Misinformation

Li, Hao
University of Toronto & University of British Columbia

Wei Li
University of California, Riverside & University of British Columbia

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Abstract

A candidate has private information about her own quality and about her rival's quality. She can run an informative campaign which generates a public signal about either her quality (positive campaign) or that of her rival (negative campaign). Both the type of campaign and its informativeness are used by the voters to form interim beliefs to evaluate the campaign signal. Misinformation occurs when the interim beliefs diverge from the candidate's private beliefs. In a separating equilibrium, a candidate runs a positive campaign if she has a good signal about her own quality and a negative campaign if she has a bad signal about her rival's quality. When the candidate has both good news about herself and bad news about her rival, the type of the campaign generally depends both on the accuracies of her private signals and on relative accuracy, but with a small marginal cost of running more informative campaigns, the candidate chooses a positive campaign if her good news is more accurate than her bad news and a negative campaign if the reverse is true. Competing campaigns are less informative in equilibrium than unopposed campaigns, because the value of misinformation is reduced by countering campaigns. When the candidate cares only about the probability of winning instead of the expected margin of a win or loss, misinformation can occur in equilibrium.

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1 Introduction

Much attention has been paid to the issue of positive versus negative advertising in political campaigns, in both academia and popular press. The focus is on which type of campaign — praising one's own qualifications or discrediting that of the rival's — is more effective in influencing the voters. The effectiveness of a campaign, however, also depend on the informativeness of the campaign. Some campaign advertisements are "feel-good" ads that do not reveal much about the candidate's quality, while others give more specific information about a candidate's records and character. The informativeness of a campaign may be thought of as how much a candidate is willing to let the voters learn about either herself or her rival. Therefore both the type and the informativeness of campaign may matter to the voters' decision.

This paper considers a model in which a candidate uses the informativeness as well as the type of the campaign to influence a representative voter's beliefs, which determine the election outcome. In the basic model, a candidate has private information about herself and a rival candidate and wants to maximize the expected margin in a win and minimize the loss. She can choose one campaign, modeled here as a costly public information structure, but not its realization, to convey her private information about her quality or her rival's to the voter. She controls both the target of the campaign, which can be either her own quality (*positive campaign*) or her rival's (*negative campaign*), as well as the informativeness of the campaign, which is the accuracy of the public signal the voter observes. Since the candidate has private information, her private beliefs about the types of both candidates are generally different from the *interim beliefs* that the voter forms after observing her campaign choices and then uses to evaluate the realized signal generated by the campaign. In this case, we say that *misinformation* occurs.

The value of misinformation to the candidate arises from misleading the voter into forming a higher belief about her own quality than her private belief, or a lower belief about her rival's quality. However, since the candidate does not control the realization of the campaign signal, misinformation is necessarily imperfect as the voter's interim beliefs after observing the candidate's campaign choice are partially corrected by the realized campaign signal. In particular, if the voter has a higher interim belief about the candidate's quality given her strategy than her private belief, then she prefers to "obfuscate the negative" by running an uninformative campaign because in expectation, any informative public signal is likely to lower the voter's ex post belief of her quality. In this case, the marginal value of misinformation is negative, because the realized campaign signal agrees with her private information in expectation, leading to a higher opinion of the rival's quality. In the opposite case when the voter has a lower interim belief about the candidate's quality given her strategy than her private belief, the candidate prefers to "accentuate the positive" by running an informative campaign about her own quality. The marginal value of misinformation is positive.

In our basic model the candidate can only run one campaign. One natural question is therefore whether a candidate should run an informative campaign about herself or about her rival. In each campaign, the candidate presents direct evidence about the target of the campaign, which we label *direct campaigning*. We show that the value of misinformation through positive direct campaigning is less for a candidate with bad news about her own quality than for one with the good news, and symmetrically the value of misinformation through negative direct campaigning is less for a candidate with good news about her rival's quality than for one with the bad news. As a result, there is *horizontal separation* in direct campaigning: the candidate runs a positive informative campaign about herself if she has good news about her own quality, while she runs a negative informative campaign about her rival if she has bad news about her rival's quality. This result holds regardless of the accuracies of her private signals about herself and her rival. If the candidate has both good news about her own quality and bad news about her rival's quality, whether she should run a positive campaign or a negative campaign in equilibrium depends on the *relative* value of misinformation. Running a positive campaign avoids the payoff loss due to misinformation by the candidate type that actually has good news about her own quality, while incurring the campaign cost due to misinformation by the type that has bad news instead. Similarly, running a negative campaign avoids a payoff loss equal to the value of misinformation by the type with bad news about the rival at a cost equal to the value of misinformation by the type with good news instead. When the good news about her own quality and bad news about her rival are inaccurate, she runs a positive campaign if the form is relatively more accurate and a negative campaign otherwise.

We extend our analysis to a model of competing campaigns. In order to make this competing model comparable to the model with single campaign in terms of the amount of information available, we assume that the private signals about either candidate are perfectly correlated across the two candidates. We show that competition reduces the value of misinformation, and thus candidates runs less informative campaigns in comparison with the single campaign case. There are two reasons for this result. First, there is less incentive for a candidate to run an informative campaign to misinform the voter about the his own quality or about the his rival's quality, because with a positive probability the misinformation is countered by a signal resulting from the rival's campaign. Second, for the informativeness of the campaign, the voter is less likely to be swayed by any misinformation because the candidate's choice of campaign becomes a less effective way of changing the voter's interim beliefs before the voter observes the realized campaign signals.

Unlike the model of single campaigns, the model of competing campaigns exhibits *vertical separation*, in the sense that a candidate with both good news about her own quality and bad news about her rival runs more informative campaigns than both the candidate type with only good news about herself and the type with only bad news about her rival. The highly informative campaign is used to suggest, without presenting any direct evidence, that her rival's quality is low in a positive informative campaign, or that her own quality is high in a negative campaign. We call this *indirect campaigning*. In the presence of another active campaign run by the rival, the value of misinformation through indirect campaigning depends on the campaign choices made by the rival, in contrast to the case of single campaign. In particular, the assumption that the candidates have correlated private signals implies that their campaign choices are correlated. As a result, the value of misinformation through indirect campaigning to the candidate type with only good news about herself or to the type with only bad news about her rival is reduced by the direct campaigning of the rival. In comparison, there is a greater payoff loss due to misinformation by candidate type with both good news about herself and bad news about her rival, as the rival is likely to run no campaign.

When candidates care only about the probability of winning instead of the expected margin of a win or loss, misinformation can occur on the equilibrium path as well off the path. In such winner-take-all model with a single active campaign, we construct pooling equilibria in which the candidate's private beliefs differ from the beliefs that the voter uses in evaluating the campaign signals. As a result of equilibrium misinformation, the "wrong" candidate may be chosen by the voter in equilibrium. The reason for equilibrium misinformation is that, unlike in the basic model where candidates care about the expected margin of a win or loss and thus the value of information is zero if the candidate's private beliefs are common knowledge, when the candidate cares only about the probability winning, there are incentives to run informative campaigns even if her private beliefs are common knowledge. Furthermore, the incentives may be the strongest when the candidate has both bad news about her own quality and good news about her rival. Pooling, and hence misinformation, may therefore occur in equilibrium if the campaign cost is relatively low.

In existing papers, advertising, political or otherwise, is either modeled as directly informative because they contain hard information about the candidate's own (Nelson 1974, Coate 2004), or indirectly informative (Milgrom and Roberts 1986, Prat 2002). Milgrom and Roberts (1986) shows that the amount of money a firm spends on advertising can signal its product quality. Prat (2002) considers an electoral campaign setting where voters do not learn useful information from the campaign advertising itself. Rather, they use the amount of money spent on the campaign, in terms of campaign contributions, as a costly signal of the interest groups who have private information about the candidate's quality. Kotowitz and Mathewson (1979) study a setting where consumers' purchase decision based on their expectations of the quality of an experience good. They show that a monopolist can use advertising to mislead the consumers into having higher expectations than the actual quality of the good, at least in the short run.

Mostly closely related to our paper, Polborn and Yi (2006) considers a model in which each candidate has two characteristics, of which he can only reveal one. He runs an informative and truthful campaign, which can be a positive one about one good characteristic of his own or a negative one about one bad characteristic of his rival. The voters rationally estimate the characteristic not revealed. Thus a candidate is more likely to choose a positive campaign when his own characteristic is good and/or his rival's is good too. Our model differs from Polborn and Yi (2006) in that the candidate chooses the informativeness of the campaign signal the voter receives, which influences the voters' beliefs about his own quality and his rival's quality.

