because the Pareto efficient decision for two type $H$ members is a coin flip. The argument is based on a contradiction, and is particularly intuitive when $y_{[n+1]} = 1$. We truncate the game after round $[n+1]$ and replace the continuation with a coin flip. The equilibrium payoff $U_{[n+1]}$ for type $L$ from conceding is unchanged, as long as type $H$ still persists with probability one. The deviation payoff for type $L$ from persisting in round $[n+1]$ is decreased: in the original continuation after the deviation in round $[n+1]$ it is a dominant strategy for type $L$ to concede to type $H$, with a payoff strictly higher than the coin-flip payoff, as type $H$ persists with a strictly positive probability after round $[n+1]$. As a result, it remains an equilibrium for type $L$ to play in the same way as in the original equilibrium. On the other hand, truncating the game after round $[n+1]$ increases the continuation payoff of type $H$, because in the original continuation after round $[n+1]$, type $H$'s maximum symmetric payoff from playing against each other is the same as flipping a coin.[14]

**Proposition 1.** *Any optimal delay mechanism has a finite number of active rounds and a deadline round, and the low type does not fully concede before the deadline.*

## 4.2. Efficient deadline concession

Our next characterization result is that in any optimal dynamic mechanism type $L$ must concede in the deadline round with probability one. That is, if a delay mechanism has $n$ active rounds with $n \geq 1$ and is optimal, then not only round $[n+1]$ is the deadline round

[13] If $y_{[n+1]} = x_{[n+1]} = 0$, type $H$ would strictly prefer persisting to conceding regardless of $\delta_{[n+1]}$ or $\mu_{[n+1]}$ in round $[n+1]$, which is a contradiction.

[14] The proofs of all propositions in Section 4 are relegated to the appendix.

due to Proposition 1, but also $x_{[n+1]} = 0$. Further, since the proof of Proposition 1 applies without change to rule out $x_{[n+1]} = 0$ and $y_{[n+1]} < 1$ in the deadline round,[15] the deadline outcome is Pareto efficient. Thus, we have the result of *efficient deadline concession*.

Proposition 2 below establishes the efficient deadline concession result using a localized variations method. Suppose that $x_{[n+1]} > 0$. We insert an auxiliary round $s$ between the last active round $(n)$ and the deadline round $[n+1]$, and construct an equilibrium with $x_s \in (0,1)$ so that type $L$ concedes a little more after round $(n)$. This would increase the continuation payoff of type $L$ at round $(n)$. In order to avoid having to characterize the equilibrium before and after the modification, we insert another round between $(n)$ and $s$ with delay equal to the increase in $U_{(n)}$. Then, setting $t = (n)$ and $t' = [n+1]$ in equation (2) of the Linkage Lemma, we have

$$\gamma_{(n)} V_{(n)} = \mu_{(n)} U_{(n)} - \gamma_{(n)} \mu_{(n)} \prod_{\tau=(n)}^{[n+1]} x_\tau (\lambda_{HL} - \lambda_{LL}) + \left(\mu_{(n)} - \gamma_{(n)}\right) \sum_{\tau=(n)}^{[n+1]} \delta_\tau + \text{constant.}$$

Since $\lambda_{HL} \geq \lambda_{LL}$ under Assumption 2, and since $\gamma_{(n)} < \mu_{(n)}$ by Assumption 1 and the Screening Lemma, $V_{(n)}$ is increased as the modifications increase the total delay $\sum_{\tau=(n)}^{[n+1]} \delta_\tau$ after round $(n)$ and simultaneously reduce the total persistence $\prod_{\tau=(n)}^{[n+1]} x_\tau$. Because the equilibrium play before round $(n)$ is unaffected by this localized variation, an induction argument establishes that an increase in $V_{(n)}$ implies an increase in $V_1$, while $U_1$ remains unchanged. Thus ex ante welfare increases with this localized variation.

**Proposition 2.** *Any optimal delay mechanism with at least two rounds has efficient deadline concession.*

### 4.3. Front-loading

In any optimal mechanism, round 1 is active. Our next result shows that incentives for type $L$ to concede in an optimal delay mechanism with at least two rounds are *front-loaded*, in that round 1 is not only active but also induces maximal concession by type $L$.

There are two cases. In the first case, which is more involved and uses a localized variations construction, there are $n \geq 2$ active rounds before the deadline round. Suppose that there is slack in the first round. In the modified mechanism, the effective delay in the first round is raised to lower the probability of persistence $x_1$. Simultaneously, the effective delay in the second active round is reduced to increase the probability of persistence $x_{(2)}$

[15] In the proof we truncate the game by replacing round $[n+1]$ with a coin flip. It does not matter whether round $[n+1]$ is an exiting round or a deadline round.

to keep the belief after the first two active rounds constant. Lowering $x_1$ improves the expected payoff $U_1$ of type $L$ in round 1, but there is no previous round for which we can increase the effective delay to compensate for the improvement. Nonetheless, we localize the changes to the equilibrium by keeping unchanged the continuation payoff $U_{(2)}$. This allows us to use the following version of the Linkage Lemma:

$$\gamma_1 V_1 = \mu_1 U_1 + (\mu_1 - \gamma_1)\left(\sigma_1 + \sigma_{(2)}\right) + \text{constant}.$$

We show that the positive effect on $V_1$ through the increase in $U_1$ outweighs the negative effect through the decrease in $\sigma_{(2)}$. With $\sigma_1$ also increased, the overall effect is positive.

**Proposition 3.** *Any optimal mechanism with at least two rounds has no slack in the first round.*

### 4.4. Efficient deadline belief

Proposition 2 implies that the deadline belief of type $L$ cannot exceed $\gamma^*$, which is the highest belief to induce type $L$ to concede with probability one in the deadline round. We now establish that an optimal delay mechanism with $n \geq 2$ active rounds has an *efficient deadline belief*, in that the deadline belief $\gamma_{[n+1]}$ is exactly $\gamma^*$. Type $L$ is not induced to concede more than necessary to achieve a Pareto efficient decision in the deadline round.

Our contradiction argument starts with $\gamma_{[n+1]} < \gamma^*$, and modifies the delay mechanism so that type $L$ concedes less in the last active round $(n)$ before $[n+1]$. We do so by increasing the belief of type $L$ in round $[n+1]$ as much as possible without causing it to go above $\gamma^*$, so that type $L$ will still concede with probability one in the deadline round. Although we are not making infinitesimal changes to the mechanism, the key is still localizing the changes to the original equilibrium by keeping unchanged the expected payoff to type $L$ throughout the modification, so we can evaluate the payoff impact on type $H$ alone.[16] This is done by reducing the effective delay in round $(n)$, which results in a decrease of the expected payoff to type $L$, and neutralizing this change by reducing the delay in the penultimate active round $(n-1)$, so that the expected payoff $U_{(n-1)}$ to type $L$ is unaffected. This is why we require there are at least two active rounds.[17]

[16] For infinitesimal changes to the original mechanism, the argument in the proof of Proposition 4 is insufficient for us to increase type $H$'s equilibrium payoff in the penultimate round $(n-1)$ without changing type $L$'s payoff.

[17] In an optimal delay mechanism, we can indeed have a single active round before $[n+1]$ and thus $\gamma_{[n+1]} < \gamma^*$; this is Case (b) in Theorem.

The version of the Linkage Lemma we use is:

$$\gamma_{(n-1)}V_{(n-1)} = \mu_{(n-1)}U_{(n-1)} + \gamma_{(n-1)}\mu_{(n-1)} \prod_{\tau=(n-1)}^{[n+1]-1} x_\tau \lambda_{LL} + \left(\mu_{(n-1)} - \gamma_{(n-1)}\right) \sum_{\tau=(n-1)}^{[n+1]-1} \delta_\tau$$
$$+ \gamma_{(n-1)} \left(1 - \mu_{(n-1)}\right) V_{[n+1],H} + \text{constant},$$

where we have set $t = (n-1)$ and $t' = [n+1]-1$ in equation (2), and used the evaluations $U_{t'+1,L} = \phi_{LL}$, $U_{t'+1,H} = \underline{\pi}_{LH}$ and $V_{t'+1,L} = \overline{\pi}_{HL}$ because type $L$ concedes with probability one in round $[n+1]$. For any fixed beliefs $\gamma_{(n-1)}$ and $\mu_{(n-1)}$ in our local modifications, the positive effect on the payoff $V_{(n-1)}$ to type $H$ in round $(n-1)$ comes from the increase in the total persistence $\prod_{\tau=(n-1)}^{[n+1]-1} x_\tau$ before the deadline round, and the negative effect comes from the decrease in the total delay $\sum_{\tau=(n-1)}^{[n+1]-1} \delta_\tau$. We show that the positive effect dominates the negative effect, and therefore $V_{(n-1)}$ is increased.

**Proposition 4.** *Any optimal delay mechanism with at least two active rounds has an efficient deadline belief.*

### 4.5. Start-and-stop

For any optimal delay mechanism with at least two active rounds, our results so far have established the game ends for both type $H$ and type $L$ simultaneously in the deadline round, with type $L$ conceding with probability one and type $H$ persisting with probability one, and with the efficient deadline belief $\gamma^*$ for type $L$. Before the deadline round, type $L$ may concede with positive probabilities while type $H$ always persists. We now characterize the dynamics of concessions made by type $L$.

We first establish the key start-and-stop property of optimal delay mechanisms with at least two active rounds. We show that a mechanism with slack in two consecutive active rounds, say round $(i)$ and round $(i+1)$, cannot be optimal. This is because the presence of slack in $(i)$ and $(i+1)$ means that it is possible, by appropriately changing the effective delays, to change the probabilities of persistence $x_{(i)}$ and $x_{(i+1)}$ of type $L$ in the two rounds in any direction we want. In the localized variations argument used to prove Proposition 5 below, we modify the effective delays in both rounds while maintaining total persistence $x_{(i)}x_{(i+1)}$ constant. From repeated applications of Bayes' rule, this guarantees that the belief after the regular disagreement in round $(i+1)$, and hence the continuation equilibrium, is left unchanged. Finally, an appropriate change in the effective delay in round $(i-1)$ keeps the continuation payoff after a regular disagreement in that round constant; this is possible because, by Proposition 3, we can assume that round $(i)$ is not the first active round. As a result, the original equilibrium remains unchanged in the modified mechanism before round $(i)$ and after round $(i+1)$. Setting $t = (i-1)$ and $t' = (i+1)$ in (2) in the Linkage Lemma, we have

$$\mu_{(i-1)}V_{(i-1)} = \gamma_{(i-1)}U_{(i-1)} + \left(\mu_{(i-1)} - \gamma_{(i-1)}\right) \sum_{\tau=(i-1)}^{(i+1)} \sigma_\tau + \text{constant}.$$

Since $\gamma_{(i-1)} < \mu_{(i-1)}$ by Assumption 1 and the Screening Lemma, we have the desired contradiction if we show that the total delay from round $(i-1)$ to $(i+1)$ is increased.

One can think of this localized variations exercise as choosing $\gamma_{(i+1)}$ to maximize the total delay, while holding fixed $\gamma_{(i)}$ and $\gamma_{(i+2)}$ (as well as the continuation payoff in round $(i-1)$). From the Maximal Concession Lemma, the feasible set for $\gamma_{(i+1)}$ is

$$\left[\max\left\{\gamma_{(i+2)}, \Gamma(\gamma_{(i)})\right\}, \min\left\{\gamma_{(i)}, \Gamma^{-1}(\gamma_{(i+2)})\right\}\right].$$

In the proof of Proposition 5, we show that the total delay is a strictly convex function of $\gamma_{(i+1)}$. When there is slack in both round $(i)$ and round $(i+1)$, the belief $\gamma_{(i+1)}$ is in the interior of the feasible set, implying that the mechanism is not optimal.

Proposition 5 below establishes that which one of two consecutive active rounds is given the slack is payoff-irrelevant (so long as the slack is not given to the first round). As a result, there can be at most one active round with slack.[18] By the Maximal Concession Lemma, the probability of concession by type $L$ is maximized in active rounds without slack. The efficient deadline belief $\gamma^*$ is reached in the least possible number of active rounds. Moreover, after each round $(i)$ with no slack and maximal concession, type $L$ makes no concession, with $x_{(i)+1} = 1$. After that, in the next active round $(i+1)$, they will make the maximal concession again provided there is no slack in round $(i+1)$, followed by no concession. Proposition 5 thus shows that equilibrium play exhibits a start-and-stop pattern, alternating between maximal concession and no concession.

**Proposition 5.** *Any optimal delay mechanism with at least two active rounds has at most one active round with slack.*

[18] If there are two active rounds with slack, we can always rearrange the active rounds to create two consecutive rounds with slack. From our earlier result the rearranged mechanism cannot be optimal, but since the rearrangement does not affect the payoffs $U_1$ and $V_1$, the original mechanism cannot be optimal.

## 5. Optimal Delay Mechanisms

### 5.1. The four candidate mechanisms

By Proposition 1, only four types of mechanisms can be possibly optimal, corresponding to the four cases in Theorem. We first use the characterization results of the previous section to construct the candidate optimal mechanism for each case.

*Case (a): One-round mechanism.*

A one-round delay mechanism can be optimal only if type $L$ is indifferent between persisting and conceding. As we raise $\delta_1$, the indifference condition of type $L$ implies that $x_1$ decreases, and thus $U_1$ increases. By the Linkage Lemma, $V_1$ increases as $U_1$ increases and $x_1$ decreases. Thus, for any $\gamma_1 \geq \gamma^*$, it is optimal to set $\delta_1 = \Delta$, with resulting payoffs:

$$\begin{aligned} U_1^{(a)} &= \gamma_1 \underline{\pi}_{LL} + (1-\gamma_1)\underline{\pi}_{LH} + \gamma_1(1-x_1)\lambda_{LL}, \\ V_1^{(a)} &= \mu_1 \overline{\pi}_{HL} + (1-\mu_1)\phi_{HH} - \mu_1 x_1 \lambda_{HL} - (\mu_1 x_1 + 1 - \mu_1)\Delta; \end{aligned} \tag{5}$$

where

$$x_1 = \frac{\gamma_1 \lambda_{LL} - (1-\gamma_1)(-\lambda_{LH} + \Delta)}{\gamma_1 \Delta}. \tag{6}$$

We need $x_1 < 1$. This is equivalent to $\gamma_1 < (-\lambda_{LH} + \Delta)/(\lambda_{LL} - \lambda_{LH})$.

*Case (b): A dynamic mechanism with a single active round.*

By Proposition 1, the deadline round is [2]. Proposition 2 requires type $L$ to concede with probability one in round [2], so this case is valid only if $\gamma_1 \leq \Gamma^{-1}(\gamma^*)$, or $n^* = 1$. By Proposition 3, we have $x_1 = \chi(\gamma_1)$ and $\gamma_{[2]} = \Gamma(\gamma_1)$. Since there is no slack in round 1 and type $L$ concedes with probability one in round [2],

$$\sigma_1 = \Delta + \gamma_{[2]} \lambda_{LL} = \gamma_1(\lambda_{LL} + \Delta). \tag{7}$$

We have the following optimized expected payoffs:

$$\begin{aligned} U_1^{(b)} &= \gamma_1 \underline{\pi}_{LL} + (1-\gamma_1)\underline{\pi}_{LH} + \frac{\lambda_{LL}\Delta}{\lambda_{LL} + \Delta}, \\ V_1^{(b)} &= \mu_1 \overline{\pi}_{HL} + (1-\mu_1)\phi_{HH} - \gamma_1 \lambda_{LL} + (\mu_1 - \gamma_1)\Delta - (1-\mu_1)\delta_{[2]}; \end{aligned} \tag{8}$$

where $\delta_{[2]} = 0$ if $\gamma_1 \leq \Gamma^{-1}(\gamma_*)$, and is otherwise given by

$$\delta_{[2]} = \frac{\Gamma(\gamma_1)}{1 - \Gamma(\gamma_1)} \lambda_{LL} + \lambda_{LH}. \tag{9}$$

*Case (c): A dynamic mechanism with at least two active rounds.*

In this case, since the deadline belief must be $\gamma^*$ by Proposition 4, and since there can be at most one round with slack by Proposition 5, there are exactly $n^*$ active rounds in an optimal mechanism and $n^* \geq 2$. Unless the "residue" $\eta$ defined in (4) happens to be $(\lambda_{LL} + \Delta)/\lambda_{LL}$, there are $n^* - 1$ rounds with no slack, and one round with slack. Denote the round with slack as $j$. By Proposition 3, we have $j \geq 2$; by Proposition 5, it is payoff-irrelevant which of the other values that $j$ takes. We assume below that $j = n^*$.

For each $i = 1, \ldots, n^* - 2$, we have $\delta_{(i)} = \Delta$. In round $(i+1)$, the equilibrium belief of type $L$ is $\gamma_{(i+1)} = \Gamma(\gamma_{(i)})$, and the probability of persisting is $x_{(i+1)} = \chi(\gamma_{(i+1)})$. The condition that there is no slack in round $(i)$ requires the low type to be indifferent between persisting and conceding when the opponent is playing $x_{(i)+1} = 1$ in round $(i) + 1$:

$$\begin{aligned} U_{(i)+1} &= -\sum_{\tau=(i)+1}^{(i+1)-1} \delta_\tau + \gamma_{(i+1)} \left( \underline{\pi}_{LL} + \left(1 - \chi(\gamma_{(i+1)})\right) \lambda_{LL} \right) + \left(1 - \gamma_{(i+1)}\right) \underline{\pi}_{LH} \\ &= \gamma_{(i+1)} \underline{\pi}_{LL} + \left(1 - \gamma_{(i+1)}\right) \underline{\pi}_{LH}. \end{aligned}$$

Thus, the total delay between $(i)$ and $(i+1)$ is constant, given by

$$\sum_{\tau=(i)+1}^{(i+1)-1} \delta_\tau = \gamma_{(i+1)} \left(1 - \chi(\gamma_{(i+1)})\right) \lambda_{LL} = \frac{\lambda_{LL}\Delta}{\lambda_{LL} + \Delta}. \tag{10}$$

The penultimate active round $(n^* - 1)$ has no slack, so again $\delta_{(n^*-1)} = \Delta$, and

$$\sum_{\tau=(n^*-1)+1}^{(n^*)-1} \delta_\tau = \gamma_{(n^*)} \left(1 - x_{(n^*)}\right) \lambda_{LL}.$$

Since $\gamma_{(n^*)+1} = \gamma^*$, and since $(1 - \gamma^*)/(1 - \gamma_{(n^*)}) = \eta$, we obtain:

$$x_{(n^*)} = \frac{1 - \left(1 - \gamma_{(n^*)}\right) \eta}{\gamma_{(n^*)} \eta},$$

and thus the effective delay in round $(n^*-1)+1$ is given by

$$\sum_{\tau=(n^*-1)+1}^{(n^*)-1} \delta_\tau = \frac{(\eta-1)\lambda_{LL}}{\eta}. \tag{11}$$

In the last active round $(n^*)$, there is generally slack. We have already found $x_{(n^*)}$ above, and by construction the belief after a regular disagreement would become $\gamma^*$. To find the effective delay in round $(n^*)$, we use $U_{[n^*+1]} = \gamma^*\phi_{LL} + (1-\gamma^*)\underline{\pi}_{LH}$, and the indifference condition in round $(n^*)$ to obtain:

$$\sum_{\tau=(n^*)}^{[n^*+1]-1} \delta_\tau = \frac{\gamma_{(n^*)}(1-\gamma^*)\lambda_{LL}}{1-\gamma_{(n^*)}} = (\eta-1+\gamma^*)\lambda_{LL}. \tag{12}$$

Finally, in the deadline round $[n^*+1]$, given $\gamma_{[n^*+1]} = \gamma^*$, choosing $\delta_{[n^*+1]} = \Delta$ induces $x_{[n^*+1]} = 0$, which always ends the game with the Pareto efficient decision.

The optimized expected payoffs are given by:

$$U_1^{(c)} = \gamma_1\underline{\pi}_{LL} + (1-\gamma_1)\underline{\pi}_{LH} + \frac{\lambda_{LL}\Delta}{\lambda_{LL}+\Delta}, \tag{13}$$
$$V_1^{(c)} = \frac{\mu_1}{\gamma_1}U_1^{(c)} + \mu_1(\overline{\pi}_{HL} - \overline{\pi}_{LL}) + \frac{\mu_1-\gamma_1}{\gamma_1}\sum_{t=1}^{[n^*+1]}\delta_t + (1-\mu_1)\phi_{HH} - \frac{\mu_1(1-\gamma_1)}{\gamma_1}\phi_{LH};$$

where we have used the Maximal Concession Lemma to compute $U_1^{(c)}$, which takes the same form as $U_1^{(b)}$, and the Linkage Lemma for $V_1^{(c)}$. The total delay is given by

$$\sum_{t=1}^{[n^*+1]}\delta_t = (n^*-2)\left(\Delta + \frac{\lambda_{LL}\Delta}{\lambda_{LL}+\Delta}\right) + \left(\Delta + \frac{(\eta-1)\lambda_{LL}}{\eta}\right) + (\eta-1+\gamma^*)\lambda_{LL} + \Delta. \tag{14}$$

*Case (d): Flipping a coin.*

The payoffs for the two types are:

$$U_1^{(d)} = \gamma_1\phi_{LL} + (1-\gamma_1)\phi_{LH}; \qquad V_1^{(d)} = \mu_1\phi_{HL} + (1-\mu_1)\phi_{HH}. \tag{15}$$

### 5.2. Optimal delay mechanisms

Now we complete the derivation of optimal delay mechanisms given in Theorem by comparing the ex ante payoff $W_1$ for the four cases derived above. The proof involves straightforward calculations given the payoff expressions of (5) to (15).

Case (a) applies when $\gamma_1$ is sufficiently close to $\gamma^*$, so that the benefit from a dynamic delay mechanism given in Case (b) does not justify the additional cost of delay relative to a static delay mechanism. At the other end, when $\gamma_1$ is sufficiently to close to 1, the benefit from a Pareto efficient decision through a delay mechanism given in Case (b) or Case (c) is overwhelmed by the cost of delay incurred, so the coin flip of Case (d) is optimal. Away from $\gamma_1 = \mu_1 = 1$, either Case (b) or Case (c) dominates Case (d) if $\gamma_1 < \overline{g}(\mu_1)$, depending on whether $\gamma_1 \leq \Gamma^{-1}(\gamma^*)$ or not. The boundary $\underline{g}(\mu_1)$ between Case (a) and Case (b) is decreasing in $\mu_1$ (for $\mu_1 \geq \underline{\mu}$), while the boundary $\overline{g}(\mu_1)$ between Case (b) or Case (c) and Case (d) is increasing in $\mu_1$; see Figure 1. Thus, for given $\gamma_1$, due to Assumption 1, a greater $\mu_1$ makes it more likely that dynamic mechanisms (Case (b) and Case (c)) are optimal. This is intuitive, because a higher belief $\mu_1$ of type $H$ that his opponent is of type $L$ makes it less likely that the former is playing a pure war-of-attrition game with another type $H$, in which case there is no benefit to delaying the decision.[19]

**Proposition 6.** *There exist $\underline{\mu}$ and $\overline{\mu}$ satisfying $\gamma^* < \underline{\mu} < \overline{\mu} < 1$; $\underline{g}(\mu_1)$ satisfying $\underline{g}(\mu_1) = \mu_1$ for $\mu_1 \in [\gamma^*, \underline{\mu}]$, $\underline{g}'(\mu_1) < 0$ for $\mu_1 \in (\underline{\mu}, 1]$ and $\underline{g}(1) > \gamma^*$; and $\overline{g}(\mu_1)$ satisfying $\overline{g}(\mu_1) = \mu_1$ for $\mu_1 \in [\gamma^*, \overline{\mu}]$, $\overline{g}'(\mu_1) > 0$ for $\mu_1 \in (\overline{\mu}, 1]$ and $\overline{g}(1) < 1$, such that optimal delay mechanisms are given by Case (a) of Theorem if $\gamma_1 < \underline{g}(\mu_1)$, by Case (b) if $\gamma_1 \in [\underline{g}(\mu_1), \overline{g}(\mu_1)]$ and $n^* = 1$, by Case (c) if $\gamma_1 \in [\underline{g}(\mu_1), \overline{g}(\mu_1)]$ and $n^* \geq 2$, and by Case (d) if $\gamma_1 \in (\overline{g}(\mu_1), \mu_1)$.*

Between the two cases of dynamic delay mechanisms, Case (b) and Case (c) in Theorem, which case applies depends only on whether or not it takes a single active round without slack or more rounds to bring down $\gamma_1$ to $\gamma^*$ or below. From the definition of $n^*$ in equation (3), this is equivalent to whether $\gamma_1 \leq \Gamma^{-1}(\gamma^*)$ or not. Therefore, for a fixed belief $\mu_1$ of type $H$, there are beliefs $\gamma_1$ of type $L$ to which Case (c) applies if and only if $\Gamma^{-1}(\gamma^*) < \overline{g}(\mu_1)$. Since start-and-stop cycles are the unique feature of Case (c) in Theorem, we want to know under what conditions they are part of an optimal mechanism.

The following proposition shows that the boundary functions, in particular $\overline{g}(\mu_1)$, increases with $\Delta$. There is a critical value $\underline{\Delta}$, such that for all $\Delta < \underline{\Delta}$, we have $\overline{\mu} > \Gamma^{-1}(\gamma^*)$. Since $\overline{g}'(\mu_1) > 0$ for $\mu_1 > \overline{\mu}$, there are initial beliefs $\gamma_1$ of type $L$ for which the dynamic

[19] The proofs of Proposition 6 and Proposition 7 involve straightforward calculations and are provided in an online appendix.

mechanism of Case (c) dominates Case (d) for all $\mu_1 > \gamma_1$, and is optimal. In other words, if the bound $\Delta$ on delay per-round is sufficiently tight, there are committee problems with the initial belief $\gamma_1$ of type $L$ between $\Gamma^{-1}(\gamma^*)$ and $\overline{\mu}$ such that start-and-stop cycles will always be present regardless of the initial belief $\mu_1$ of type $H$. This is the case shown in Figure 1, where the boundary function $\overline{g}(\mu_1)$ above the 45-degree line lies entirely to the right of the vertical line of $\gamma_1 = \Gamma^{-1}(\gamma^*)$. At the other end, there is a critical value $\overline{\Delta}$, such that $\overline{g}(1) \leq \Gamma^{-1}(\gamma^*)$ for all $\Delta > \overline{\Delta}$. Since $\overline{g}'(\mu_1) > 0$ for $\mu_1 > \overline{\mu}$, the dynamic mechanism of Case (c), whenever it is applicable with $\gamma_1 > \Gamma^{-1}(\gamma^*)$, is dominated by the coin flip of Case (d). The boundary function $\overline{g}(\mu_1)$ lies entirely to the left of the vertical line of $\gamma_1 = \Gamma^{-1}(\gamma^*)$, and optimal delay mechanisms will not have start-and-stop cycles regardless of the beliefs $\gamma_1$ and $\mu_1$. For intermediate values of $\Delta$, the boundary $\overline{g}(\mu_1)$ cuts through the vertical line, so there are initial beliefs $\gamma_1$ of type $L$ for which the dynamic mechanism of Case (c) is optimal for $\mu_1$ close to 1 but not for $\mu_1$ close to $\gamma_1$.

**Proposition 7.** *The boundary functions $\underline{g}(\mu_1)$ and $\overline{g}(\mu_1)$ strictly increase in $\Delta$ for $\mu_1 > \underline{\mu}$ and for $\mu_1 > \overline{\mu}$, respectively. Moreover, there exist $\underline{\Delta}$ and $\overline{\Delta}$, satisfying $0 < \underline{\Delta} < \overline{\Delta}$, such that $\overline{\mu} \geq \Gamma^{-1}(\gamma^*)$ if $\Delta \leq \underline{\Delta}$, $\overline{\mu} < \Gamma^{-1}(\gamma^*) \leq \overline{g}(1)$ if $\underline{\Delta} < \Delta \leq \overline{\Delta}$, and $\overline{g}(1) < \Gamma^{-1}(\gamma^*)$ if $\Delta > \overline{\Delta}$.*

### 5.3. Continuous-time limit

We have seen in Section 2.1 that in the absence of costly delay, there is no incentive compatible mechanism that Pareto dominates a coin flip in our committee model for sufficiently high initial beliefs of type $L$. That is, if the uniform upper bound $\Delta$ on expected delay is identically zero, flipping a coin is the only symmetric incentive compatible outcome if $\gamma_1 > \gamma_*$. On the other hand, our Theorem applies so long as $\Delta > 0$. The set of incentive compatible outcomes is discontinuous at $\Delta = 0$. We now examine which features of optimal mechanisms disappear, and which ones persist.