2 Information Campaigns

We may generally define an *information campaign* as an observable choice of distribution of a publicly verifiable signal about some state by a privately informed agent. To put our model of misinformation in context, throughout the paper we refer to the agent as a political candidate for some office, and the state as the qualification of either the candidate herself or that of her rival. The public is represented by a single median voter, or simply the voter. The assumption that an information campaign is an observable choice means that the privately informed candidate uses it as a signal. We will assume that the publicly verifiable signal generated by an information campaign, referred to as "campaign signals," are independent from the candidate's private signal conditional on the state. This distinguishes information campaigns from earlier models of information disclosure (Milgrom and Roberts 1986), in which the agent is assumed to have full control of the information content. The assumption of conditional independence also means that an information campaign is not cheap talk (Crawford and Sobel 1982).

For simplicity, we adopt a binary model of information. Let the two candidates be A and B. Each candidate i, $i = A, B$, is either qualified, denoted as $i = 1$, or unqualified, denoted as $i = 0$. It is common knowledge between the candidates that the prior probability that $i = 1$ is 0.5, and that the candidates' qualities are independent. Candidate i receives an imperfect binary signal $t_i^i \in \{h, l\}$ about her own quality, and another signal $t_j^i \in \{h, l\}$ about her rival j's quality. We assume that the signal structures are symmetric:

$$\Pr(t_i^i = h | i = 1) = \Pr(t_i^i = l | i = 0) = q,$$

and

$$\Pr(t_j^i = h | j = 1) = \Pr(t_j^i = l | j = 0) = r,$$

with $q, r \in (0.5, 1)$. We allow q and r to differ, representing the idea that candidate i's information about her own quality and her rival's quality may have different accuracies. The two signals t_i^i and t_j^i are assumed to be independent. We refer to the vector (t_i^i, t_j^i) as the "type" of candidate i, which is private information to i, and write the four types as lh, hh, ll and hl. For example, type $t = lh$ is when candidate i has a bad signal $t_A = l$ about her own quality and a good signal $t_B = h$ about her rival's quality. We may allow the candidates' signals about i, t_i^i and t_i^j, to be correlated conditional on the quality of i.

Each candidate may choose to run an information campaign. Candidate i may choose one of two kinds of campaigns: a *positive* campaign, modeled as an information structure of a binary public signal about the candidate's own quality, and a *negative* campaign, which is an information structure of a public signal about the rival's quality. The informativeness of the campaign, is referred to as the *level* of the campaign.

Specifically, if $s_p \in \{+, -\}$ is the public signal from a positive campaign run by candidate i, the level κ_p^i of the campaign is

$$\kappa_p^i = \Pr(s_p = +|i = 1) = \Pr(s_p = -|i = 0).$$

Similarly, if $s_n \in \{+, -\}$ is the public signal from a negative campaign, the level κ_n^i of the campaign is

$$\kappa_n^i = \Pr(s_n = +|j = 1) = \Pr(s_n = -|j = 0).$$

Candidate i must choose both the kind, positive or negative, and the level, κ_p^i or κ_n^i, of the campaign. Note that the candidate does not control the realization of the public signal. Without loss of generality, we assume that κ_p^i and κ_n^i are at least as great as 0.5, with levels strictly higher than 0.5 referred to informative campaigns and $\kappa_p^i = 0.5$ or $\kappa_n^i = 0.5$ as an uninformative campaign. When there are two campaigns about the same candidate, we assume that the campaign signals are independent from each other conditional on the state.

Information campaigns are assumed to be costly. Let $f(\kappa)$ be the cost to the candidate of running a campaign at level κ. We implicitly assume that the cost is independent of the kind of campaign; this is to highlight our later results about the choice between positive and negative campaigns. We adopt the normalization that $f(0.5) = 0$, and for simplicity assume that f is differentiable with $f'(\kappa) > 0$ for all κ. We may need further assumptions on f for some of our results.

The voter's prior belief is the same as the candidates': candidate i is qualified with probability 0.5, and the candidates' qualities are independent. After observing the candidates' choices of campaigns, the voter updates his belief about the two candidates' qualities. These beliefs are referred to as "interim beliefs." The campaign signals are then realized and observed by the voter. Let π^i denote the voter's posterior belief that candidate i's quality is 1.

We mainly consider two payoff specifications. In the *expected margin model*, each candidate maximizes the difference of the voter's posterior beliefs about herself over her rival. Candidate i's payoff is thus

$$u^i = \pi^i - \pi^j - f(\kappa^i).$$

In the opposite model of *winner-take-all model*, each candidate maximizes the chance of winning, or the probability that the voter's posterior belief is higher about herself than about the rival. Without loss of generality, we assume that the tie-breaking rule favors candidate A.[1] Then, candidate A's payoff is

$$u^A = \begin{cases} 1 - f(\kappa^A), & \text{if } \pi^A \geq \pi^B \\ -f(\kappa^A), & \text{if } \pi^A < \pi^B. \end{cases}$$

Candidate B's payoff is analogously defined. We also briefly consider the hybrid *winning-margin model*, where each candidate cares about difference of the voter's posterior beliefs only when she wins. Candidate i's payoff in this model is

$$u^i = \begin{cases} \pi^i - \pi^j - f(\kappa^i), & \text{if } \pi^i \geq \pi^j \\ -f(\kappa^i), & \text{if } \pi^i < \pi^j. \end{cases}$$

In the context of information campaigns, the standard notions of value of information can be easily adapted to our game. In any information campaign, positive or negative, we think of the value of information to the candidate as the difference between her expected payoff computed with the posterior beliefs resulting from the campaign signal, and her payoff with the posterior beliefs equal to her private beliefs. According to this definition, the value of information in an uninformative campaign is zero, and thus the definition excludes the candidate's private information from the calculation of the value of information. However, the voter's posterior beliefs are calculated from the interim beliefs, which are endogenous in the game. To define the value of information, we specify that the interim beliefs are identical to the private beliefs of the candidate.

We say that *misinformation* occurs when the voter's interim beliefs differ from the candidate's private beliefs. Such differences may arise either on the equilibrium path when different types pool on the same campaign choices, or off the path when types contemplate their deviations. The *value of misinformation* in an information campaign of any level κ may be defined in a way analogous to the value of information, as the difference in the candidate's payoff computed under the posterior beliefs resulting from the campaign, and the payoff when the posterior beliefs are equal to her private beliefs. Thus, in calculating the value of misinformation associated with any campaign, we use the same reference payoff as in calculating the value of information: any difference in the value of misinformation and the value of information arises because the voter's interim beliefs after observing the candidate's campaign choices differ from the candidate's

[1] The payoff specification below implies that there is a discontinuity at symmetric configurations of $q = r$ when both candidates run uninformative campaigns: the payoffs do not correspond to the limits as either q approaches r from above or from below. This discontinuity affects the following calculations of value of information (though not the marginal values), but turns out not to matter to our equilibrium analysis.

private beliefs.[2]

We conclude the model section with a few remarks about the setup. First, we have assumed that a candidate cannot run both a positive and a negative campaign. This assumption allows us to focus on the choice of the kind of campaign. We will show later that our results are robust when the candidates can simultaneously choose both types of campaign, so long as the campaigns are additively separable from each other. Second, the voter in our setup forms his posterior beliefs about the two candidates based on the campaign choices as well as the realized public campaign signals. Since a candidate does not control the realization of the public signal generated by her campaign, the level of campaign is an important ingredient in our analysis of the signaling equilibrium. The assumption that the voter directly observes the informativeness of a campaign is of course an abstraction, but in practice the public may be able to indirectly infer it from campaign spending disclosures, or how easily campaign claims can be verified. Third, we have chosen to model the payoffs of the two candidates in a reduced form, instead of modeling the voting game after the campaigns. The reduced form can be made more general without affecting the results qualitatively. The three payoff specifications have natural interpretations in the context of political campaigns. The expected-margin model is suitable in a proportional representation political system, while the winner-take-all model is more appropriate in a plurality system. The hybrid winning-margin model captures the winning candidate's additional concerns for mandate. Outside the political context, the three models may also be appropriate for different objectives in advertising campaigns.