As $\Delta$ becomes arbitrarily small, a one-round mechanism or a dynamic mechanism with a single active round loses bite. In the limit, only Case (c) and Case (d) of Theorem are relevant. This limit has the natural interpretation of a continuous-time model, with cumulative delay as "time." Each start-and-stop cycle in Case (c), starting at some instant $t$ with belief $\gamma(t)$ of type $L$, consists of an interval of time of length $\Delta$, where type $L$ makes the maximal concession of $1 - \chi(\gamma(t))$, and an interval of length $\lambda_{LL}\Delta/(\lambda_{LL} + \Delta)$, where type $L$ makes no concession. As $\Delta$ goes to 0, the average rate of concession over the cycle by type $L$, is given by

$$\lim_{\Delta \to 0} \frac{1 - \chi(\gamma(t))}{\Delta + \lambda_{LL}\Delta/(\lambda_{LL} + \Delta)} = \frac{1}{2\gamma(t)\lambda_{LL}}.$$

The evolution of belief $\gamma(t)$ of type $L$ over the cycle in the limit satisfies:

$$\lim_{\Delta \to 0} \frac{\Gamma(\gamma(t)) - \gamma(t)}{\Delta + \lambda_{LL}\Delta/(\lambda_{LL} + \Delta)} = -\frac{1 - \gamma(t)}{2\lambda_{LL}}.$$

In the limit, the start-and-stop cycles are "smoothed out." At any instant $t$, with belief $\gamma(t)$, type $L$ concedes at a flow rate of $1/(2\gamma(t)\lambda_{LL})$, with $\gamma(t)$ satisfying a differential equation given by

$$\dot{\gamma}(t) = -\frac{1 - \gamma(t)}{2\lambda_{LL}}.$$

As $\Delta$ converges to zero, the second-best belief $\gamma^*$ converges to the first-best belief $\gamma_*$. The total number of rounds it takes for $\gamma_1$ to go down to $\gamma_*$ goes to infinity, but from the definition of $n^*$ (equation 3), we have

$$\lim_{\Delta \to 0} 2n^*\Delta = 2\lambda_{LL} \log\left(\frac{1 - \gamma_*}{1 - \gamma_1}\right).$$

The above is the total length of the start-and-stop cycles, or equivalently, the exact time it takes for $\gamma_1$ to reach $\gamma_*$ so that the Pareto efficient decision can be made. The limit of the total delay (14) is

$$\lim_{\Delta \to 0} \sum_{t=1}^{T} \delta_t = \lambda_{LL}\left(2\log\left(\frac{1 - \gamma_*}{1 - \gamma_1}\right) + \gamma_*\right),$$

which is the optimal deadline in the continuous-time limit of $\Delta$ going to zero. This is the same result as in Damiano, Li and Suen (2012), after adjusting for the special payoff structure used in that paper.

## 6. Extensions

### 6.1. Discounting

We have assumed that the cost of delay is explicit and additive. The alternative to this money-burning model of costly delay is discounting. There is a constant discount rate $r$, and an upper bound $\Delta$ on the length of time between two adjacent rounds. The implied lower bound on the discount factor is $B = e^{-r\Delta}$.

A critical component of our analysis of the money-burning model is Lemma 1, which establishes that the high type has a greater incentive to persist than the low type. Under money burning, the payoff loss resulting from any length of delay is the same for the two types, and more importantly, is independent of future decisions. So by imitating the low type the high type guarantees the same payoff loss from delay and the same decisions in terms of probability of getting his favorite, and Lemma 1 follows from Assumption 1 and Assumption 2. These two assumptions are no longer sufficient under discounting. For the imitation argument to work under discounting, we need additionally that the high type's payoff is less than or equal to the low type's from implementing the opponent's favorite decision regardless of the latter's type. More precisely, we need

$$\underline{\pi}_{H\theta} \leq \underline{\pi}_{L\theta} \tag{16}$$

for each $\theta = H, L$. This is a cross-type assumption on the payoffs, instead of one on the payoff differences as the current Assumption 2.

The first-best outcome in the simple committee problem given in Section 2.1 of course remains the same when we switch from money-burning to discounting. For the second-best, further assumptions on preferences can be used to establish that when the low type's belief is above the first best, it is optimal to minimize the discount factor, and thus maximize delay, subject to the low type being indifferent between persisting and conceding. In additional to Assumption 1 and Assumption 2, it is sufficient if the payoff ratio of the sum of payoffs from implementing one's favorite and implementing the opponent's favorite when the opponent type switches from high to low is smaller for the high type than for the low type.[20] More precisely, we need

$$\frac{\phi_{HH}}{\phi_{HL}} \leq \frac{\phi_{LH}}{\phi_{LL}}. \tag{17}$$

Under this condition, we can show that the payoff difference between persisting and conceding for the high type is decreasing in the discount factor whenever the low type is indifferent (as a greater discount factor is countered by a greater probability of persisting). Since the payoff from conceding for the high type is decreasing in the discount factor as greater delay increases the probability of concession by the low type, and since the high type strictly prefer to persist, we have the result that the second-best outcome is reached by choosing the minimum discount factor so that the low type concedes with probability one but is indifferent between conceding and persisting.

Lemma 3 goes through with a different formula for the maximum concession probability as a function of the current belief of the low type. The counterpart of $\chi(\gamma)$, the

[20] We also assume that the decision payoffs $\overline{\pi}_{\theta\theta'}$ and $\underline{\pi}_{\theta\theta'}$ are strictly positive for all $\theta, \theta' = L, H$.

lowest probability of persistence when the current belief of the low type is $\gamma$, is

$$\chi_B(\gamma) \equiv \frac{\gamma\lambda_{LL} - (1-B)(1-\gamma_t)\underline{\pi}_{LH}}{\gamma_t(\lambda_{LL} + (1-B)\underline{\pi}_{LL})},$$

and the counterpart of $\Gamma(\gamma)$, the lowest continuation belief of the low type after one round of maximal concession is

$$\Gamma_B(\gamma) \equiv \frac{\gamma\lambda_{LL} - (1-B)(1-\gamma)\underline{\pi}_{LH}}{\lambda_{LL} - (1-B)(1-\gamma)(\underline{\pi}_{LH} - \underline{\pi}_{LL})}.$$

It remains true that maximizing concession by the low type at a given round requires no slack—the discount factor is minimized between this round and the next round, and in the next round the low type is indifferent between persisting and conceding but chooses persisting with probability one.

Lemma 2 has to be modified under discounting. In the model of money-burning, the payoffs of the high type and the low type are linked through a sufficient statistic which is the sum of delays. Under discounting, the sequence of discount factors interacts with the sequence of persisting probabilities; the sufficient statistic linking the payoffs turns out be the product of the discount factors. Suppose that $x_\tau > 0$ for $\tau = t, \ldots, t'$. Since Lemma 1 holds, $y_\tau = 1$. By repeatedly using the payoff from persisting, we obtain the following formula for the payoff of the low type at the beginning of round $t$:

$$U_t = \gamma_t \prod_{\tau=t}^{t'} (\beta_\tau x_\tau) \frac{U_{t'+1}}{\gamma_{t'+1}} + \gamma_t D_t^{t'} \overline{\pi}_{LL},$$

where

$$D_t^{t'} = \sum_{\tau'=t}^{t'} \prod_{\tau=t}^{\tau'-1} (\beta_\tau x_\tau)(1 - x_{\tau'})$$

is the total discounted probability of receiving concessions from a low-type opponent from round $t$ through to round $t'$. The corresponding formula for type $H$ is

$$V_t = \mu_t \prod_{\tau=t}^{t'} (\beta_\tau x_\tau) \frac{V_{t'+1}}{\mu_{t'+1}} + \mu_t D_t^{t'} \overline{\pi}_{HL}.$$

The counterpart of Lemma 2 is

$$\frac{U_t}{\gamma_t \overline{\pi}_{LL}} - \frac{V_t}{\mu_t \overline{\pi}_{HL}} = \prod_{\tau=t}^{t'} (\beta_\tau x_\tau) \left( \frac{U_{t'+1}}{\gamma_{t'+1} \overline{\pi}_{LL}} - \frac{V_{t'+1}}{\mu_{t'+1} \overline{\pi}_{HL}} \right). \quad (18)$$

Thus, if in a localized variations exercise we fix the starting belief $\gamma_t$ and the ending belief $\gamma_{t'+1}$, hence the total persistence $\prod_{\tau=t}^{t'} x_\tau$, and fix the associated payoffs $U_t$ and $U_{t'+1}$, the payoff of the high type $V_t$ varies one-to-one with the the product of discount factors $\prod_{\tau=t}^{t'} \beta_\tau$.

With the above modifications, we can show that our characterization results in Theorem are qualitatively robust to discounting. Instead of redoing all the results here, we select the most interesting one, namely start-and-stop cycles, and show that it continues to hold under discounting. As in the proof of Proposition 5, we establish that there cannot be two consecutive active rounds $(i)$ and $(i+1)$ with slack by a contradiction argument. When there is slack in both round $(i)$ and round $(i+1)$, the belief $\gamma_{(i+1)}$ is in the interior of a feasible set, when we hold as fixed $\gamma_{(i)}$ and $\gamma_{(i+2)}$. Additionally, we fix $\gamma_{(i-1)}$, and hence $x_{(i-1)}$ through Bayes' rule and $U_{(i-1)}$ through the low type's indifference between persisting and conceding in round $(i-1)$, and we also fix the low type's continuation payoff $U_{(i+2)}$. Applying (18) to round $(i-1)$ to $(i+1)$, we have that $V_{(i-1)}$ is quasi-convex in $\gamma_{(i+1)}$, and thus having slacks in both round $(i)$ and round $(i+1)$ is sub-optimal, if $\beta_{(i-1)}\beta_{(i)}\beta_{(i+1)}$ is quasi-concave in $\gamma_{(i+1)}$, and if

$$\frac{U_{(i+2)}}{\gamma_{(i+2)}\overline{\pi}_{LL}} > \frac{V_{(i+2)}}{\mu_{(i+2)}\overline{\pi}_{HL}}.$$

In the proof of the following result, we use backward induction, together with the two additional preference assumptions (16) and (17), to establish the above inequality. The quasi-concavity of $\beta_{(i-1)}\beta_{(i)}\beta_{(i+1)}$ in $\gamma_{(i+1)}$ is shown to follow from the low type's indifference conditions in the active rounds $(i-1)$, $(i)$ and $(i+1)$ in our local variations exercise. Using the above characterization of maximal concession under discounting, we also establish a corresponding payoff-equivalence result: the value of $\beta_{(i-1)}\beta_{(i)}\beta_{(i+1)}$ as a function of $\gamma_{(i+1)}$, and hence the value of $V_{(i-1)}$, is the same at the two corners of the feasibility set, namely, $\gamma_{(i+1)} = \Gamma_B(\gamma_{(i)})$ and $\gamma_{(i+1)} = \Gamma_B^{-1}(\gamma_{(i+2)})$. The proof of the following counterpart of Proposition 5 under discounting is relegated to the online appendix.

**Proposition 8.** *Suppose that (16) and (17) hold. If an optimal delay mechanism under discounting has at least two active rounds, then there is slack in at most one active round.*

### 6.2. Redesign-proof

The form of limited commitment in this paper has to do with Assumption 3, namely that the amount of delay after each regular disagreement is at most equal to some $\Delta > 0$. The commitment is however unlimited in terms of the number of rounds. In this subsection, we impose another form of limit to the commitment power: the number of rounds of delays is at most equal to some $\Omega \geq 1$. We imagine that an opportunity for redesign occurs after the last round of committed delays has been imposed (if there is no commitment to ending the game with a coin flip). However, we do not model explicitly "renegotiation" between the two committee members over the continuation sequences of delay. Instead, we take a reduced-form approach and require the optimal delay mechanism to be redesign-proof after $\Omega$ rounds. The analysis in this subsection can be seen as a robustness exercise with respect to the optimal delay mechanism characterized in Theorem.

We assume that the fixed length $\Omega$ of delay sequences that can be committed to is at least 1, and allow $\Omega$ to be arbitrarily large. The assumption that $\Omega \geq 1$ amounts to the minimum commitment power required for single-round delay mechanisms; the case of $\Omega = \infty$ corresponds to the maximum commitment power underlining Theorem. We say that a correspondence $M_\Omega$ from a pair of initial beliefs to a set of ex ante average payoffs is "$\Omega$-redesign proof," if for any $(\gamma_1, \mu_1)$ and for any $m_1 \in M_\Omega(\gamma_1, \mu_1)$: (i) there is a finite[21] sequence of delays $\{\delta_t\}_{t=1}^T$ satisfying $0 \leq \delta_t \leq \Delta$ for each $t = 1, \ldots, T$, with an equilibrium $\left(\{(\gamma_t, \mu_t), (U_t, V_t)\}_{t=1}^{T+1}; \{(x_t, y_t)\}_{t=1}^T\right)$ of the delay game $\left((\gamma_1, \mu_1); \{\delta_t\}_{t=1}^T\right)$ with deadline round $T$ such that

$$m_1 = \frac{\mu_1}{1 - \gamma_1 + \mu_1} U_1 + \frac{1 - \gamma_1}{1 - \gamma_1 + \mu_1} V_1;$$

and (ii) there is an increasing sequence of integers $\{k_i\}_{i=0}^I$ with $k_0 = 0$, $k_i - k_{i-1} \leq \Omega$ for each $i = 1, \ldots, I$, and $0 \leq T - k_I \leq \Omega$, such that

$$m_{k_i+1} \leq \frac{\mu_{k_i+1}}{1 - \gamma_{k_i+1} + \mu_{k_i+1}} U_{k_i+1} + \frac{1 - \gamma_{k_i+1}}{1 - \gamma_{k_i+1} + \mu_{k_i+1}} V_{k_i+1}$$

for all $i = 1, \ldots, I$ and for all $m_{k_i+1} \in M_\Omega(\gamma_{k_i+1}, \mu_{k_i+1})$.

Part (i) of the definition ensures that there is an equilibrium that achieves the requisite average payoff $m_1$ for the initial beliefs $(\gamma_1, \mu_1)$. Part (ii) applies a total of $I$ tests to ensure that this equilibrium is robust against redesign along the path when the maximum length of delay sequences that can be committed to is $\Omega$. Each test compares the equilibrium average continuation payoff with all other equilibria at the same continuation beliefs that are themselves robust against redesign. This feature of self-reference is why our definition applies to the correspondence $M_\Omega$ rather than an individual payoff $m_1 \in M_\Omega(\gamma_1, \mu_1)$ for some initial beliefs $(\gamma_1, \mu_1)$. Nonetheless, because in any equilibrium the beliefs de-

[21] We assume that $T$ is finite for simplicity of the definition. Using the same truncation argument as in Proposition 1, we can show that in characterizing $\overline{M}_\Omega(\gamma_1, \mu_1)$ (defined below) for any $\Omega$ and any $(\gamma_1, \mu_1)$, it is without loss to restrict to delay games with a finite deadline round.

crease along the path, part (ii) of the definition is recursive in the sense that for each $i = 1, \ldots, I$, the continuation equilibrium $\left(\{(\gamma_t, \mu_t), (U_t, V_t)\}_{t=k_i+1}^{T+1}; \{(x_t, y_t)\}_{t=k_i+1}^{T}\right)$ is also robust against redesign.

For any $(\gamma_1, \mu_1)$, the set $M_\Omega(\gamma_1, \mu_1)$ is non-empty. This is because the definition allows delay games with a deadline round $T \leq \Omega$ in part (i) and $I = 0$ in part (ii) for any equilibrium of such a delay game, making the requirement for the equilibrium to be robust against redesign vacuous. We can then define a mapping $\overline{M}_\Omega$ from a pair of initial beliefs to an ex ante average payoff as the "optimal $\Omega$-redesign proof" payoff function if

$$\overline{M}_\Omega(\gamma_1, \mu_1) = \max M_\Omega(\gamma_1, \mu_1)$$

for any $(\gamma_1, \mu_1)$. For any finite $\Omega$ and any $(\gamma_1, \mu_1)$, if $m \in M_\Omega(\gamma_1, \mu_1)$ then $m \in M_{\Omega'}(\gamma_1, \mu_1)$ for all $\Omega'$ sufficiently large, because for any equilibrium that achieves $m$ in part (i) of the definition we can set $\Omega' \geq T$ and choose $I = 0$ in part (ii). As a result,

$$\overline{M}_\Omega(\gamma_1, \mu_1) \leq \overline{M}_\infty(\gamma_1, \mu_1), \tag{19}$$

where $\overline{M}_\infty(\gamma_1, \mu_1)$ is achieved by the optimal delay mechanism given by Theorem.

Combining the upper-bound given by (19) with the recursive structure of tests in part (ii) of the definition of $\Omega$-redesign proof, we have the following algorithm for showing that the optimal delay mechanisms in Theorem are robust in the sense that the same ex ante average payoffs are achievable when the number of rounds of delays that can be credibly committed to is at most a relatively small number $\Omega$. For any fixed initial beliefs $(\gamma_1, \mu_1)$, we find the smallest number $\Omega$, which may depend on $(\gamma_1, \mu_1)$, such that there is a delay game $((\gamma_1, \mu_1); \{\delta_t\}_{t=1}^{T})$ with an equilibrium $\left(\{(\gamma_t, \mu_t), (U_t, V_t)\}_{t=1}^{T+1}; \{(x_t, y_t)\}_{t=1}^{T}\right)$ and an increasing sequence of integers $\{k_i\}_{i=0}^{I}$ with $k_0 = 0$, $k_i - k_{i-1} \leq \Omega$ for each $i = 1, \ldots, I$, and $0 \leq T - k_I \leq \Omega$, satisfying

$$\frac{\mu_{k_i+1}}{1 - \gamma_{k_i+1} + \mu_{k_i+1}} U_{k_i+1} + \frac{1 - \gamma_{k_i+1}}{1 - \gamma_{k_i+1} + \mu_{k_i+1}} V_{k_i+1} = \overline{M}_\infty(\gamma_{k_i+1}, \mu_{k_i+1}) \tag{20}$$

for all $i = 0, \ldots, I$. The above conditions (20) imply that the equilibrium passes each test for robustness against redesign (for $i = 1, \ldots, I$), and that it achieves the upper-bound (19) on the ex ante average payoff (for $i = 0$). By definition, the function $\overline{M}_\infty$ is optimal $\Omega'$-redesign proof for any $\Omega'$ greater than or equal to the maximum number $\Omega$ found across all $(\gamma_1, \mu_1)$. We now apply the algorithm by dividing initial beliefs $(\gamma_1, \mu_1)$ into the same four cases as in Theorem. Let $\omega^*$ be the smallest integer $\omega$ such that

$$\omega\Delta \geq \gamma^* \lambda_{LL}.$$

For initial beliefs that fall into Cases (a) and (d) of Theorem, the optimal delay mechanism involves a single round. This mechanism is clearly optimal 1-redesign proof. Only the minimum commitment with regard to the number of rounds is needed when $\gamma_1$ is sufficiently low or high.

For initial beliefs that fall into Case (b) of Theorem, the optimal delay mechanism constructed in Section 5.1 is dynamic. It has a single active round with no slack and a deadline round $T = [2]$. Since the effective delay $\sigma_1$ in round 1 satisfies (7), and since $\gamma_{[2]} < \gamma^*$, we can implement this equilibrium using a delay mechanism with $\omega^* + 2$ rounds, where $\delta_1 = \Delta$, $\delta_t = (\sigma_1 - \Delta)/\omega^* < \Delta$ for each $t = 2, \ldots, \omega^* + 1$, and $\delta_{\omega^*+2}$ is given by (9). This equilibrium then satisfies conditions (20) with $I = 1$ and $k_1 = \omega^* + 1$, because the single test occurs at the deadline round with efficient concession by type $L$, and because (19) holds. It follows that the equilibrium is optimal $(\omega^* + 1)$-redesign proof. By the definition $\gamma^*$, if $\lambda_{LH}$ is close to 0 then $\omega^*$ is equal to 1, so the requirement for commitment with regards to the number of rounds is not overly stringent. Nonetheless, we require at least 2 rounds of commitment because the optimal delay mechanism in this case is dynamic. Having no slack in the first round means that after a regular disagreement and the maximum delay $\Delta$, an effective delay of up to $\omega^*\Delta$ is incurred when there is no concession by type $L$. Part or the whole of this effective delay would be "redesigned away" if there were no commitment all the way to the beginning of the deadline round.

For Case (c) of Theorem, in the equilibrium constructed in Section 5.1, for $n^* \geq 2$ given by (3), the first $n^* - 1$ active rounds have no slack, round $n^*$ generally has slack, and $T = [n^* + 1]$. We can implement this equilibrium with $2n^* + \omega^*$ rounds, where $\delta_{2t-1} = \Delta$ and $\delta_{2t} < \Delta$ is given by (10) for each $t = 1, \ldots, n^* - 2$, corresponding to the $t$-th start-and-stop cycle; $\delta_{2n^*-3} = \Delta$ and $\delta_{2n^*-2} < \Delta$ is given by (11), corresponding to $(n^* - 1)$-th start-and-stop cycle; $\delta_{2n^*-1} = (\eta - 1)\lambda_{LL} \leq \Delta$ and $\delta_t = \gamma^* \lambda_{LL}/\omega^* \leq \Delta$ for each $t = 2n^*, \ldots, 2n^* + \omega^* - 1$ so that $\sum_{t=2n^*-1}^{2n^*+\omega^*-1} \delta_t$ is given by (12) and corresponds to the last active round with slack; and finally $\delta_{2n^*+\omega^*} = \Delta$, corresponding to the deadline round. Let the sequence of integers be $k_i = 2i$ for $i = 1, \ldots, n^* - 2$ and $k_{n^*-1} = 2n^* + \omega^* - 1$, with $I = n^* - 1$. The length of the test is 2 for the first $n^* - 3$ tests, $\omega^* + 3$ for the $(n^* - 2)$-th test, and 1 for the last test. By construction, the continuation equilibrium at the last test is 1-redesign proof (because of efficient deadline concession by type $L$); the continuation beliefs at the beginning of each of first $n^* - 2$ tests fall into Case (c) of Theorem and the corresponding equilibrium is optimal $(\omega^* + 3)$-redesign proof. All conditions (20) are satisfied, and so the mechanism constructed above is optimal $(\omega^* + 3)$-redesign proof.

Compared to Case (b) above, we require 2 more rounds of commitment. This reason is that $n^* \geq 2$ in Case (c) means that there is generally one active round with slack in any optimal delay mechanism.[22] By Proposition 3, the active round with slack cannot be the first round, but this also implies that it may not be the first round in any test for redesign proof-ness. In the above construction, the length of $(n^* - 2)$-th test is $\omega^* + 3$, which includes not only the last active round without slack (round $2n^* - 3$ and round $2n^* - 2$) but also the one with slack (round $2n^* - 1$ through to round $2n^* + \omega^* - 1$). Imagine that we add one test at the end of round $2n^* - 2$, and attempt to show the continuation equilibrium is $(\omega^* + 1)$-redesign proof. Since $\gamma_{2n^*-1}$ is equal to $\gamma_{(n^*-1)}$, which as given in the construction of the optimal delay mechanism in Section 5.1 lies between $\gamma^*$ and $\Gamma^{-1}(\gamma^*)$ and therefore belongs to either Case (a) or Case (b) analyzed above, the highest continuation average payoff is given by either (5) or (8), and can be achieved with an $(\omega^* + 1)$-redesign proof equilibrium. Since round $2n^* - 1$ has slack, By Proposition 3, the continuation equilibrium average payoff $W_{2n^*-1}$ falls short of the highest continuation payoff that is implementable with an $(\omega^* + 1)$-redesign proof mechanism. Therefore, the equilibrium we have constructed fails the test for $(\omega^* + 1)$-redesign proof.

The definition of $\Omega$-redesign proof we have come up with has a recursive structure that is similar to the characterization of subgame perfect equilibrium payoffs in repeated games (Abreu, Pearce and Stacchetti, 1990). Instead of discounting, the recursive structure we have is based on the evolution of the beliefs. For $\Omega = 1$, our definition is also related to the literature on the "ratchet effect" in dynamic contracts (Freixas, Guesnerie and Tirole, 1985; Laffont and Tirole, 1988). The complete characterization of $\Omega$-redesign proof payoffs for any fixed $\Omega \geq 1$ in the committee-decision making and related contexts is beyond the scope of the present paper. We leave it to future research.

[22] When the initial belief $\gamma_1$ is such that $\eta = (\lambda_{LL} + \Delta)/\Delta$, all active rounds have no slack, and the above mechanism is $(\omega^* + 1)$-redesign proof.

# 7. Appendix

## 7.1. Proof of Proposition 1

First, suppose that there is an equilibrium with an infinite number of active rounds. Then, $x_t > 0$ for all $t$. By Lemma 1, $y_t = 1$ for all $t$. In any such mechanism $\gamma_t$ decreases in $t$. Since it is bounded from below by 0, $\gamma_t$ converges and persisting for all $t$ is optimal for type $L$. If the limit of $\gamma_t$ is strictly positive, Bayes' rule implies $\lim_{n\to\infty} \prod_{t=\tau}^{\tau+n} x_t$ can be made arbitrarily close to 1 by taking $\tau$ sufficiently large. However, in round $\tau$, always persisting results in no alternative being implemented with probability close to 1 and yields a payoff to type $H$ strictly lower than the payoff from implementing any alternative, contradicting the equilibrium condition. If $\gamma_t$ converges to zero instead, then for $t$ large enough the expected payoff to type $L$ from conceding is close to the highest possible payoff of $\underline{\pi}_{LH}$, while the strategy of persisting from $t$ onward leads to no alternative being implemented with probability close to 1, again a contradiction.

Next, suppose that there is a finite number, say $n$, of active rounds followed by an exiting round $[n+1]$. Since type $L$ weakly prefers conceding to persisting in round $[n+1]$,

$$\begin{aligned} &\gamma_{[n+1]}\phi_{LL} + \left(1-\gamma_{[n+1]}\right)\left(y_{[t+1]}\underline{\pi}_{LH} + \left(1-y_{[n+1]}\right)\phi_{LH}\right) \\ &\geq \gamma_{[n+1]}\overline{\pi}_{LL} + \left(1-\gamma_{[n+1]}\right)\left(y_{[t+1]}\left(-\delta_{[n+1]}+\hat{U}\right) + \left(1-y_{[n+1]}\right)\overline{\pi}_{LH}\right), \end{aligned}$$

where $\hat{U}$ is type $L$'s continuation payoff after deviating to persisting in round $[n+1]$. Since round $[n+1]$ is the exiting round for type $L$, and since type $L$ can always concede with probability one in the round after $[n+1]$, we have $\hat{U} \geq \phi_{LH}$. By the definition of $\gamma^*$, the above implies that $\gamma_{[n+1]} \leq \gamma^*$. Since $\gamma_1 > \gamma^*$ by assumption, we have $n \geq 1$. We have already made the truncation argument for the case of $y_{[n+1]} = 1$ in the text, so we now assume $y_{[n+1]} < 1$. There are two cases.

Case (i): $\gamma_{(n)} > \gamma_*$.

From the indifference condition of type $L$ in round $(n)$, we have

$$\left(\gamma_{(n)}x_{(n)} + 1 - \gamma_{(n)}\right)\sigma_{(n)} = \gamma_{(n)}\lambda_{LL} + \left(1-\gamma_{(n)}\right)\left(1-y_{[n+1]}\right)\lambda_{LH}.$$

Since $\gamma_{(n)} > \gamma_*$ and $y_{[n+1]} > 0$, there exists $\tilde{\sigma}_{(n)} \in (0, \sigma_{(n)})$ such that

$$\left(\gamma_{(n)}x_{(n)} + 1 - \gamma_{(n)}\right)\tilde{\sigma}_{(n)} = \gamma_{(n)}\lambda_{LL} + \left(1-\gamma_{(n)}\right)\lambda_{LH}.$$

Now, we reduce the effective delay in round $(n)$ to $\tilde{\sigma}_{(n)}$ and replace the exiting round $[n+1]$ with a coin flip. Then, in the modified mechanism, assuming that $\tilde{y}_{(n)} = 1$, the above equation implies that it is an equilibrium for type $L$ to persist in round $(n)$ with the same probability $x_{(n)}$, with the same payoff $U_{(n)}$ as in the original equilibrium. By Lemma 1, it remains an equilibrium for type $H$ to persist in round $(n)$, so indeed we have $\tilde{y}_{(n)} = 1$. Since $y_{(n)} = 1$ and $y_{[n+1]} \in (0, 1)$, the expected payoff $V_{(n)}$ to type $H$ in the original equilibrium, from persisting in round $(n)$ and conceding in round $[n+1]$, is

$$\mu_{(n)} \left( x_{(n)} \left( -\sigma_{(n)} + \phi_{HL} \right) + (1 - x_{(n)}) \overline{\pi}_{HL} \right) + (1 - \mu_{(n)}) \left( -\sigma_{(n)} + \phi_{HH} - y_{[n+1]} \lambda_{HH} \right).$$

This is strictly less than what type $H$ gets in the modified mechanism, given by

$$\tilde{V}_{(n)} = \mu_{(n)} \left( x_{(n)} \left( -\tilde{\sigma}_{(n)} + \phi_{HL} \right) + \left( 1 - x_{(n)} \right) \overline{\pi}_{HL} \right) + (1 - \mu_{(n)}) \left( -\tilde{\sigma}_{(n)} + \phi_{HH} \right).$$

Case (ii): $\gamma_{(n)} \leq \gamma_*$.

We can modify the delay mechanism by reducing the delay $\sigma_{(n)}$ in round $(n)$ to 0, with $\tilde{x}_{(n)} = 0$, and truncating the game afterwards. The expected payoff to type $L$ in round $(n)$ increases as a result, and we can neutralize the change by adding to the effective delay in round $(n-1)$. We omit the the computations as they are similar to case (i).

### 7.2. Proof of Proposition 2

Fix a delay mechanism with a finite number $n$ of active rounds and a deadline round $[n+1]$, with $n \geq 1$. Suppose that $x_{[n+1]} > 0$. By Lemma 1, we have $y_{[n+1]} = 1$. For now, we assume that either round $(n)$ has slack, or $(n) + 1 = [n+1]$.

Case (i): Type $L$ strictly prefers to persist in round $[n+1]$.