3 Positive or Negative?

In this section and the next one, candidates maximize the winning margin. The value of information is zero for any type of candidate and any information campaign. This follows from a simple application of the law of iterated expectations. For example, for type lh, an informative positive campaign of level κ_p generates the campaign signal $s_p = +$ and a posterior belief about the candidate herself

$$\frac{(1-q)\kappa_p}{q(1-\kappa_p)+(1-q)\kappa_p} > 1-q$$

[2] Note also that in contrast to the value of information, our definition potentially allows the value of misinformation in an uninformative campaign to be non-zero.

with probability $q(1-\kappa_p)+(1-q)\kappa_p$, and $s_p = -$ and a posterior belief

$$\frac{(1-q)(1-\kappa_p)}{q\kappa_p+(1-q)(1-\kappa_p)} < 1-q,$$

with probability $q\kappa_p+(1-q)(1-\kappa_p)$. The expectation of the posterior belief remains $1-q$.

In this section only candidate A runs an active campaign. The payoff specification is expected-margin. Since this is a signaling game with candidate A as the only player, we drop the superscripts whenever there is no confusion. A strategy of A specifies, for each of the four private types, the kind of campaign, positive or negative, and the corresponding level, κ_p or κ_n. We adopt perfect Bayesian equilibrium as the solution concept.

3.1 Direct campaigning and horizontal separation

Consider first a positive campaign of level κ_p. Denote as χ as the private belief of candidate A that she is qualified and $\tilde{\chi}$ as the voter's interim belief. The candidate's private belief χ can be directly computed from her type t using Bayes' rule; the interim belief $\tilde{\chi}$ is part of equilibrium description to be given later. But for now, suppose that χ and $\tilde{\chi}$ are fixed. Then, candidate A's expectation of the voter's posterior belief π^A about her own quality is given by

$$\pi(\chi,\tilde{\chi};\kappa_p) = (\chi\kappa_p+(1-\chi)(1-\kappa_p))\frac{\tilde{\chi}\kappa_p}{\tilde{\chi}\kappa_p+(1-\tilde{\chi})(1-\kappa_p)}+(\chi(1-\kappa_p)+(1-\chi)\kappa_p)\frac{\tilde{\chi}(1-\kappa_p)}{\tilde{\chi}(1-\kappa_p)+(1-\tilde{\chi})\kappa_p},$$

where the first fraction is the voter's posterior belief about the candidate's quality after observing a plus public signal ($s_p = +$), and the second fraction is the voter's posterior after observing a minus public signal ($s_p = -$). The above can be rewritten as

$$\pi(\chi,\tilde{\chi};\kappa_p) = \tilde{\chi}+(\chi-\tilde{\chi})(2\kappa_p-1)\left(\frac{\tilde{\chi}\kappa_p}{\tilde{\chi}\kappa_p+(1-\tilde{\chi})(1-\kappa_p)}-\frac{\tilde{\chi}(1-\kappa_p)}{\tilde{\chi}(1-\kappa_p)+(1-\tilde{\chi})\kappa_p}\right).$$

Note that if χ and $\tilde{\chi}$ respectively denote the private belief and the interim belief that the rival is qualified, then for any negative campaign of level κ_n, the expected posterior belief about candidate B is given by the same function $\pi(\chi,\tilde{\chi};\kappa_n)$. We have the following result immediately, where the distinction between positive and negative campaigns is dropped.

Lemma 1 (Value of Misinformation I) *(i) $\pi(\chi,\tilde{\chi};\kappa)=\chi$ if $\tilde{\chi}=\chi$; (ii) $\pi(\chi,\tilde{\chi};\kappa)$ decreases in κ if $\chi<\tilde{\chi}$; and (iii) $\pi(\chi,\tilde{\chi};\kappa)$ increases in κ if $\chi>\tilde{\chi}$.*

Both a positive campaign to change the voter's perception of one's own quality and a negative campaign to change the voter's perception of her rival's quality are *direct campaigning*, because the candidate allows the voter to receive a direct public signal about the target of the campaign. We can interpret the above lemma as a characterization of the *marginal value of misinformation* through direct campaigning. By the above lemma, if the candidate is privately more confident about her quality than the voter is, then she can increase her average perceived type by choosing a more informative positive campaign. That is, the marginal value of misinformation through direct campaigning is positive. In this case, an informative positive campaign about the candidate highlights the good news. On the flip side, a candidate can hide the bad news by reducing the informativeness of her campaign signal if she is privately less confident about her own quality than the voter. In this case, the marginal value of misinformation through direct campaigning is negative. Naturally, the opposite holds for a negative campaign: the candidate lowers the voter's perception about her rival by running an informative campaign if she has worse news about the rival than the voter.

Part (i) of Lemma 1 confirms that the value of information is zero in the expected-margin model. It applies on the equilibrium path of any separating equilibrium, because the private beliefs of each type t about her own quality and the rival's quality are equal to the interim beliefs of the voter on the equilibrium path. As a result, in any separating equilibrium type lh candidate runs an uninformative campaign because she can obtain her complete information payoff from doing so, while running any informative campaign, positive or negative, is costly but has no effect on either the candidate's own quality perceived by the voter, or her rival's perceived quality.

In our model, there is *horizontal separation* between types hh and ll through direct campaigning in the following sense. In any separating equilibrium, type hh strictly prefers running its equilibrium level κ_p of informative positive campaign to misinforming the voter with the negative campaign of level κ_n run by type ll in the equilibrium, and vice versa for type ll. That is, the kind of campaign is associated with whether the candidate has good news about herself or bad news about her rival. In the proposed separating equilibrium, type hh succeeds in communicating to the voter her private belief that she is qualified with probability $\Pr(A = 1|t = hh) = q$ by running a positive campaign of level κ_p. By deviating to the negative campaign of level κ_n run by type ll, the candidate only succeeds in understating her private belief about her own quality, leading the voter to believe that her own quality is 1 with probability $\Pr(A = 1|t = lh) = 1 - q$ instead of q. This loss in payoff is the same as when type hh deviates from running the equilibrium level of positive campaign to running an uninformative campaign. By part (iii) of Lemma 1, just as type lh, this candidate fails to lower the voter's perception about her rival's quality to the level achieved by type ll in equilibrium, because type hh's private belief about the rival's quality is $\Pr(B = 1|t = hh) = r$, which is higher than the voter's interim belief $\Pr(B = 1|t = ll) = 1 - r$.

It may seem counterintuitive that horizontal separation in our model does not depend on the relative accuracy of the candidate's private signal about her own quality and about her rival's quality. After all, if q is just above 0.5 and $r > q$, then type hh has little to say about herself, but accurate information about her rival. Thus it may suggest discrediting the rival is more important than signaling one's own quality. However, since the candidate can only run one campaign, type hh has more to lose from pretending to be type ll than type lh, $2q - 1$ versus 0, in terms of a missed opportunity of informing the voter of her own quality; but the same to gain, $r - \pi(r, 1 - r; \kappa_n)$, in terms of an expected decrease in the voter's posterior belief about her rival's quality. If running a negative campaign of level κ_n is sufficient for type ll to deter type lh from imitating, type hh strictly prefers running a positive campaign of level κ_p, even though doing so acknowledges that her rival is qualified as well.

Since in any separating equilibrium type lh runs an uninformative campaign, and types hh and ll respectively run informative positive and negative campaigns, Lemma 1 suggests that we can construct out-of-equilibrium beliefs to achieve *vertical separation* of type lh from types hh and ll. Define $\underline{\kappa}_p$ as the level of informative positive campaign that makes type lh indifferent between running the campaign and running an uninformative campaign, assuming that the former choice leads to the interim belief of q that the candidate is qualified and r that the candidate's rival is qualified. This is given by

$$1 - q - r = \pi(1 - q, q; \underline{\kappa}_p) - r - f(\underline{\kappa}_p). \tag{1}$$

The above equation determines a level of positive campaign that depends only on the accuracy q of the candidate's private signal about her own quality. Furthermore, for any q there is a unique $\underline{\kappa}_p$ that satisfies the above equation: the right-hand-side of equation (1) ranges between $1 - q - r - f(1)$, when $\underline{\kappa}_p = 1$, and $q - r$, when $\underline{\kappa}_p = 0.5$, and is decreasing by part (ii) of Lemma 1.

Lemma 1 implies that the least costly level of the informative positive campaign in any separating equilibrium is $\underline{\kappa}_p$. This is because, in any separating equilibrium of level κ_p, the voter's belief after observing the choice of campaign is that candidate A is qualified with probability q. By part (ii) of Lemma 1, the expected posterior belief of the voter about A's quality is decreasing in the level of positive campaign for type lh. Thus, any level $\kappa_p < \underline{\kappa}_p$ is insufficient to deter type lh from deviation. Moreover, since in the separating equilibrium of level $\underline{\kappa}_p$, type lh is indifferent between the campaign and an uninformative campaign, type hh strictly prefers the latter to the former. This follows because $\pi\left(\chi, \tilde{\chi}; 0.5\right) = \tilde{\chi}$ regardless of χ, and part (iii) of Lemma 1 implies that the maximum gain for type lh in deviating to any informative positive campaign, in terms of changing the voter's posterior belief about her quality, is $q-(1-q)$, which is the loss for type hh in a separating equilibrium from running an uninformative campaign.