First, we add an auxiliary round $s$ between $(n)$ and $(n)+1$, with delay

$$\delta_s = U_{[n+1]} - \left( \gamma_{[n+1]} \underline{\pi}_{LL} + \left( 1 - \gamma_{[n+1]} \right) \underline{\pi}_{LH} \right) - \sum_{t=(n)+1}^{[n+1]-1} \delta_t,$$

and subtract $\delta_s$ from $\delta_{(n)}$. We claim that this is feasible because $\delta_s < \delta_{(n)}$. To see this, we write the indifference condition of type $L$ in round $(n)$ as

$$\begin{aligned} &\gamma_{(n)} \left( x_{(n)} \left( -\sigma_{(n)} + U_{[n+1],L} \right) + \left( 1 - x_{(n)} \right) \overline{\pi}_{LL} \right) + \left( 1 - \gamma_{(n)} \right) \left( -\sigma_{(n)} + U_{[n+1],H} \right) \\ = &\gamma_{(n)} \left( x_{(n)} \underline{\pi}_{LL} + \left( 1 - x_{(n)} \right) \phi_{LL} \right) + \left( 1 - \gamma_{(n)} \right) \underline{\pi}_{LH}, \end{aligned}$$

where $U_{[n+1],L} = -\delta_{[n+1]} + \phi_{LL}$ and $U_{[n+1],H} = -\delta_{[n+1]} + \phi_{LH}$. Since $\lambda_{LL} > \lambda_{LH}$ by Assumption 2, we have $U_{[n+1],L} - U_{[n+1],H} > \underline{\pi}_{LL} - \underline{\pi}_{LH}$. For fixed $\gamma_{(n)}$, both the left-hand side and the right-hand side are decreasing in $x_{(n)}$, but the left-hand side decreases faster. For $x_{(n)} \in (0,1)$ to be an equilibrium, the left-hand side must be strictly less than the right-hand side at $x_{(n)} = 1$:

$$\gamma_{(n)}\left(-\sigma_{(n)} + U_{[n+1],L}\right) + \left(1-\gamma_{(n)}\right)\left(-\sigma_{(n)} + U_{[n+1],H}\right) < \gamma_{(n)}\underline{\pi}_{LL} + \left(1-\gamma_{(n)}\right)\underline{\pi}_{LH}.$$

Since $x_{(n)} \in (0,1)$ implies $\gamma_{(n)} > \gamma_{[n+1]}$, and since $U_{[n+1],L} - U_{[n+1],H} > \underline{\pi}_{LL} - \underline{\pi}_{LH}$, the above inequality implies:

$$\gamma_{[n+1]}\left(-\sigma_{(n)} + U_{[n+1],L}\right) + \left(1-\gamma_{[n+1]}\right)\left(-\sigma_{(n)} + U_{[n+1],H}\right) < \gamma_{[n+1]}\underline{\pi}_{LL} + \left(1-\gamma_{[n+1]}\right)\underline{\pi}_{LH}.$$

On the other hand, by construction, we have

$$\gamma_{[n+1]}\left(-\sigma_s + U_{[n+1],L}\right) + \left(1-\gamma_{[n+1]}\right)\left(-\sigma_s + U_{[n+1],H}\right) = \gamma_{[n+1]}\underline{\pi}_{LL} + \left(1-\gamma_{[n+1]}\right)\underline{\pi}_{LH}.$$

Thus, $\sigma_s < \sigma_{(n)}$, which equivalent to $\delta_s < \delta_{(n)}$. (If there is no slack in round $(n)$, we have $\delta_s = 0$ and $\delta_{(n)} = \Delta$.) It remains an equilibrium for type $L$ to play the same strategy up to round $(n)$, and then persist with probability one in round $s$, followed by persisting in round $(n)+1$ to $[n+1]$. Moreover, by construction, type $L$ is indifferent between persisting and conceding in round $s$, with $\gamma_s = \gamma_{[n+1]}$.

Second, we marginally increase $\delta_s$, which is feasible because $\delta_s < \delta_{(n)} \leq \Delta$ initially, and construct a new continuation equilibrium for fixed $\gamma_s$, with probability $x_s$ marginally lower than one. We claim that it is possible for type $L$ to remain indifferent between persisting and conceding in round $s$, so a lower $x_s$ means a higher $U_s$, obtained from conceding. For any fixed $\gamma_{[n+1]}$ in the original mechanism, since type $L$ strictly prefers to persist in round $(n)+1$ to $[n+1]-1$, the indifference condition of type $L$ in round $s$ is

$$\gamma_{[n+1]}\left(x_s\left(-\delta_s - \sum_{t=(n)+1}^{[n+1]-1}\delta_t + U_{[n+1],L} - \underline{\pi}_{LL}\right) + (1-x_s)(\overline{\pi}_{LL} - \phi_{LL})\right)$$
$$= \left(1-\gamma_{[n+1]}\right)\left(\delta_s + \sum_{t=(n)+1}^{[n+1]-1}\delta_t - U_{[n+1],H} + \underline{\pi}_{LH}\right).$$

A marginal decrease in $x_s$ results a marginally lower deadline belief $\tilde{\gamma}_{[n+1]}$, but type $L$ still strictly prefers to persist in round $(n)+1$ to round $[n+1]$. Thus, $U_{[n+1],L} = -\delta_{[n+1]} + \phi_{LL}$, and taking derivative of the indifference condition with respect to $x_s$, we have

$$\left(\gamma_{[n+1]}x_s + 1 - \gamma_{[n+1]}\right)\frac{\mathrm{d}\delta_s}{\mathrm{d}x_s} = -\gamma_{[n+1]}\left(\delta_s + \sum_{t=(n)+1}^{[n+1]}\delta_t\right) < 0.$$

Third, we add to the delay in round $(n)$ by the amount of the increase in $U_s$ (insert another auxiliary round between $(n)$ and $s$ if $\delta_s = 0$ and $\delta_{(n)} = \Delta$ in the first step), so that it remains an equilibrium for type $L$ to play in the same way up to round $(n)$ as in the original mechanism, with the same belief $\gamma_s$ after round $(n)$ and the same expected payoff. The desired contradiction then follows from the Linkage Lemma.

Case (ii): Type $L$ is indifferent between persisting and conceding in round $[n+1]$.

We follow the same two steps as in case (i): first use the indifference condition of type $L$ in round $(n)$ to show that $\delta_s < \delta_{(n)}$, and then use the indifference condition in round $s$ to show $\mathrm{d}\delta_s/\mathrm{d}x_s < 0$. We no longer have explicit expressions for $U_{[n+1],L}$ and $U_{[n+1],H}$; instead, we must use the indifference condition of type $L$ in round $[n+1]$. Other than this, the calculations are similar to case (i) and thus omitted.

Finally, if round $(n)$ has no slack and there is at least one round, $(n)+1$, before the deadline round $[n+1]$, then the auxiliary round $s$ is inserted after $(n)+1$. Since type $L$ is indifferent between persisting and conceding in round $(n)+1$ with $x_{(n)+1} = 1$ and hence $\gamma_{(n)+1} = \gamma_{[n+1]}$, in the first step we have $\delta_s = \delta_{(n)+1}$. If $\delta_{(n)+1} < \Delta$, the second step is feasible and the rest of the argument goes through without change. If instead $\delta_{(n)+1} = \Delta$, we insert another auxiliary round $s'$ after $s$ and in the second step we increase $\delta_{s'}$ marginally from zero. The rest of the argument goes through again.

### 7.3. Proof of Proposition 3

Case (i): There are $n \geq 2$ active rounds.

Suppose that there is slack in round 1. Using the payoff from concession in round (2), we can write the indifference condition in round 1 for type $L$ as

$$\gamma_1\left(1 - x_1x_{(2)}\right)\lambda_{LL} = (1 - \gamma_1 + \gamma_1x_1)\sigma_1.$$

For fixed $\gamma_1$ and $x_1x_{(2)}$, we have

$$\frac{\mathrm{d}\sigma_1}{\mathrm{d}x_1} = -\frac{\gamma_1^2\left(1 - x_1x_{(2)}\right)\lambda_{LL}}{(1 - \gamma_1 + \gamma_1x_1)^2} < 0.$$

Lowering $x_1$ thus requires raising $\sigma_1$. Since there is slack in round 1, raising $\sigma_1$ is feasible. Similarly, the indifference condition in round $(2)$ can be written as

$$\gamma_1 x_1 \lambda_{LL} + \gamma_1 x_1 x_{(2)} \lambda_{LL} = \left(1 - \gamma_1 + \gamma_1 x_1 x_{(2)}\right) \left(\sigma_{(2)} - U_{(2)}\right) + \gamma_1 \overline{\pi}_{LL} + (1 - \gamma_1) \underline{\pi}_{LH}.$$

For fixed $\gamma_1$, $x_1 x_{(2)}$, and $U_{(2)}$, we have

$$\frac{\mathrm{d}\sigma_{(2)}}{\mathrm{d}x_1} = \frac{\gamma_1 \lambda_{LL}}{1 - \gamma_1 + \gamma_1 x_1 x_{(2)}} > 0.$$

Lowering $x_1$ thus requires reducing $\sigma_{(2)}$. Since $\sigma_{(2)} > 0$ in the original mechanism, this step is also feasible. The effect on the payoff to type $L$ is

$$\frac{\mathrm{d}U_1}{\mathrm{d}x_1} = -\gamma_1 \lambda_{LL} < 0.$$

Thus, lowering $x_1$, while holding $x_1 x_{(2)}$ constant, increases the payoff to type $L$.

By Lemma 2, the effect of a change in $x_1$, while holding $x_1 x_{(2)}$ constant, on the payoff of type $H$ is given by:

$$\frac{\mathrm{d}V_1}{\mathrm{d}x_1} = \frac{\mu_1}{\gamma_1} \frac{\mathrm{d}U_1}{\mathrm{d}x_1} + \frac{\mu_1 - \gamma_1}{\gamma_1} \left( \frac{\mathrm{d}\sigma_1}{\mathrm{d}x_1} + \frac{\mathrm{d}\sigma_{(2)}}{\mathrm{d}x_1} \right) = \frac{\mu_1 - \gamma_1}{\gamma_1} \frac{\mathrm{d}\sigma_1}{\mathrm{d}x_1} + \left( -\mu_1 + \frac{\mu_1 - \gamma_1}{1 - \gamma_1 + \gamma_1 x_1 x_{(2)}} \right) \lambda_{LL},$$

which is strictly negative. Hence, both type $H$ and type $L$ are better off.

Case (ii): There is a single active round before deadline round [2].

By Proposition 2, we have $x_{[2]} = 0$ and $\gamma_{[2]} \leq \gamma^*$. It is straightforward to use the indifference condition for type $L$ to show that $U_1$ is maximized when $x_1$ is minimized. Given that $x_{[2]} = 0$, it is optimal to set $\delta_{[2]}$ such that type $L$ is indifferent between conceding and persisting at $x_{[2]} = 0$. This gives an expression for $\delta_{[2]}$, which we can then use to derive an expression for $V_1$ and show that it is maximized when $x_1$ is minimized.

### 7.4. Proof of Proposition 4

Fix a delay mechanism with an equilibrium where type $L$ concedes in deadline round $[n+1]$ at deadline belief $\gamma_{[n+1]} < \gamma^*$. Denote the last two active rounds before $[n+1]$ as $(n-1)$ and $(n)$. We modify the original mechanism by decreasing the delay in round $(n)$. From type $L$'s indifference condition in round $(n)$, for the same belief $\gamma_{(n)}$, this would result in a marginally greater probability of persistence $\tilde{x}_{(n)}$. Since type $L$'s payoff from conceding decreases as $\tilde{x}_{(n)}$ increases, the equilibrium payoff $\tilde{U}_{(n)}$ in round $(n)$ becomes lower. As a result, it is possible to reduce the delay in round $(n-1)$ to neutralize the increase in type $L$'s payoff in round $(n)$, keeping the equilibrium play of type $L$ unchanged up to round $(n-1)$, with the same $x_{(n-1)}$ and hence the same $\gamma_{(n)}$. Furthermore, for the same $\gamma_{(n)}$, type $L$'s belief entering round $[n+1]$ increases as $\tilde{x}_{(n)}$ increases. So long as the new belief $\tilde{\gamma}_{[n+1]} \leq \gamma^*$, we can raise the delay in round $[n+1]$ to make it an equilibrium for type $L$ to concede with probability one. By Lemma 2, we have

$$\frac{\tilde{V}_{(n-1)} - V_{(n-1)}}{\mu_{(n-1)}} = x_{(n-1)} \left(\tilde{x}_{(n)} - x_{(n)}\right) \lambda_{LL} + \left(\frac{1}{\gamma_{(n-1)}} - \frac{1}{\mu_{(n-1)}}\right) \left(\tilde{\sigma}_{(n-1)} - \sigma_{(n-1)} + \tilde{\sigma}_{(n)} - \sigma_{(n)}\right)$$
$$+ \left(\frac{1}{\mu_{(n-1)}} - 1\right) \left(\tilde{V}_{[n+1],H} - V_{[n+1],H}\right).$$

To evaluate the above equation, we express the changes in $x_{(n)}$, $\sigma_{(n-1)}$, $\sigma_{(n)}$, and $V_{[n+1],H}$ in terms of $\tilde{\gamma}_{[n+1]}$, while keeping $U_{(n-1)}$, $\gamma_{(n-1)}$ and $x_{(n-1)}$, and hence $\gamma_{(n)}$, at their respective original equilibrium values.

By Bayes' rule, the increase in $x_{(n)}$ is given by

$$\tilde{x}_{(n)} - x_{(n)} = \frac{1-\gamma_{(n)}}{\gamma_{(n)}} \left(\frac{\tilde{\gamma}_{[n+1]}}{1-\tilde{\gamma}_{[n+1]}} - \frac{\gamma_{[n+1]}}{1-\gamma_{[n+1]}}\right).$$

For fixed belief $\gamma_{(n)}$, the decrease in type $L$'s equilibrium payoff in round $(n)$ is

$$\tilde{U}_{(n)} - U_{(n)} = -\gamma_{(n)} \left(\tilde{x}_{(n)} - x_{(n)}\right) \lambda_{LL}.$$

To keep $U_{(n-1)}$ unchanged with the same $x_{(n-1)}$, we reduce the effective delay in $(n-1)$:

$$\tilde{\sigma}_{(n-1)} - \sigma_{(n-1)} = \tilde{U}_{(n)} - U_{(n)} = -\left(1-\gamma_{(n)}\right) \left(\frac{\tilde{\gamma}_{[n+1]}}{1-\tilde{\gamma}_{[n+1]}} - \frac{\gamma_{[n+1]}}{1-\gamma_{[n+1]}}\right) \lambda_{LL}.$$

By construction, $\tilde{x}_{[n+1]} = 0$, with

$$\tilde{U}_{[n+1]} = \tilde{\gamma}_{[n+1]}\phi_{LL} + \left(1-\tilde{\gamma}_{[n+1]}\right) \underline{\pi}_{LH}.$$

Using the above expression and Bayes' rule, we can write the indifference condition for type $L$ in round $(n)$ as

$$\tilde{\sigma}_{(n)} = \gamma_{(n)} \frac{1-\tilde{\gamma}_{[n+1]}}{1-\gamma_{(n)}} \lambda_{LL}.$$

Thus, the decrease in the effective delay in round $(n)$ is

$$\tilde{\sigma}_{(n)} - \sigma_{(n)} = -\gamma_{(n)} \frac{\tilde{\gamma}_{[n+1]} - \gamma_{[n+1]}}{1 - \gamma_{(n)}} \lambda_{LL}.$$

Finally, since in round $[n+1]$ type $L$ weakly prefers conceding to persisting under the original mechanism, if we make type $L$ indifferent under the new mechanism, the increase in the delay in round $[n+1]$ is

$$\tilde{\delta}_{[n+1]} - \delta_{[n+1]} \leq \left( \frac{\tilde{\gamma}_{[n+1]}}{1 - \tilde{\gamma}_{[n+1]}} - \frac{\gamma_{[n+1]}}{1 - \gamma_{[n+1]}} \right) \lambda_{LL}.$$

Thus, the decrease in the payoff to type $H$ against $H$ in round $[n+1]$ is

$$\tilde{V}_{[n+1],H} - V_{[n+1],H} = -\left( \tilde{\delta}_{[n+1]} - \delta_{[n+1]} \right) \geq -\left( \frac{\tilde{\gamma}_{[n+1]}}{1 - \tilde{\gamma}_{[n+1]}} - \frac{\gamma_{[n+1]}}{1 - \gamma_{[n+1]}} \right) \lambda_{LL}.$$

Adding up the terms in $(\tilde{V}_{(n-1)} - V_{(n-1)})/\mu_{(n-1)}$ that depend on $\mu_{(n-1)}$, we have

$$\begin{aligned} &\tilde{\sigma}_{(n-1)} - \sigma_{(n-1)} + \tilde{\sigma}_{(n)} - \sigma_{(n)} + \tilde{\delta}_{[n+1]} - \delta_{[n+1]} \\ &= \left( \frac{1}{\left(1 - \tilde{\gamma}_{[n+1]}\right)\left(1 - \gamma_{[n+1]}\right)} - \frac{1}{1 - \gamma_{(n)}} \right) \frac{\gamma_{(n)}}{1 - \gamma_{(n)}} \lambda_{LL}. \end{aligned}$$

If $\gamma_{(n)} \leq \gamma^*$, we set $\tilde{\gamma}_{[n+1]} = \gamma_{(n)}$; in this case, the above change in the sum of the delays from round $(n-1)$ to $[n+1]$ is clearly positive. If instead $\gamma_{(n)} > \gamma^*$, we set $\tilde{\gamma}_{[n+1]} = \gamma^*$; in this case, the change in the total delay has the same sign as

$$\left(1 - \gamma_{(n)}\right) - (1 - \gamma^*) \left(1 - \gamma_{[n+1]}\right) \geq \left(1 - \gamma_{(n)}\right) - (1 - \gamma^*) \left(1 - \Gamma(\gamma_{(n)})\right) > 0,$$

where the first inequality follows from $\gamma_{[n+1]} \geq \Gamma(\gamma_{(n)})$, and the second from the definitions of $\gamma^*$ and $\Gamma$. Since $\gamma_{(n-1)} < \mu_{(n-1)}$ by Assumption 1 and Lemma 1, evaluating the three terms in $(\tilde{V}_{(n-1)} - V_{(n-1)})/\mu_{(n-1)}$ at $\mu_{(n-1)} = \gamma_{(n-1)}$, and thus discarding the second term, we have

$$\frac{\tilde{V}_{(n-1)} - V_{(n-1)}}{\mu_{(n-1)}} > \left( x_{(n-1)} \frac{1 - \gamma_{(n)}}{\gamma_{(n)}} - \frac{1 - \gamma_{(n-1)}}{\gamma_{(n-1)}} \right) \left( \frac{\tilde{\gamma}_{[n+1]}}{1 - \tilde{\gamma}_{[n+1]}} - \frac{\gamma_{[n+1]}}{1 - \gamma_{[n+1]}} \right) \lambda_{LL}.$$

The right-hand side is zero by Bayes' rule, implying that $\tilde{V}_{(n-1)} > V_{(n-1)}$.

### 7.5. Proof of Proposition 5

We first show that no optimal delay mechanism can have two consecutive active rounds with slack. Using the indifference condition in round $(i)$ and applying Bayes' rule we have

$$\sigma_{(i)} = \frac{\gamma_{(i)}\left(1 - x_{(i)}x_{(i+1)}\right)}{1 - \gamma_{(i)} + \gamma_{(i)}x_{(i)}}\lambda_{LL} = \frac{\left(\gamma_{(i)} - \gamma_{(i+2)}\right)\left(1 - \gamma_{(i+1)}\right)}{\left(1 - \gamma_{(i)}\right)\left(1 - \gamma_{(i+2)}\right)}\lambda_{LL}.$$

Similarly, for fixed $\gamma_{(i)}$ and $\gamma_{(i+2)}$, using the indifference condition in round $(i+1)$ and applying Bayes' rule repeatedly we have

$$\begin{aligned}\sigma_{(i+1)} &= \frac{\gamma_{(i)}x_{(i)}}{1 - \gamma_{(i)} + \gamma_{(i)}x_{(i)}x_{(i+1)}}\lambda_{LL} + \frac{\gamma_{(i)}x_{(i)}x_{(i+1)}}{1 - \gamma_{(i)} + \gamma_{(i)}x_{(i)}x_{(i+1)}} + U_{(i+2)} - 1 \\ &= \frac{\gamma_{(i+1)}\left(1 - \gamma_{(i+2)}\right)}{1 - \gamma_{(i+1)}}\lambda_{LL} + \text{constant}.\end{aligned}$$

Finally, to keep $\gamma_{(i)}$ fixed, we need to adjust $\sigma_{(i-1)}$ to make sure that the continuation value $U_{(i)} - \sigma_{(i-1)}$ is held constant. This implies that

$$\sigma_{(i-1)} = -\gamma_{(i)}x_{(i)}\lambda_{LL} + \text{constant} = -\frac{\gamma_{(i+1)}\left(1 - \gamma_{(i)}\right)}{1 - \gamma_{(i+1)}}\lambda_{LL} + \text{constant}.$$

Summing up the three terms, we obtain

$$\sum_{t=(i-1)}^{(i+1)} \sigma_t = \frac{\gamma_{(i)} - \gamma_{(i+2)}}{1 - \gamma_{(i+2)}}\left(\frac{1 - \gamma_{(i+1)}}{1 - \gamma_{(i)}} + \frac{1 - \gamma_{(i+2)}}{1 - \gamma_{(i+1)}}\right)\lambda_{LL} + \text{constant}.$$

This is a strictly convex function of $\gamma_{(i+1)}$. If there is slack in both $(i)$ and $(i+1)$, then $\gamma_{(i+1)}$ is in the interior of the feasible set. As a result, it is possible to increase the total delay $\sum_{t=(i-1)}^{(i+1)} \sigma_t$ from $(i-1)$ through to $(i+1)$, without changing either the total probability $\prod_{\tau=(i-1)}^{(i+1)} x_\tau$ that type $L$ persists or his payoff $U_{(i-1)}$. By Lemma 2, this cannot be optimal.

Evaluating the above expression of the total delay $\sum_{t=(i-1)}^{(i+1)} \sigma_t$ at $\gamma_{(i+1)} = \Gamma^{-1}(\gamma_{(i+2)})$, with no slack in round $(i+1)$, and at $\gamma_{(i+1)} = \Gamma(\gamma_{(i)})$, with no slack in round $(i)$, we obtain the same value of

$$\sum_{t=(i-1)}^{(i+1)} \sigma_t = \frac{\lambda_{LL} + \Delta}{\lambda_{LL}} + \frac{1 - \gamma_{(i+2)}}{1 - \gamma_{(i)}} \frac{\lambda_{LL}}{\lambda_{LL} + \Delta} + \text{constant.}$$

By Lemma 2, it is payoff-equivalent whether to give the slack to round $(i)$ or $(i+1)$. It follows that there can be at most one active round with slack.

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## Online Appendix to "Optimal Delay in Committees"

### A. Proof of Proposition 6

We compare the payoffs in the four cases of Theorem.

*Case (a) vs. Case (b)*

(i) Suppose $\gamma_1 \in [\gamma^*, \Gamma^{-1}(\gamma_*)]$. Note that $\Gamma^{-1}(\gamma_*) < (-\lambda_{LH} + \Delta)/(-\lambda_{LH} + \lambda_{LL})$. Thus, in Case (a) we have $x_1 < 1$ from (6). In Case (b) we have $\delta_{[2]} = 0$ from (9). Using the payoff formulas (5) and (8), we have

$$U_1^{(a)} - U_1^{(b)} = \gamma_1(1 - x_1)\lambda_{LL} - \frac{\lambda_{LL}\Delta}{\lambda_{LL} + \Delta},$$
$$V_1^{(a)} - V_1^{(b)} = -\mu_1 x_1 \lambda_{HL} - (\mu_1 x_1 + 1 - \mu_1)\Delta + \gamma_1 \lambda_{LL} - (\mu_1 - \gamma_1)\Delta.$$

At $\gamma_1 = \gamma^*$, we have $x_1 = 0$. Then,

$$U_1^{(a)} - U_1^{(b)} = \left(\gamma^* - \frac{\Delta}{\lambda_{LL} + \Delta}\right)\lambda_{LL} > 0,$$
$$V_1^{(a)} - V_1^{(b)} = -(1 - \gamma^*)\Delta + \gamma^* \lambda_{LL} > 0.$$

Therefore, $W_1^{(a)} > W_1^{(b)}$ at $\gamma_1 = \gamma^*$ for all $\mu_1$.

At $\gamma_1 = \Gamma^{-1}(\gamma_*)$, we have $x_1 = \chi(\gamma_1)$. Therefore $U_1^{(a)} = U_1^{(b)}$. Furthermore,

$$\begin{aligned} V_1^{(a)} - V_1^{(b)} &= -\mu_1 x_1 \lambda_{HL} - \mu_1 x_1 \Delta + \gamma_1 \lambda_{LL} - (1 - \gamma_1)\Delta \\ &= x_1 \left(-\mu_1 \lambda_{HL} - \mu_1 \Delta + \gamma_1(\lambda_{LL} + \Delta)\right) < 0. \end{aligned}$$

Thus, $W_1^{(a)} < W_1^{(b)}$ at $\gamma_1 = \Gamma^{-1}(\gamma_*)$ for all $\mu_1 > \gamma_1$.

Using (1) for the average payoff, we take the derivative of $(1 - \gamma_1 + \mu_1)(W_1^{(a)} - W_1^{(b)})$ with respect to $\gamma_1$ to get

$$\begin{aligned} &\mu_1(1 - x_1)\lambda_{LL} + \mu_1 x_1 \lambda_{HL} + (\mu_1 x_1 + 1 - \mu_1)\Delta + (1 - 2\gamma_1)\lambda_{LL} + (1 - \gamma_1 + \mu_1 - \gamma_1)\Delta \\ &\quad - (\mu_1 \gamma_1 \lambda_{LL} + \mu_1(1 - \gamma_1)\lambda_{HL} + \mu_1(1 - \gamma_1)\Delta) \frac{\mathrm{d}x_1}{\mathrm{d}\gamma_1} \end{aligned}$$

When $W_1^{(a)} = W_1^{(b)}$, we can substitute out $(\mu_1 x_1 + 1 - \mu_1)\Delta$. Also, using the expression (6) for $x_1$ in Case (a), we can substitute out $\mathrm{d}x_1/\mathrm{d}\gamma_1$. The derivative at the point $W_1^{(a)} = W_1^{(b)}$

can be written as

$$
\begin{aligned}
&\frac{\mu_1}{1-\gamma_1}(1-x_1)\lambda_{LL} + (1-\gamma_1)\lambda_{LL} - \frac{\mu_1}{1-\gamma_1}\frac{\Delta}{\lambda_{LL}+\Delta}\lambda_{LL} + (1-\gamma_1)\Delta \\
&\quad - \left(\frac{\mu_1}{\gamma_1}\lambda_{LL} + \frac{\mu_1(1-\gamma_1)}{\gamma_1^2}\lambda_{HL} + \frac{\mu_1(1-\gamma_1)}{\gamma_1^2}\Delta\right)\frac{-\lambda_{LH}+\Delta}{\Delta} \\
&= \frac{\mu_1}{1-\gamma_1}\lambda_{LL}\frac{(1-\gamma_1)(-\lambda_{LH}+\Delta)(\lambda_{LL}+\Delta)-\gamma_1\lambda_{LL}^2}{\gamma_1(\lambda_{LL}+\Delta)\Delta} + (1-\gamma_1)(\lambda_{LL}+\Delta) \\
&\quad - \left(\frac{\mu_1}{\gamma_1}\lambda_{LL} + \frac{\mu_1(1-\gamma_1)}{\gamma_1^2}(\lambda_{HL}+\Delta)\right)\frac{-\lambda_{LH}+\Delta}{\Delta} \\
&= -\frac{\mu_1\lambda_{LL}^3}{(1-\gamma_1)(\lambda_{LL}+\Delta)\Delta} + (1-\gamma_1)\left((\lambda_{LL}+\Delta) - (\lambda_{HL}+\Delta)\frac{-\lambda_{LH}+\Delta}{\Delta}\frac{\mu_1}{\gamma_1^2}\right) < 0.
\end{aligned}
$$

This shows that $W_1^{(a)} - W_1^{(b)}$ is single-crossing in $\gamma_1$ from above.

The derivative of $(1-\gamma_1+\mu_1)\left(W_1^{(a)} - W_1^{(b)}\right)$ with respect to $\mu_1$ is

$$
\gamma_1(1-x_1)\lambda_{LL} - \frac{\Delta}{\lambda_{LL}+\Delta}\lambda_{LL} - (1-\gamma_1)x_1\lambda_{HL} - (1-\gamma_1)x_1\Delta.
$$

Evaluated at the point where $W_1^{(a)} = W_1^{(b)}$, the above expression is equal to

$$
\begin{aligned}
&(1-\gamma_1)\left(x_1 + \frac{1-\mu_1}{\mu_1}\right)\Delta - (1-\gamma_1)\frac{\gamma_1}{\mu_1}\lambda_{LL} + (1-\gamma_1)\frac{\mu_1-\gamma_1}{\mu_1}\Delta - (1-\gamma_1)x_1\Delta \\
&= -\frac{1-\gamma_1}{\mu_1}(\gamma_1\lambda_{LL} - (1-\gamma_1)\Delta) < 0.
\end{aligned}
$$

where the last inequality follows because $\chi(\gamma_1) > 0$. Thus, $W_1^{(a)} - W_1^{(b)}$ is single-crossing in $\mu_1$ from above.