Similarly, we can define $\underline{\kappa}_n$ as the level of informative negative campaign that makes type lh indifferent between running and not running a campaign, assuming that running the negative campaign of level $\underline{\kappa}_n$ leads to the interim belief of $1-r$ that the candidate's rival is qualified and $1-q$ that the candidate herself is qualified. This is given by

$$1 - q - r = 1 - q - \pi(r, 1-r; \underline{\kappa}_n) - f(\underline{\kappa}_n). \tag{2}$$

A similar argument as above then implies that the least costly level of the informative negative campaign in any separating equilibrium is $\underline{\kappa}_n$, at which type ll strictly prefers the campaign to an uninformative campaign. It can be easily verified that

$$\pi(\chi, 1-\chi; \kappa) = 1 - \pi(1-\chi, \chi; \kappa),$$

for any $\chi \in [0, 1]$ and any κ, so the equation for $\underline{\kappa}_n$ takes a symmetric form as equation (1). For convenience, we denote the least costly level as a function $\underline{\kappa}(\chi)$, with $\underline{\kappa}(q) = \underline{\kappa}_p$ and $\underline{\kappa}(r) = \underline{\kappa}_n$.

Finally, we turn to the question of what campaign choices the candidate must make to credibly signal both good news for her own quality and bad news for her rival's. There is no a priori reason why type hl should choose a positive or a negative campaign. But clearly, if she chooses to run a positive campaign for example, she must run a campaign with a level higher than $\underline{\kappa}_p$ to separate from type hh. In this case, type hl runs a more informative campaign about her own quality and does not provide any direct evidence about the her rival's quality. We refer to it as *indirect campaigning*. Similarly, type hl can run a negative campaign with a level higher than $\underline{\kappa}_n$ to suggest that her own quality is high without giving the voter direct evidence, a public signal about herself.

Define $\overline{\kappa}_p$ as the level of informative positive campaign that makes type hh indifferent between running this campaign and running her equilibrium positive campaign of the level $\underline{\kappa}_p$, assuming that running the positive campaign of level $\overline{\kappa}_p$ leads to the interim belief of q that herself is qualified and $1-r$ that her rival is qualified. This is given by:

$$q - r - f(\underline{\kappa}_p) = q - (1-r) - f(\overline{\kappa}_p). \tag{3}$$

For both the campaign at the level $\underline{\kappa}_p$ and the campaign at the level $\overline{\kappa}_p$, the candidate's expectation of the voter's belief about her own quality does not depend on whether her type is hh or hl. By part (i) of Lemma 1, the expectation is q from her direct campaigning. The indirect campaigning about the quality of her rival does not depend on whether her type is hh or hl either, because she does not provide the voter with a public signal. As a result, the higher is r, the stronger is type hh's incentive to use a higher level of positive campaign as an indirect signal of her rival's low quality. Similarly, define $\overline{\kappa}_n$ as the level of informative negative campaign that makes type ll indifferent between running this campaign and running her equilibrium negative campaign of the level $\underline{\kappa}_n$. This is given by

$$1 - q - (1-r) - f(\underline{\kappa}_n) = q - (1-r) - f(\overline{\kappa}_n). \tag{4}$$

From the perspective of type hl, we have $f(\overline{\kappa}_p) < f(\overline{\kappa}_n)$ if and only if

$$f(\underline{\kappa}_p) + r - (1-r) < f(\underline{\kappa}_n) + q - (1-q),$$

which is equivalent to

$$r - \pi(1-r, r; \underline{\kappa}(r)) < q - \pi(1-q, q; \underline{\kappa}(q)), \tag{5}$$

by the definitions of $\underline{\kappa}_p$ and $\underline{\kappa}_n$ in equation (1) and (2). The above condition can be interpreted as the relative value of misinformation to type lh who has both bad news for herself and good news for the rival. The left-hand-side is the value of misinformation through negative indirect campaigning, $r-(1-r)$, relative to the value of misinformation through negative direct campaigning of level $\underline{\kappa}(r)$, given by $r - \pi(r, 1-r; \underline{\kappa}(r)))$. Similarly, the right-hand-side is the value of misinformation through positive indirect campaigning, $q-(1-q)$, relative to the value of misinformation through positive direct campaigning of level $\underline{\kappa}(q)$, given by $\pi(1-q;q;\underline{\kappa}(q))-(1-q)$. Note that the relative value of misinformation is always positive. Thus, for type hl to credibly inform the voter that she has both good news about her own quality and bad news about her rival's quality, a positive campaign of level $\overline{\kappa}(q)$ is less costly than a negative campaign of level $\overline{\kappa}(r)$, if relative to direct campaigning, misinformation through indirect campaigning is less effective when the target is the rival than when the target is the candidate herself.

3.2 The least costly separating equilibrium

We now proceed to state the least costly separating equilibrium.

Proposition 1 *In the least costly separating equilibrium, type lh candidate runs an uninformative campaign. Type hh runs an informative positive campaign of level $\underline{\kappa}(q)$ while type ll runs an informative negative campaign of level $\underline{\kappa}(r)$. Type hl runs a positive campaign of level $\overline{\kappa}_p$ if condition (5) holds, and otherwise a negative campaign of level $\underline{\kappa}_n$.*

A natural set of out-of-equilibrium-path beliefs is: the candidate is of type lh if $\kappa_p \in \left(0.5, \underline{\kappa}_p\right)$ or $\kappa_n \in (0.5, \underline{\kappa}_n)$; regardless of whether type hl runs a positive or a negative campaign in the equilibrium, the candidate is of type hh if $\kappa_p \in (\underline{\kappa}_p, \overline{\kappa}_p)$ and type hl if $\kappa_p \geq \overline{\kappa}_p$, and is of type ll if $\kappa_n \in (\underline{\kappa}_n, \overline{\kappa}_n)$ and type hl if $\kappa_n \geq \overline{\kappa}_n$. Under the above set of beliefs, type lh has no incentive to deviate to any positive campaign of level $\kappa_p \in (0.5, \underline{\kappa}(q))$, or any negative campaign of level $\kappa_n \in (0.5, \underline{\kappa}(r))$. For example, the deviation payoff from a positive campaign is $1-q-r-f(\kappa_p)$ by part (i) of Lemma 1, which is strictly less than its equilibrium payoff of $1-q-r$. We claim that type hh has no incentive to deviate to a positive campaign of any level $\kappa_p < \underline{\kappa}(q)$, and symmetrically type ll has no incentive to deviate to a negative campaign of any level $\kappa_n < \underline{\kappa}(r)$. To establish this claim, note that given the above out-of-equilibrium-path beliefs, the deviation payoff for type hh is

$$\pi(q, 1-q; \kappa_p) - r - f(\kappa_p) = 1 - \pi(1-q, q; \kappa_p) - r - f(\kappa_p).$$

Since $\kappa_p < \underline{\kappa}(q)$, from part (ii) of Lemma 1 we have that the deviation payoff is strictly less than $1-\pi(1-q,q;\underline{\kappa}(q))-r-f(\kappa_p)$, which by the definition of $\underline{\kappa}(q)$ is equal to $q-f(\underline{\kappa}(q))-r-f(\kappa_p)$.

Thus, the deviation payoff for type hh is strictly less than her equilibrium payoff. A symmetric argument establishes that type ll has no incentive to deviate to a negative campaign of any level $\kappa_n < \underline{\kappa}(r)$. We have the following proposition.