The above results imply that there exist a value $\underline{\mu} \in (\gamma^*, \Gamma^{-1}(\gamma_*))$ and a function $\underline{g}(\mu_1) \in (\gamma^*, \Gamma^{-1}(\gamma_*))$ defined for all $\mu_1 \in (\gamma^*, 1]$, with $\underline{g}(\mu_1) = \mu_1$ for $\mu_1 \in (\gamma^*, \underline{\mu})$ and $\underline{g}'(\mu_1) < 0$ for $\mu_1 \in [\underline{\mu}, 1]$, such that $W_1^{(a)} \geq W_1^{(b)}$ if $\gamma_1 \in [\gamma^*, \underline{g}(\mu_1)]$, and the opposite inequality holds if $\gamma_1 \in \left(\underline{g}(\mu_1), \min\left\{\Gamma^{-1}(\gamma_*), \mu_1\right\}\right]$.

(ii) Suppose that $\gamma_1 \in \left(\Gamma^{-1}(\gamma_*), \min\{\Gamma^{-1}(\gamma^*), (-\lambda_{LH}+\Delta)/(\lambda_{LL}-\lambda_{LH})\}\right]$. In this region, for Case (a) we have $x_1 > \chi(\gamma_1)$ from (6). Furthermore, for Case (b) we have $\delta_{[2]} > 0$ from (9). From the payoff formulas (5) and (8),

$$
U_1^{(a)} - U_1^{(b)} = \gamma_1(\chi(\gamma_1) - x_1)\lambda_{LL} < 0.
$$

Furthermore,

$$\begin{aligned}
V_1^{(a)} - V_1^{(b)} &= -\mu_1 x_1 \lambda_{HL} - \mu_1 x_1 \Delta + \gamma_1 \lambda_{LL} - (1-\gamma_1)\Delta + (1-\mu_1)\delta_{[2]} \\
&< -\gamma_1 x_1 \lambda_{LL} - \gamma_1 x_1 \Delta + \gamma_1 \lambda_{LL} - (1-\gamma_1)\Delta + (1-\mu_1)\delta_{[2]} \\
&= -\gamma_1 x_1 \lambda_{LL} - (\gamma_1 \lambda_{LL} - (1-\gamma_1)(-\lambda_{LH} + \Delta)) + \gamma_1 \lambda_{LL} - (1-\gamma_1)\Delta \\
&\quad + (1-\mu_1)\left(\frac{\gamma_1}{1-\gamma_1}\chi(\gamma_1)\lambda_{LL} + \lambda_{LH}\right) \\
&= -\gamma_1 x_1 \lambda_{LL} - (1-\gamma_1)\lambda_{LH} + ((1-\gamma_1) - (\mu_1 - \gamma_1))\left(\frac{\gamma_1}{1-\gamma_1}\chi(\gamma_1)\lambda_{LL} + \lambda_{LH}\right) \\
&= (\chi(\gamma_1) - x_1)\gamma_1 \lambda_{LL} - (\mu_1 - \gamma_1)\delta_{[2]} < 0.
\end{aligned}$$

Therefore, $W_1^{(a)} < W_1^{(b)}$ for all $\gamma_1$ and $\mu_1$ in this region.

*Case (b) vs. Case (d)*

(i) Suppose $\gamma_1 \in \left[\gamma^*, \Gamma^{-1}(\gamma_*)\right]$. For Case (b) we have $\delta_{[2]} = 0$ from (9). From the payoff formulas (8) and (15), we have

$$U_1^{(b)} - U_1^{(d)} = -\gamma_1 \lambda_{LL} - (1-\gamma_1)\lambda_{LH} + \frac{\lambda_{LL}\Delta}{\lambda_{LL} + \Delta}.$$

The above is decreasing in $\gamma_1$ and is equal to 0 at $\gamma_1 = \Gamma^{-1}(\gamma_*)$. Thus, $U_1^{(b)} > U_1^{(d)}$ for all $\gamma_1 \in (\gamma^*, \Gamma^{-1}(\gamma_*))$. Furthermore,

$$V_1^{(b)} - V_1^{(d)} = \mu_1 \lambda_{HL} - \gamma_1 \lambda_{LL} + (\mu_1 - \gamma_1)\Delta > 0,$$

because $\mu_1 > \gamma_1$ and $\lambda_{HL} > \lambda_{LL}$. We conclude $W_1^{(b)} > W_1^{(d)}$ for all $\gamma_1, \mu_1$ in this region.

(ii) Suppose $\gamma_1 \in \left(\Gamma^{-1}(\gamma_*), \Gamma^{-1}(\gamma^*)\right]$. For Case (b) we have $\delta_{[2]} > 0$ from (9). From the payoff formulas (8) and (15), we have $U_1^{(b)} - U_1^{(d)}$ is the same as given in (i) above, while

$$\begin{aligned}
V_1^{(b)} - V_1^{(d)} &= \mu_1 \lambda_{HL} - \gamma_1 \lambda_{LL} + (\mu_1 - \gamma_1)\Delta \\
&\quad - (1-\mu_1)\left(\frac{\gamma_1 \lambda_{LL} - (1-\gamma_1)\Delta}{(1-\gamma_1)(\lambda_{LL} + \Delta)}\lambda_{LL} + \lambda_{LH}\right).
\end{aligned}$$

Thus, we have

$$\begin{aligned}
(1-\gamma_1+\mu_1)\left(W_1^{(b)} - W_1^{(d)}\right) &= -\gamma_1 \lambda_{LL} - (1-\gamma_1)\lambda_{LH} + \frac{\Delta}{\lambda_{LL}+\Delta}\lambda_{LL} + \mu_1(1-\gamma_1)\lambda_{HL} \\
&\quad - \gamma_1(1-\gamma_1)\lambda_{LL} + (1-\gamma_1)(\mu_1-\gamma_1)\Delta. \qquad \text{(A.1)}
\end{aligned}$$

The above expression is decreasing in $\gamma_1$ and increasing in $\mu_1$. This means $W_1^{(b)} - W_1^{(d)}$ is single-crossing from above in $\gamma_1$ and from below in $\mu_1$.

*Case (c) vs. Case (d)*

Case (c) is relevant only for $\gamma_1 \geq \Gamma^{-1}(\gamma^*)$. Using (13) and (15), we have

$$(1-\gamma_1+\mu_1)\left(W_1^{(c)} - W_1^{(d)}\right) = \mu_1(\gamma_1 - 1 - \chi(\gamma_1))\lambda_{LL} - \frac{\mu_1(1-\gamma_1)}{\gamma_1}\lambda_{LH} + \mu_1(1-\gamma_1)\lambda_{HL} + \frac{(1-\gamma_1)(\mu_1-\gamma_1)}{\gamma_1}\sum_{t=1}^{[n^*+1]}\delta_t. \tag{A.2}$$

The derivative of $(1-\gamma_1+\mu_1)(W_1^{(c)} - W_1^{(d)})$ with respect to $\mu_1$ is

$$(\gamma_1 - 1 - \chi(\gamma_1))\lambda_{LL} - \frac{1-\gamma_1}{\gamma_1}\lambda_{LH} + (1-\gamma_1)\lambda_{HL} + \frac{1-\gamma_1}{\gamma_1}\sum_{t=1}^{[n^*+1]}\delta_t.$$

Evaluated at the point $W_1^{(c)} = W_1^{(d)}$, the above is equal to

$$\frac{1-\gamma_1}{\mu_1}\sum_{t=1}^{[n^*+1]}\delta_t > 0.$$

Thus, $W_1^{(c)} - W_1^{(d)}$ is single-crossing in $\mu_1$ from below.

Multiply the expression for $(1-\gamma_1+\mu_1)(W_1^{(c)} - W_1^{(d)})$ by $\gamma_1$ and taking derivative with respect to $\gamma_1$, we obtain

$$\mu_1((2\gamma_1-2)\lambda_{LL} + \lambda_{LH} + (1-2\gamma_1)\lambda_{HL}) - (1-\gamma_1+\mu_1-\gamma_1)\sum_{t=1}^{[n^*+1]}\delta_t + (1-\gamma_1)(\mu_1-\gamma_1)\frac{\mathrm{d}}{\mathrm{d}\gamma_1}\left(\sum_{t=1}^{[n^*+1]}\delta_t\right).$$

Evaluating the above at $W_1^{(c)} = W_1^{(d)}$, we can eliminate $\mu_1(1-\gamma_1)\lambda_{HL}$, and from (14) we can get

$$\frac{\mathrm{d}}{\mathrm{d}\gamma_1}\left(\sum_{t=1}^{[n^*+1]}\delta_t\right) = \left(\frac{1}{\eta^2}+1\right)\lambda_{LL}\frac{\mathrm{d}\eta}{\mathrm{d}\gamma_1} = \left(\frac{1}{\eta}+\eta\right)\frac{\lambda_{LL}}{1-\gamma_1}.$$

We can therefore rewrite the derivative of $\gamma_1(1-\gamma_1+\mu_1)(W_1^{(c)} - W_1^{(d)})$ with respect to

$\gamma_1$ as

$$-\mu_1(1-\chi(\gamma_1))\lambda_{LL}-\mu_1\gamma_1(\lambda_{HL}-\lambda_{LL})+\frac{\mu_1}{\gamma_1}\lambda_{LH}-\frac{\mu_1-\gamma_1^2}{\gamma_1}\sum_{t=1}^{[n^*+1]}\delta_t+(\mu_1-\gamma_1)\left(\frac{1}{\eta}+\eta\right)\lambda_{LL}.$$

From the indifference condition of type $L$ in round 1 and the payoff formula (13) for $U_1^{(c)}$, since the total delay $\sum_{t=1}^{[n^*+1]}\delta_t$ is greater than the expected delay when type $L$ plays against another type $L$, we have

$$\sum_{t=1}^{[n^*+1]}\delta_t > \gamma_1(1+\chi(\gamma_1))\lambda_{LL}+(1-\gamma_1)\lambda_{LH}.$$

Thus, the derivative of $\gamma_1(1-\gamma_1+\mu_1)(W_1^{(c)}-W_1^{(d)})$ with respect to $\gamma_1$ is less than

$$\begin{aligned}
&-\mu_1\gamma_1(\lambda_{HL}-\lambda_{LL})-\left(\mu_1(1-\chi(\gamma_1))+(\mu_1-\gamma^2)(1+\chi(\gamma_1))-(\mu_1-\gamma_1)\frac{\eta+1}{\eta}\right)\lambda_{LL}\\
&\quad+\left(\frac{\mu_1}{\gamma_1}-\frac{\mu_1-\gamma^2}{\gamma_1}(1-\gamma_1)\right)\lambda_{LH}\\
&=-\mu_1\gamma_1(\lambda_{HL}-\lambda_{LL})-\left(\mu_1\frac{\eta-1}{\eta}+2\gamma_1(1-\gamma_1)\right)\lambda_{LL}+\left(\mu_1+\gamma_1-\gamma_1^2\right)\lambda_{LH}<0.
\end{aligned}$$

We conclude that the derivative of $\gamma_1(1-\gamma_1+\mu_1)(W_1^{(c)}-W_1^{(d)})$ with respect to $\gamma_1$ is negative when $W_1^{(c)}=W_1^{(d)}$. That is, $W_1^{(c)}-W_1^{(d)}$ is single-crossing in $\gamma_1$ from above.

*Case (b) or Case (c) vs. Case (d)*

Evaluating (A.1), at $\gamma_1=\Gamma^{-1}(\gamma^*)$ and $\mu_1=1$, we find that $W_1^{(b)}<W_1^{(d)}$ if and only if

$$\Gamma^{-1}(\gamma^*)(\lambda_{LL}+\Delta)>\lambda_{HL}. \tag{A.3}$$

At $\mu_1=\gamma_1$, we have the right-hand side of (A.1) is equal to

$$-\gamma_1\lambda_{LL}-(1-\gamma_1)\lambda_{LH}+\frac{\Delta}{\lambda_{LL}+\Delta}\lambda_{LL}+\gamma_1(1-\gamma_1)\lambda_{HL}-\gamma_1(1-\gamma_1)\lambda_{LL}.$$

The above expression is concave in $\gamma_1$ and is non-negative at $\gamma_1=\Gamma^{-1}(\gamma_*)$. Furthermore, it is positive at $\gamma_1=\Gamma^{-1}(\gamma^*)$ if and only if

$$\lambda_{LL}+\frac{\Delta}{\Gamma^{-1}(\gamma^*)}<\lambda_{HL}. \tag{A.4}$$

Evaluating (A.2), at $\gamma_1 = \mu_1 = 1$ we have $W_1^{(c)} < W_1^{(d)}$. At $\gamma_1 = \Gamma^{-1}(\gamma^*)$, by continuity we have $W_1^{(c)} = W_1^{(b)}$.

Since both $W_1^{(c)} - W_1^{(d)}$ and $W_1^{(b)} - W_1^{(d)}$ are single-crossing from above in $\gamma_1$ and from below in $\mu_1$, we have the following: there exist a value $\overline{\mu} \in \left(\Gamma^{-1}(\gamma_*), 1\right)$ and a function $\overline{g}(\mu_1)$ defined for all $\mu_1 \in (\gamma^*, 1]$, with $\overline{g}(\mu_1) = \mu_1$ for all $\mu_1 \in (\gamma^*, \overline{\mu}]$, $\overline{g}'(\mu_1) > 0$ for all $\mu_1 \in (\overline{\mu}, 1]$ and $\overline{g}(1) < 1$, such that $W_1^{(b)} > W_1^{(d)}$ for $\gamma_1 \in \left[\gamma_*, \min\left\{\Gamma^{-1}(\gamma^*), \overline{g}(\mu_1)\right\}\right)$ and $W_1^{(c)} > W_1^{(d)}$ for $\gamma_1 \in \left[\Gamma^{-1}(\gamma^*), \overline{g}(\mu_1)\right)$; $W_1^{(b)} < W_1^{(d)}$ for $\gamma_1 \in \left[\overline{g}(\mu_1), \min\left\{\Gamma^{-1}(\gamma^*), \mu_1\right\}\right)$ and $W_1^{(c)} < W_1^{(d)}$ for $\gamma_1 \in \left(\max\left\{\Gamma^{-1}(\gamma^*), \overline{g}(\mu_1)\right\}, \mu_1\right)$. Furthermore, $\overline{\mu} > \Gamma^{-1}(\gamma^*)$ if and only if (A.4) holds, and $\overline{g}(1) > \Gamma^{-1}(\gamma^*)$ if and only if (A.3) holds.