As is standard in a signaling game, there are other separating equilibria involving higher levels of campaigns for type hh and type ll, and equilibria involving pooling by type lh and other types can be constructed by carefully choosing out-of-equilibrium beliefs. However, such separating equilibria can be ruled out by applying the standard out-of-equilibrium-path belief refinements such as the "Intuitive Criterion" of Cho and Kreps (1987) and D1 of Banks and Sobel (1987). For example, in a separating equilibrium under consideration, since type hh strictly prefers her equilibrium campaign choice of running a positive campaign of level $\underline{\kappa}(q)$ to any lower level, there are values of κ_p greater than $\underline{\kappa}_p(q)$ that are equilibrium levels of positive campaign for type hh as part of a separating equilibrium, supported by the out-of-equilibrium belief that the candidate is of type lh if she runs a positive campaign of any level $\tilde{\kappa}_p$ below κ_p. Since the separating level is $\kappa_p > \underline{\kappa}_p$, by part (ii) of Lemma 1, for any $\tilde{\kappa}_p \in (\underline{\kappa}_p, \kappa_p)$, type lh strictly prefers an uninformative campaign to running a positive campaign of level $\tilde{\kappa}_p$, regardless of the out-of-equilibrium-path belief about her own quality associated with $\tilde{\kappa}_p$. However, by part (i) of Lemma 1, type hh benefits from such a deviation if the interim belief is that she is type hh. Thus, there is no out-of-equilibrium-path belief that satisfies the Intuitive Criterion that supports the separating level of $\kappa_p > \underline{\kappa}(q)$. To rule out pooling equilibria, and hence equilibrium misinformation, observe that type lh can always receive the complete-information payoff by running an uninformative campaign. This means that in any pooling equilibrium under consideration, the interim beliefs on the equilibrium path are more favorable to type lh than her private beliefs, and are thus less favorable to types that pool with lh. Then, by part (i) and part (ii) of Lemma 1, the set of out-of-equilibrium beliefs that would make it profitable for type lh to deviate to a slightly higher level of positive (negative) campaign would be strictly contained in the set of beliefs for type hh and type hl (type ll and type hl). Thus, these pooling equilibria are ruled out by an application of D1.

In contrast to the vertical separation of types hh and ll from type lh, there is no vertical separation of type hl from types hh and ll. More precisely, if in equilibrium type hl runs an informative positive campaign, both type hl and type hh are indifferent between running a campaign of level $\overline{\kappa}_p$ and $\underline{\kappa}_p$. This occurs because the voter's interim and posterior belief about the candidate given either $\underline{\kappa}_p$ or $\overline{\kappa}_p$ are both q. The gain for type hh to pretend to be type hl, which is $r - (1 - r)$, is precisely the loss for type hl to pretend to be type hh. Given the common campaign cost function and the linear payoff structure, both types are indifferent between running a positive campaign of level $\underline{\kappa}_p$ and level $\overline{\kappa}_p$. Similarly, if in equilibrium, type hl runs an informative negative campaign, type ll is also indifferent between a campaign of level $\underline{\kappa}_n$ and $\overline{\kappa}_n$.

It is straightforward to modify the basic model to generate vertical separation, for example, by assuming that type hl faces a smaller marginal campaign cost than type hh, or that payoff complementarity exist so that type hl has more to lose from overstating the rival's quality than type hh to value of understating it. Further, such modifications may be natural in some circumstances. We will not attempt these modifications however, because the focus of this section is on the equilibrium choice of type hl between positive and negative campaigns, which is unaffected by the absence of the vertical separation.[3] We now try to find sufficient conditions for (5). To do so, we first provide some comparative statics properties of the value of misinformation.

Lemma 2 (Value of Misinformation II) *Suppose that* $\chi > 0.5$. *(i)* $\pi(1 - \chi, \chi; \kappa) - (1 - \chi)$ *increases in* χ*; and (ii) there exists* $\xi(\kappa) \in (0.5, 1)$ *such that* $\chi - \pi(1 - \chi, \chi; \kappa)$ *increases in* χ *if* $\chi \in [0.5, \xi(\kappa))$*, and decreases if* $\chi \in [\xi(\kappa), 1]$.

As mentioned previously, $\pi(1 - \chi, \chi; \kappa) - (1 - \chi)$ represents the value of misinformation to type lh through direct campaigning. Part (i) of the above lemma states that the value of misinformation is increasing in the accuracy of the candidate's private information. To see this why this is true, consider the case of positive direct campaigning. Rewrite $\pi(1 - q, q; \kappa)$ as

$$\frac{q\kappa}{q\kappa + (1 - q)(1 - \kappa)} + \frac{q(1 - \kappa)}{q(1 - \kappa) + (1 - q)\kappa} - q,$$

where the first fraction is the voter's posterior belief about the candidate's quality after observing $s_p = +$, while the second fraction is his posterior belief after $s_p = -$. Thus, an increase in q raises the value of

[3] In the next section we will introduce competing campaigns to deliver the desired vertical separation. The discussion section contains an extension of the single-campaign model to illustrate the key to the vertical separation is that the information from the rival's campaign is correlated with the candidate's campaign choices.

misinformation $\pi(1-q,q;\kappa)-(1-q)$ by increasing both posterior beliefs. It then follows immediately from equations (1) and (2) that the least costly separating level $\underline{\kappa}(\chi)$ is strictly increasing in the accuracy of the candidate's private beliefs, and in particular, $\underline{\kappa}_p \geq \underline{\kappa}_n$ if $q \geq r$. Note that this result holds even if the marginal cost of direct campaigning f' is arbitrarily small. It may seem counterintuitive that the more accurate is a candidate's private information about herself or about her rival, the more informative a campaign — and thus a more costly one — she has to run. One may think that a better candidate should spend less on campaign activities, expecting to be supported by public campaign signals that the voter observes later. Such intuition is misguided by the value of information; our result that $\underline{\kappa}(\chi)$ is increasing in χ follows instead from a characterization of the value of misinformation, because in our model the voter's evaluation of the campaign signals depends on the candidate's campaign choices. In particular, if the candidate has a more accurate signal that she is qualified, the voter forms a higher interim belief after observing her campaign choices, which increases the incentive for a low type candidate to misinform and makes it necessary for a qualified candidate to choose a higher level of positive campaign to discourage imitation.

To understand part (ii) of Lemma 2, recall that $\chi-\pi(1-\chi,\chi;\kappa)$ is the value of misinformation through indirect campaigning relative to direct campaigning. Consider positive campaigns for example. Let us write the relative value of misinformation as

$$\left(\chi-\frac{\chi(1-\kappa)}{\chi(1-\kappa)+(1-\chi)\kappa}\right)-\left(\frac{\chi\kappa}{\chi\kappa+(1-\chi)(1-\kappa)}-\chi\right).$$

Observe that the first difference is how much the voter adjusts downwards his posterior belief about the candidate's quality upon seeing $s_p=-$, while the second difference is how much he adjusts upwards his posterior belief upon seeing $s_p=+$. That the value of misinformation is greater through indirect campaigning than through direct campaigning means that she has more to lose when $s_p=-$ than when $s_p=+$, because the voter's interim belief is already above 0.5. Furthermore, the second difference always decreases in χ, that is, the higher is the voter's interim belief, the less responsive his posterior belief is to $s_p=+$. In contrast, the first difference is not monotone in χ: the decrease in the posterior belief after $s_p=-$ is large when the interim belief χ is close to 0.5 but becomes smaller when χ increases. By taking derivatives, we can verify that for any $\kappa\in(0.5,1)$, the difference of the differences, $\chi-\pi(1-\chi,\chi;\kappa)$, is a concave function of χ, starting at 0 when $\chi=0.5$, increasing and reaching a maximum at $\chi=\xi(\kappa)$, and then decreasing to 0 when $\chi = 1$. Thus, when the candidate's private information about her own quality (or about her rival's quality) is not accurate, an improvement in the accuracy increases the relative value of misinformation through indirect campaigning, because a greater accuracy in this case means that direct campaigning is riskier, as the voter's posterior belief about the candidate's quality (or about the rival's quality) is more responsive when $s_p = -$ for the candidate (or $s_p = +$ for the rival).