**B. Proof of Proposition 7**

Recall that $\underline{g}(\mu_1)$ is defined by $W_1^{(a)} - W_1^{(b)} = 0$. We have already shown in the proof of Proposition 6 that the left-hand-side of this equation is decreasing in $\gamma_1$ when it crosses zero. It is straightforward to verify that $\mathrm{d}\chi(\gamma_1)/\mathrm{d}\Delta > \mathrm{d}x_1/\mathrm{d}\Delta$, where $x_1$ in Case (a) is given by (6). Since $U_1^{(a)} - U_1^{(b)} = \gamma_1(\chi(\gamma_1) - x_1)\lambda_{LL}$, we have $U_1^{(a)} - U_1^{(b)}$ is increasing in $\Delta$. Furthermore, taking derivatives of $V_1^{(a)} - V_1^{(b)}$ with respect to $\Delta$, we obtain

$$\begin{aligned}
&- \mu_1(\lambda_{HL} + \Delta)\frac{\mathrm{d}x_1}{\mathrm{d}\Delta} - \mu_1 x - (1-\gamma_1) \\
=& \mu_1(\lambda_{HL} + \Delta)\frac{\gamma_1\lambda_{LL} + (1-\gamma_1)\lambda_{LH}}{\gamma_1\Delta} - \mu_1\frac{\gamma_1\lambda_{LL} - (1-\gamma_1)(-\lambda_{LH} + \Delta)}{\gamma_1\Delta} - (1-\gamma_1) \\
=& \mu_1\frac{\gamma_1\lambda_{LL} + (1-\gamma_1)\lambda_{LH}}{\gamma_1\Delta}\frac{\lambda_{HL}}{\Delta} + \frac{(1-\gamma_1)(\mu_1 - \gamma_1)}{\gamma_1} > 0.
\end{aligned}$$

Hence, $W_1^{(a)} - W_1^{(b)}$ is increasing in $\Delta$. The implicit function theorem then implies that the boundary $\underline{g}(\mu_1)$ shifts to the right when $\Delta$ increases.

Next, we consider $\overline{g}(\mu_1)$, which is defined by the condition that $W_1^{(b)} - W_1^{(d)} = 0$ for $\gamma_1 \leq \Gamma^{-1}(\gamma^*)$, or $W_1^{(c)} - W_1^{(d)} = 0$ for $\gamma_1 > \Gamma^{-1}(\gamma^*)$. We have already shown in the proof of Proposition 6 that the left-hand-side of either equation is single-crossing in $\gamma_1$ from above. Furthermore, since raising $\Delta$ relaxes the constraint of in the optimal delay problem, a larger $\Delta$ increases $W_1^{(b)}$ and $W_1^{(c)}$ but does not affect $W_1^{(d)}$. By the implicit function theorem, $\overline{g}(\mu_1)$ also shifts to the right when $\Delta$ increases.

The critical values $\underline{\Delta}$ and $\overline{\Delta}$ are determined by (A.4) and (A.3) holding as equalities, respectively. It is straightforward to verify $\overline{\mu} \geq \Gamma^{-1}(\gamma^*)$ if $\Delta \leq \underline{\Delta}$, $\overline{\mu} < \Gamma^{-1}(\gamma^*) \leq \overline{g}(1)$ if $\underline{\Delta} < \Delta \leq \overline{\Delta}$, and $\overline{g}(1) < \Gamma^{-1}(\gamma^*)$ if $\Delta > \overline{\Delta}$.

### C. Proof of Proposition 8

First we show that in any delay mechanism,

$$\frac{U_t}{\gamma_t \overline{\pi}_{LL}} > \frac{V_t}{\mu_t \overline{\pi}_{HL}}$$

holds for any round $t$. We use backward induction on (18). Using similar arguments as in Proposition 1, we can show that under discounting, there is a finite number of active rounds and a deadline round, denoted as $[n+1]$, after which a coin flip follows. (We no longer need the assumption that never implementing any alternative gives a payoff that is strictly lower than implementing any alternative to any member in any state.) As a result, under any delay mechanism, the equilibrium payoff of type $L$ at any round $t$ can be obtained by persisting with probability one, from round $t$ onward to either the deadline round $[n+1]$ if $x_{[n+1]} > 0$, or to the round $[n+1]-1$ just before the deadline round. There are thus two cases.

If $x_{[n+1]} > 0$, type $L$'s belief $\gamma_{[n+2]}$ after the deadline round $[n+1]$ is strictly positive, and $U_{[n+2]}$ and $V_{[n+2]}$ are both given by the coin-flip:

$$U_{[n+2]} = \gamma_{[n+2]}\phi_{LL} + (1-\gamma_{[n+2]})\phi_{LH},$$

and

$$V_{[n+2]} = \mu_{[n+2]}\phi_{HL} + (1-\mu_{[n+2]})\phi_{HH}.$$

We have

$$\begin{aligned}
\frac{U_{[n+2]}}{\gamma_{[n+2]}\overline{\pi}_{LL}} &= \frac{\phi_{LL}}{\overline{\pi}_{LL}}\left(1+\frac{1-\gamma_{[n+2]}}{\gamma_{[n+2]}}\frac{\phi_{LH}}{\phi_{LL}}\right) \\
&> \frac{\phi_{LL}}{\overline{\pi}_{LL}}\left(1+\frac{1-\mu_{[n+2]}}{\mu_{[n+2]}}\frac{\phi_{LH}}{\phi_{LL}}\right) \\
&\geq \frac{1}{2}\left(1+\frac{\underline{\pi}_{LL}}{\overline{\pi}_{LL}}\right)\left(1+\frac{1-\gamma_{[n+2]}}{\gamma_{[n+2]}}\frac{\phi_{HH}}{\phi_{HL}}\right) \\
&\geq \frac{1}{2}\left(1+\left(1+\frac{\lambda_{HL}}{\underline{\pi}_{HL}}\right)^{-1}\right)\left(1+\frac{1-\mu_{[n+2]}}{\mu_{[n+2]}}\frac{\phi_{HH}}{\phi_{HL}}\right) \\
&= \frac{V_{[n+2]}}{\mu_{[n+2]}\overline{\pi}_{HL}},
\end{aligned}$$

where the first inequality follows from Assumption 1, the second from (17), the third from combining $\underline{\pi}_{LL} \geq \underline{\pi}_{HL}$ from (16) with $0 < \lambda_{LL} \leq \lambda_{HL}$ from Assumption 2.

If $x_{[n+1]} = 0$, the payoffs $U_{[n+1]}$ and $V_{[n+1]}$ are given by type $L$ conceding with probability one:

$$U_{[n+1]} = \gamma_{[n+1]}\phi_{LL} + (1 - \gamma_{[n+1]})\underline{\pi}_{LH},$$

and

$$V_{[n+1]} = \mu_{[n+1]}\overline{\pi}_{HL} + (1 - \mu_{[n+1]})\beta_{[n+1]}\phi_{HH}.$$

Since type $L$ weakly prefers conceding to persisting in round $[n+1]$, we have

$$U_{[n+1]} \geq \gamma_{[n+1]}\overline{\pi}_{LL} + (1 - \gamma_{[n+1]})\beta_{[n+1]}\phi_{LH}.$$

The rest of the argument then follows in the same way as in the case of $x_{[n+1]} > 0$.

Next, we show that $\beta_{(i-1)}\beta_{(i)}\beta_{(i+1)}$ is strictly quasi-concave in $\gamma_{(i+1)}$. For any $t$, we have

$$\beta_t = \frac{(1 - \gamma_{t+1})U_t - (\gamma_t - \gamma_{t+1})\overline{\pi}_{LL}}{(1 - \gamma_t)U_{t+1}}.$$

Define

$$R_t = \frac{(1 - \gamma_{t+1})U_t - (\gamma_t - \gamma_{t+1})\overline{\pi}_{LL}}{(1 - \gamma_{t+1})U_t}.$$

We have

$$\beta_{(i-1)}\beta_{(i)}\beta_{(i+1)} = R_{(i)}R_{(i+1)} \times \text{constant},$$

where the constant does not vary with $\gamma_{(i+1)}$ in the localized variation. Therefore, it suffices to show that $R_{(i)}R_{(i+1)}$ is quasi-concave in $\gamma_{(i+1)}$.

Writing $U_{(i)}$ in terms of the payoff from conceding,

$$U_{(i)} = \gamma_{(i)}\left(x_{(i)}\underline{\pi}_{LL} + (1 - x_{(i)})\phi_{LL}\right) + \left(1 - \gamma_{(i)}\right)\underline{\pi}_{LH},$$

and using the fact that $\mathrm{d}x_{(i)}/\mathrm{d}\gamma_{(i+1)} = x_{(i)}/(\gamma_{(i+1)}(1 - \gamma_{(i+1)}))$ (derived from Bayes' rule), we obtain

$$\frac{\mathrm{d}U_{(i)}}{\mathrm{d}\gamma_{(i+1)}} = -\frac{(1 - \gamma_{(i)})\lambda_{LL}}{(1 - \gamma_{(i+1)})^2}. \tag{A.5}$$

Similarly, writing $U_{(i+1)}$ in terms of the payoff from conceding, and using the fact that $\mathrm{d}x_{(i+1)}/\mathrm{d}\gamma_{(i+1)} = -x_{(i+1)}/(\gamma_{(i+1)}(1 - \gamma_{(i+1)}))$, we obtain

$$\frac{\mathrm{d}U_{(i+1)}}{\mathrm{d}\gamma_{(i+1)}} = \frac{-U_{(i+1)} + \phi_{LL}}{1 - \gamma_{(i+1)}}. \tag{A.6}$$

From (A.5) and (A.6), $\mathrm{d}\log(R_{(i)}R_{(i+1)})/\mathrm{d}\gamma_{(i+1)}$ is equal to

$$\frac{1-\gamma_{(i)}}{(1-\gamma_{(i+1)})^2}\left(\frac{\phi_{LL}}{U_{(i)}-\frac{\gamma_{(i)}-\gamma_{(i+1)}}{1-\gamma_{(i+1)}}\overline{\pi}_{LL}}+\frac{\lambda_{LL}}{U_{(i)}}-\frac{\frac{1-\gamma_{(i+1)}}{1-\gamma_{(i)}}\lambda_{LL}}{U_{(i+1)}-\frac{\gamma_{(i+1)}-\gamma_{(i+2)}}{1-\gamma_{(i+2)}}\overline{\pi}_{LL}}+\frac{\frac{1-\gamma_{(i+1)}}{1-\gamma_{(i)}}\phi_{LL}}{U_{(i+1)}}\right)$$

Using (A.5) and (A.6) again, the derivative of the term in brackets above is

$$\frac{1-\gamma_{(i)}}{(1-\gamma_{(i+1)})^2}\left(-\left(\frac{\phi_{LL}}{U_{(i)}-\frac{\gamma_{(i)}-\gamma_{(i+1)}}{1-\gamma_{(i+1)}}\overline{\pi}_{LL}}\right)^2+\left(\frac{\lambda_{LL}}{U_{(i)}}\right)^2\right.$$
$$\left.-\left(\frac{\frac{1-\gamma_{(i+1)}}{1-\gamma_{(i)}}\lambda_{LL}}{U_{(i+1)}-\frac{\gamma_{(i+1)}-\gamma_{(i+2)}}{1-\gamma_{(i+2)}}\overline{\pi}_{LL}}\right)^2-\left(\frac{\frac{1-\gamma_{(i+1)}}{1-\gamma_{(i)}}\phi_{LL}}{U_{(i+1)}}\right)^2\right),$$

which is negative when $\mathrm{d}\log(R_{(i)}R_{(i+1)})/\mathrm{d}\gamma_{(i+1)}=0$. This shows that $\log(R_{(i)}R_{(i+1)})$ is single-crossing from above, and therefore $R_{(i)}R_{(i+1)}$ is quasi-concave in $\gamma_{(i+1)}$.

Finally, we show that the value of $R_{(i)}R_{(i+1)}$, and hence the value of $V_{(i-1)}$, is the same at $\gamma_{(i+1)}=\Gamma_B(\gamma_{(i)})$ and at $\gamma_{(i+1)}=\Gamma_B^{-1}(\gamma_{(i+2)})$. We use the superscript $*$ to denote the values of the relevant variables when $\gamma_{(i+1)}=\Gamma_B(\gamma_i)$, and the superscript † to denote their values when $\gamma_{(i+1)}=\Gamma_B^{-1}(\gamma_{i+2})$.

By definition, if there is maximal concession in an active round $t$,

$$U_t=B\left(\gamma_t\chi_B(\gamma_t)+1-\gamma_t\right)Q\left(\Gamma_B(\gamma_t)\right)+\gamma_t\left(1-\chi_B(\gamma_t)\right)\overline{\pi}_{LL},$$

where

$$Q(\gamma)=\gamma\underline{\pi}_{LL}+(1-\gamma)\underline{\pi}_{LH}$$

is the minimum continuation payoff given updated belief $\gamma$. Substituting the formulas for $\chi_B(\gamma)$ and $\Gamma_B(\gamma)$ into the equation, we obtain

$$R_t=\frac{B(\gamma_t\chi_B(\gamma_t)+1-\gamma_t)Q(\Gamma_B(t))}{B(\gamma_t\chi_B(\gamma_t)+1-\gamma_t)Q(\Gamma_B(\gamma_t))+\gamma_t(1-\chi_B(\gamma_t))\overline{\pi}_{LL}}=\frac{B\lambda_{LL}}{B\lambda_{LL}+(1-B)\overline{\pi}_{LL}},$$

which is a constant. It follows that $R^*_{(i)}=R^\dagger_{(i+1)}$, and therefore $R^*_{(i)}R^*_{(i+1)}-R^\dagger_{(i)}R^\dagger_{(i+1)}$ is proportional to $R^*_{(i+1)}-R^\dagger_{(i)}$.

The sign of $R^*_{(i+1)} - R^\dagger_{(i)}$ is the same as

$$(\gamma_{(i)} - \gamma^\dagger_{(i-1)})(1 - \gamma_{(i+2)})U^*_{(i+1)} - (\gamma^*_{(i+1)} - \gamma_{(i+2)})(1 - \gamma^\dagger_{(i+1)})U^\dagger_{(i)}.$$

Substituting $U^*_{(i+1)}$ and $U^\dagger_{(i)}$ using the payoff from concession, and upon simplifying this is equal to

$$(\gamma_{(i)} - \gamma^\dagger_{(i+1)})(1 - \gamma^*_{(i+1)})Q(\gamma_{(i+2)}) - (\gamma^*_{(i+1)} - \gamma_{(i+2)})(1 - \gamma_{(i)})Q(\gamma^\dagger_{(i+1)}). \tag{A.7}$$

Finally, substituting $\gamma^*_{(i+1)} = \Gamma_B(\gamma_{(i)})$ and

$$\gamma^\dagger_{(i+1)} = \Gamma_B^{-1}(\gamma_{(i+2)}) = \frac{\gamma_{(i+2)}\lambda_{LL} + (1-B)Q(\gamma_{(i+2)})}{\lambda_{LL} + (1-B)Q(\gamma_{(i+2)})}$$

into (A.7), we find that (A.7) is equal to zero, and therefore $R^*_{(i+1)} - R^\dagger_{(i)} = 0$.