Lemma 2 immediately leads to the following sufficient conditions for the equilibrium choice of type hl. If $r < q < \xi(\underline{\kappa}(q))$, then

$$r - \pi(1-r, r; \underline{\kappa}(r)) < r - \pi(1-r, r; \underline{\kappa}(q)) < q - \pi(1-q, q; \underline{\kappa}(q)),$$

where the first inequality follows from part (ii) of Lemma 1 because $\underline{\kappa}(r) < \underline{\kappa}(q)$, and the second inequality follows from part (ii) of Lemma 2. Similarly, if $q < r < \xi(\underline{\kappa}(r))$, then

$$q - \pi(1-q, q; \underline{\kappa}(q)) < q - \pi(1-q, q; \underline{\kappa}(r)) < r - \pi(1-r, r; \underline{\kappa}(r)),$$

so that a negative campaign of level $\overline{\kappa}_n$ is less costly than a positive campaign of level $\overline{\kappa}_p$. To identify conditions on the parameters of the model for $q < \xi(\underline{\kappa}(q))$, we write the condition that determines $\xi(\kappa)$ for any $\kappa \in (0.5, 1)$ as

$$\frac{2}{\kappa(1-\kappa)} = \frac{1}{(\xi(\kappa)\kappa + (1-\xi(\kappa))(1-\kappa))^2} + \frac{1}{(\xi(\kappa)(1-\kappa) + (1-\xi(\kappa))\kappa)^2}.$$

Using the above condition and using the fact that $\chi - \pi(1-\chi, \chi; \kappa)$ is concave in χ and reaches the maximum at $\chi = \xi(\kappa)$, we can show that $\xi(\kappa) > \kappa$ if and only if $\kappa \in (0.5, \kappa_s)$, where $\kappa_s = 0.5 + 0.5\sqrt{\sqrt{2}-1}$. Suppose that the cost function f satisfies $\underline{\kappa}(\kappa_s) < \kappa_s$, or equivalently,

$$f(\kappa_s) < \pi(1-\kappa_s, \kappa_s; \kappa_s) - (1-\kappa_s).$$

Then, by part (i) of Lemma 2, there is a unique $q_s \in (0.5, \kappa_s)$ such that $\underline{\kappa}(q_s) = \kappa_s$. Furthermore, since $\underline{\kappa}(q_s) < \kappa_s$, we have

$$\pi(1-q_s, q_s; q_s) - f(q_s) > 1 - q_s.$$

Thus, there is a range of values of q smaller than q_s such that

$$\pi(1-q, q; q) - f(q) > 1 - q,$$

or equivalently $q < \underline{\kappa}(q)$. It follows that for any such q we have $q < \underline{\kappa}(q) < \underline{\kappa}(q_s) < \kappa_s$, and thus $q < \xi(\underline{\kappa}(q))$.

If the candidate has accurate private signals about her own quality and about her rival's quality, then whether it is more or less costly for type hl to run a strong positive campaign of level $\overline{\kappa}_p$ or a strong negative campaign of level $\overline{\kappa}_n$ depends on which of the following two opposite effects of relative accuracy of the candidate's two private signals dominates. On one hand, if $r > \xi(\underline{\kappa}(r))$ and if q is even greater than r, then by part (ii) of Lemma 2, we have

$$r - \pi(1-r, r; \underline{\kappa}(r)) > q - \pi(1-q, q; \underline{\kappa}(r)),$$

so that the value of misinformation through indirect campaigning relative to direct campaigning at the same level of $\underline{\kappa}(r)$ is greater when the target is the rival than when the target is the candidate herself. This implies that the cost for type hl to credibly inform the voter about the good news for her own quality and the bad news for her rival tends to be higher through a positive campaign than through a negative campaign. On the other hand,

$$q - \pi(1-q, q; \underline{\kappa}(r)) < q - \pi(1-q, q; \underline{\kappa}(q))$$

because $r < q$ implies $\underline{\kappa}(r) < \underline{\kappa}(q)$, so that the value of misinformation through indirect campaigning relative to a direct positive campaign is increased by the fact that the level required for the direct positive campaign is higher than the level required for the direct negative campaign. The first effect dominates if the marginal cost of campaign f' is large, so that there is little difference between $\underline{\kappa}(r)$ and $\underline{\kappa}(q)$ even though q is greater than r.

To conclude the analysis in this subsection, we provide a sufficient condition on the campaign cost function f such that the campaign choice of type hl depends only on the relative accuracy of the candidate's private signals about her own quality and about her rival's quality. Using the expression for $\underline{\kappa}(\chi)$, we can show by taking derivatives that the function $\chi - \pi(1-\chi, \chi; \underline{\kappa}(\chi))$ is increasing for any $\chi \geq 0.5$ if and only if

$$\frac{\partial \pi(1-\chi, \chi; \underline{\kappa}(\chi))/\partial \chi - 1}{\partial \pi(1-\chi, \chi; \underline{\kappa}(\chi))/\partial \chi + 1} < \frac{\partial \pi(1-\chi, \chi; \underline{\kappa}(\chi))/\partial \kappa}{\partial \pi(1-\chi, \chi; \underline{\kappa}(\chi))/\partial \kappa - f'(\underline{\kappa}(\chi))}.$$

By part (ii) of Lemma 2, the above is satisfied when $\chi < \xi(\underline{\kappa}(\chi))$ because the left-hand-side is negative and the right-hand-side is positive. If $\chi \geq \xi(\underline{\kappa}(\chi)$, the above remains true if the marginal cost of campaign

f' is sufficiently small. Thus, when informative campaigns involve some fixed cost and little marginal cost in the relevant range of levels of informativeness, the candidate credibly signals both her good news about her own quality and bad news about her rival through a strong positive campaign if her good news is more accurate than the bad news, and through a strong negative campaign if the opposite is true. Intuitively, when the marginal cost of campaign is small, the dominant force in determining the value of misinformation through indirect campaigning relative to direct campaigning, whether the target is the rival or the candidate herself, is that the level of direct campaigning, $\underline{\kappa}(\chi)$, required for successful misinformation is highly responsive to the accuracy of the candidate's private signal about the target of direct campaigning, χ. Thus, if the candidate has more accurate private information about her own quality than about her rival, misinformation through a direct positive campaign requires a high level, increasing the relative value of misinformation through indirect campaigning that targets the candidate herself. As a result, it is costly for the candidate to credibly signal both the good news about her own quality and the bad news about her rival through a strong negative campaign that involves in part indirect campaigning that targets the candidate herself. The candidate will instead choose a strong positive campaign.

4 Competing Campaigns

In this section, we add to our model in section 3 the element of competition. Candidate A and B simultaneously and independently choose a campaign which influence the voters' posterior beliefs about their quality. For simplicity, we assume throughout this section that $q = r$, that is, each candidate's signal about herself and about her rival has the same accuracy. To separate the role of competition from that of information, we focus on the limiting case when the candidate and her rival's signals are perfectly correlated. This way, the difference in their campaign choices are driven by competition, rather than the presence of additional information available to the voter.

Formally, suppose that candidate A and B's signals about A's quality are distributed such that with probability u, they are perfectly correlated conditional on A's quality, with

$$\Pr(t_A^A = h \text{ and } t_A^B = h | A = 1) = \Pr(t_A^A = l \text{ and } t_A^B = l | A = 0) = q;$$

and

$$\Pr(t_A^A = l \text{ and } t_A^B = l | A = 1) = \Pr(t_A^A = h \text{ and } t_A^B = h | A = 0) = 1 - q.$$

With probability $1-u$, candidate A and B's signals about A's quality are conditionally independent, with

$$\Pr(t_A^A = h | A = 1) = \Pr(t_A^A = l | A = 0) = \Pr(t_A^B = h | A = 1) = \Pr(t_A^B = l | A = 0) = q.$$

The candidates' signals about B's quality are symmetrically distributed. We focus on the case where u is equal to 1, and thus each candidate knows that her rival's signals are identical to her own. We construct the above model with u possibly less than 1 for technical reasons: in the ensuing equilibrium analysis, it is necessary to define out-of-equilibrium-path beliefs by considering the case of $u < 1$ and then taking the limit as u converges to 1. As in the case of single campaign, there are four private types for each candidate, lh, hh, ll and hl. By construction, regardless of the value of u, we have $\Pr(t_A^A = h | A = 1) = \Pr(A = 1 | t_A^A = h) = q$, as in the case of single campaign. Also, $\Pr(t_B^A = h | B = 1) = \Pr(B = 1 | t_B^A = h) = q$. Finally, due to perfect correlation, the belief about a candidate's quality conditional on two h signals remains q. These features make the model of competing campaigns comparable to the single campaign model in terms of the information available to the two candidates.

4.1 Indirect campaigning and vertical separation

For types lh, hh and ll, we posit equilibrium behavior similar to that in the previous section. Each candidate i runs an uninformative campaign if her private type is lh; runs a positive campaign of level $\underline{\tau}_p > 0.5$ if the private type is hh; and runs a negative campaign of level $\underline{\tau}_n > 0.5$ if the type is ll. Unlike in the single campaign case, however, we posit that type hl randomizes between a strong positive campaign of level $\overline{\tau}_p > \underline{\tau}_p$ and a strong negative campaign of level $\overline{\tau}_n > \underline{\tau}_n$ with equal probabilities.

Consider first type lh of candidate A, who runs an uninformative campaign in equilibrium. Due to perfect correlation, her rival, candidate B, has received two identical private signals and is thus randomizing between a strong positive campaign and a strong negative campaign. Given the posited equilibrium strategy, after observing an uninformative campaign by candidate A and an informative campaign of level $\overline{\tau}_p$ or $\overline{\tau}_n$ by candidate B, the voter's interim belief about A's quality is equal to $1-q$, and the interim belief about B's quality is q. Since type lh's private belief about her own quality and that about B's quality are the same as the corresponding interim beliefs of the voter, the same logic behind part (i) of Lemma 1 implies that the equilibrium payoff to type lh is $1-q-q$, regardless of the campaign choice B ends up making through randomization.

Now, consider what happens when type lh deviates to imitate type hh and runs a weak positive campaign of level $\underline{\tau}_p$. With perfect correlation of private signals across the candidates, the interim belief of the voter about candidate A's quality is undefined, because a weak positive campaign by candidate A suggests her signal is hh, contradicting B's signal lh as suggested by B's equilibrium campaign choice. However, as long as $u<1$, the belief about candidate A's quality conditional on one h signal from candidate A and one l signal from candidate B is given by

$$\Pr(A=1|t_A^A=h,t_A^B=l)=\frac{0.5(1-u)q(1-q)}{0.5(1-u)q(1-q)+0.5(1-u)q(1-q)}=0.5$$

regardless of the value of u. Therefore, in this case, we specify the out-of-equilibrium-path interim belief of the voter that A's quality is 1 as 0.5. From now on, whenever the candidates' campaign choices suggest they have different signals about a candidate, we let the voter's out-of-equilibrium-path interim belief of this candidate's quality be 0.5. Type lh of candidate A is then indifferent between running an uninformative campaign and a weak positive campaign if

$$1-q-q \quad = \quad 0.5\left(\pi\left(1-q,0.5;\underline{\tau}_p\right)+\Pi\left(1-q,0.5;\underline{\tau}_p,\overline{\tau}_n\right)\right)-q-f(\underline{\tau}_p). \tag{6}$$

As described before, with probability 0.5, candidate B chooses a strong positive campaign, and thus type lh candidate A's campaign gives the only campaign signal about herself. In her expectation, $\pi\left(1-q,0.5;\underline{\tau}_p\right)$ is the voter's posterior belief that her quality is 1 when her own private belief is $1-q$, the voter's interim belief is 0.5 and the level of her campaign is $\underline{\tau}_p$. With probability 0.5, both candidates' campaigns are about A, and $\Pi\left(1-q,0.5;\underline{\tau}_p,\overline{\tau}_n\right)$ is candidate A's expectation of the voter's posterior belief that her quality is 1 when her own private belief is $1-q$, the voter's interim belief is 0.5, and the campaign levels are $\underline{\tau}_p$ and $\overline{\tau}_n$. Formally,

$$\begin{aligned}\Pi\left(1-q,0.5;\underline{\tau}_p,\overline{\tau}_n\right) \quad &= \quad ((1-q)\underline{\tau}_p\overline{\tau}_n+q(1-\underline{\tau}_p)(1-\overline{\tau}_n))\frac{\underline{\tau}_p\overline{\tau}_n}{\underline{\tau}_p\overline{\tau}_n+(1-\underline{\tau}_p)(1-\overline{\tau}_n)} \\ &+ \quad ((1-q)\underline{\tau}_p(1-\overline{\tau}_n)+q(1-\underline{\tau}_p)\overline{\tau}_n)\frac{\underline{\tau}_p(1-\overline{\tau}_n)}{\underline{\tau}_p(1-\overline{\tau}_n)+(1-\underline{\tau}_p)\overline{\tau}_n}\end{aligned}$$

$$
\begin{aligned}
&+ \quad ((1-q)(1-\underline{\tau}_p)\overline{\tau}_n + q\underline{\tau}_p(1-\overline{\tau}_n))\frac{(1-\underline{\tau}_p)\overline{\tau}_n}{(1-\underline{\tau}_p)\overline{\tau}_n + \underline{\tau}_p(1-\overline{\tau}_n)} \\
&+ \quad ((1-q)(1-\underline{\tau}_p)(1-\overline{\tau}_n) + q\underline{\tau}_p\overline{\tau}_n)\frac{(1-\underline{\tau}_p)(1-\overline{\tau}_n)}{(1-\underline{\tau}_p)(1-\overline{\tau}_n) + \underline{\tau}_p\overline{\tau}_n}.
\end{aligned}
$$

Note that $\underline{\tau}_p$ and $\overline{\tau}_n$ enter the function $\Pi\left(1-q, 0.5; \underline{\tau}_p, \overline{\tau}_n\right)$ symmetrically.

Equation (6) defines $\underline{\tau}_p$, the least costly level of campaign that separates type lh from type hh. A symmetric expression defines $\underline{\tau}_n$, which is equal to $\underline{\tau}_p$ in this model. We look for a symmetric equilibrium, so we drop the subscripts and simply write $\underline{\tau}$. To study the properties of the function $\Pi(1-q, 0.5; \underline{\tau}, \overline{\tau})$, we rewrite it as

$$
\Pi(1-q, 0.5; \underline{\tau}, \overline{\tau}) = 0.5 - (0.5 - (1-q))\left(1 - \frac{4\underline{\tau}(1-\underline{\tau})\overline{\tau}(1-\overline{\tau})}{(\underline{\tau}\overline{\tau} + (1-\underline{\tau})(1-\overline{\tau}))(\underline{\tau}(1-\overline{\tau}) + (1-\underline{\tau})\overline{\tau})}\right).
$$

The above expression immediately implies the following lemma, which is the counterpart of Lemma 1. It imposes an upper-bound on the value of misinformation to type lh through positive direct campaigning, and establishes that the value is decreasing in both $\underline{\tau}$ and $\overline{\tau}$.[4]

Lemma 3 (Value of Misinformation III) *(i)* $\Pi(1-q, 0.5; \underline{\tau}, \overline{\tau}) < 0.5$*; and (ii)* $\Pi(1-q, 0.5; \underline{\tau}, \overline{\tau})$ *decreases with both* $\underline{\tau}$ *and* $\overline{\tau}$*.*

Now we show that as in the single-campaign model of section 3, type hh is vertically separated from type lh in that the former strictly prefers running a positive campaign of level $\underline{\tau}$ to running an uninformative campaign, and likewise for type ll. Due to perfect correlation, candidate B runs, in equilibrium, a positive campaign of level $\underline{\tau}$. By the same logic as in part (i) of Lemma 1, the equilibrium payoff for type hh of candidate A from running a positive campaign of level $\underline{\tau}$ is $q - q - f(\underline{\tau})$. If she deviates to imitate type lh and runs an uninformative campaign, then the candidates send opposing signals about A's quality. Following the same logic as above, the voter's out-of-equilibrium-path interim belief about A's quality is 0.5. Thus, type hh's deviation payoff is $0.5 - q$. From the definition of $\underline{\tau}$, type hh strictly prefers running a positive campaign of level $\underline{\tau}$ to running an uninformative campaign if and only if the payoff loss for type hh from switching to an uninformative campaign is greater than the payoff gain for type lh from

[4] While Lemma 1 is stated for arbitrary interim beliefs, for simplicity we restrict in Lemma 3 to the interim beliefs consistent with the least costly separating equilibrium to be constructed.

misinformation, or

$$q - 0.5 > 0.5\left(\pi\left(1-q, 0.5; \underline{\tau}\right) + \Pi\left(1-q, 0.5; \underline{\tau}, \overline{\tau}\right)\right) - (1-q).$$

By part (ii) of Lemma 1 and part (i) of Lemma 3, both $\pi\left(1-q, 0.5; \underline{\tau}\right)$ and $\Pi\left(1-q, 0.5; \underline{\tau}, \overline{\tau}\right)$ are smaller than 0.5, the maximum posterior belief they can induce in the voter if they run an uninformative campaign. The above inequality always holds and thus there is strict separation between type lh and type hh.

Also, as in the previous section, there is horizontal separation between type hh and ll. Suppose that type hh switches to running a negative campaign of level $\underline{\tau}$. Due to perfect correlation, her rival's private signals are identical and in equilibrium is running a positive campaign of level $\underline{\tau}$. Since their campaigns suggest different signals about both A and B, the voter's interim beliefs about them are both 0.5. The deviation payoff to type hh is then

$$0.5 - \Pi\left(q, 0.5; \underline{\tau}, \underline{\tau}\right) - f(\underline{\tau}),$$

as candidate A's private belief of B's quality is q. It is straightforward to verify that

$$\Pi\left(q, 0.5; \underline{\tau}, \underline{\tau}\right) = 1 - \Pi\left(1-q, 0.5; \underline{\tau}, \underline{\tau}\right),$$

and thus from part (i) of Lemma 3 that type hh's deviation payoff is strictly lower than her equilibrium payoff of $q - q - f(\underline{\tau})$.

In contrast with the case of single campaign, there is vertical separation between type hl and types hh and ll in competing campaigns. That is, type hl strictly prefers either the strong positive campaign of level $\overline{\tau}$ or the strong negative campaign of level $\overline{\tau}$ to imitating either type hh or ll's campaign choice. Consider first type hh of candidate A. Given the posited strategies, her equilibrium payoff from running a weak positive campaign of level $\underline{\tau}$ is $q - q - f(\underline{\tau})$. Suppose that type hh deviates to running a strong positive campaign of level $\overline{\tau}$, then similar to what we have argued in the previous subsection, upon observing the deviating campaign choice by candidate A and the equilibrium choice by candidate B, the voter's interim beliefs are such that candidate A's quality is 1 with probability q while candidate B's quality is 1 with probability 0.5. For type hh to be indifferent between her equilibrium choice of campaign and this deviation, we must have

$$q - q - f(\underline{\tau}) \quad = \quad q - \pi\left(q, 0.5; \underline{\tau}\right) - f(\overline{\tau}), \tag{7}$$

where in writing the deviation payoff on the right-hand-side we use the fact that running a strong positive campaign of level $\overline{\tau}$ does not change type hh's expectation of the voter's posterior belief about her own quality, by part (i) of Lemma 1; and the fact that type hh's private belief about her rival's quality is q.[5]

Now consider type hl of candidate A. Due to perfect correlation, her rival runs an uninformative campaign in equilibrium. Type hl's equilibrium payoff from running a strong positive campaign of level $\overline{\tau}$ is then given by $q - (1-q) - f(\overline{\tau})$. Suppose that type hl deviates and runs a weak positive campaign of level $\underline{\tau}$. Then given the weak positive campaign by candidate A and an uninformative campaign by candidate B, the voter's interim beliefs are such that candidate A's quality is 1 with probability q while candidate B's quality is 1 with probability 0.5. Type hl's deviation payoff is then

$$q - 0.5 - f(\underline{\tau}).$$

There is strict separation between type hl and type hh if the payoff loss for type hl from switching to a weak positive campaign is greater than the payoff gain for type hh from misinformation, or

$$0.5 - (1-q) > q - \pi\left(q, 0.5; \underline{\tau}\right).$$

This inequality holds because $\pi\left(q, 0.5; \underline{\tau}\right) > 0.5$: A's private belief about B's quality is higher than the voter's interim belief, and thus the minimum posterior belief of the voter is 0.5.

Intuitively, there is separation between type hl and types hh and ll because competition now affects the effectiveness of indirect campaigning, in contrast to the case of single campaign. In the competing campaigns model, because of private signal correlation, type hh receives less value of misinformation from suggesting that her rival has low quality, because it is undermined by the positive campaign run by her rival. In comparison, type hl has more to lose from not running the more informative positive campaign because in equilibrium her rival does not run an informative campaign to counter her campaign. We stress that this vertical separating result is not driven by the presence of additional information to the voter compared to the single campaign model; this point is discussed in the last section.

Due to the absence of vertical separation, in the single campaign model with symmetry, i.e., $q = r$, the equilibrium campaign choice of type hl is indeterminate in the sense that type hl may randomize arbitrarily between the strong positive campaign of level $\overline{\kappa}$ and the strong negative campaign of the same level. This

[5] We will show below that there exist values of $\underline{\tau}$ and $\overline{\tau}$ that satisfy both equation (6) and (7).

is no longer the case in the competing campaigns model. To see this, consider the two deviations that type lh may choose, a weak positive campaign of level $\underline{\tau}$ and the weak negative campaign of the same level. Suppose that candidate B chooses a strong positive campaign with probability $1-\eta \in [0,1]$, and choose a strong negative campaign with probability η. Then, by deviating to the weak positive campaign of level $\underline{\tau}$, type lh of candidate A obtains an expected payoff of

$$(1-\eta)\pi\left(1-q, 0.5; \underline{\tau}\right) + \eta\Pi\left(1-q, 0.5; \underline{\tau}, \overline{\tau}\right) - q - f(\underline{\tau}),$$

while her deviation payoff from running the weak negative campaign of level $\underline{\tau}$ is

$$(1-q) - \left((1-\eta)\Pi\left(q, 0.5; \underline{\tau}, \overline{\tau}\right) + \eta\pi\left(q, 0.5; \underline{\tau}\right)\right) - f(\underline{\tau}).$$

Thus, type lh prefers the positive campaign of level $\underline{\tau}$ to the negative campaign of the same level if and only if

$$(1-2\eta)\left(\pi\left(1-q, 0.5; \underline{\tau}\right) - \Pi\left(1-q, 0.5; \underline{\tau}, \overline{\tau}\right)\right) > 0.$$

It can be directly verified that

$$\pi\left(1-q, 0.5; \underline{\tau}\right) = \Pi\left(1-q, 0.5; \underline{\tau}, 0.5\right).$$

Since $\Pi\left(1-q, 0.5; \underline{\tau}, \overline{\tau}\right)$ is decreasing in $\overline{\tau}$ by part (ii) of Lemma 3, type lh can be indifferent between the positive campaign of level $\underline{\tau}$ and the negative campaign of the same level only when $\eta = 0.5$. It follows that in any separating equilibrium type hl must be run the positive campaign of level $\overline{\tau}$ and the negative campaign of the same level with equal probabilities.

The reason that we are able to pin down the equilibrium mixing of the highest type hl in the present model is that, with competing campaigns, type hl uses indirect campaigning to inform the voter that she has both bad news for her rival's quality in addition to good news for her own quality, influencing the effectiveness of the potential misinformation campaigns that the lowest type lh may run. In particular, suppose that $\eta < 0.5$, so that type hl is more like to run the strong positive campaign of level $\overline{\tau}$ than the strong negative campaign of the same level. Then for candidate B, who is the lowest type, the weak positive campaign becomes less effective because it is more likely to be contradicted by the strong positive campaign of A. Instead, candidate B is more attempted to deviate to a weak negative campaign. Such a link is absent in the single campaign model.

4.2 Campaign levels in competition

We can now state the following result about the least costly separating equilibrium.

Proposition 2 *Suppose that $q = r$ and $u = 1$. In the symmetric, least costly separating equilibrium of the competing campaigns model, type lh candidate runs an uninformative campaign. Type hh runs an informative positive campaign of level $\underline{\tau} > 0.5$ while type ll runs an informative negative campaign of the same level $\underline{\tau}$. Type hl randomizes between a positive campaign of level $\overline{\tau} > \underline{\tau}$ and a negative campaign of level $\overline{\tau} > \underline{\tau}$ with equal probabilities.*

To specify the out-of-equilibrium-path beliefs that support the above as an equilibrium, first note that only the beliefs after unilateral deviations are relevant. One set of such beliefs is: the deviating candidate is type lh if she runs a positive campaign of level $\tau_p \in (0.5, \underline{\tau})$ or a negative campaign of level $\tau_n \in (0.5, \underline{\tau})$; the deviating candidate is type hh if she runs a positive campaign of level $\tau_p \in (\underline{\tau}, \overline{\tau})$; and is type ll if she runs a negative campaign of level $\tau_n \in (\underline{\tau}, \overline{\tau})$; and if the candidate runs a positive campaign of level $\tau_p > \overline{\tau}$ or a negative campaign of level $\tau_n > \overline{\tau}$, she is believed to be of type hl. We are able to specify the above beliefs as in the case of single campaign, because the voter can infer who is the deviating candidate, and his beliefs about the signals of the non-deviating candidate are the equilibrium beliefs. It is straightforward to verify that the above set of beliefs supports the least costly separating equilibrium as described in the proposition. First, type lh has no incentive to deviate to any positive or negative campaign of level $\tau \in (0.5, \underline{\tau})$. This is because, due to perfect correlation the rival candidate in equilibrium runs either a strong positive campaign or a strong negative campaign of level $\overline{\tau}$, so for example, the deviation payoff from a positive campaign is

$$0.5\,(1 - q + \Pi(1 - q, 1 - q; \tau, \overline{\tau})) - q - f(\tau),$$

which is equal to $1 - q - q - f(\tau)$ and is strictly less than its equilibrium payoff. Second, type hh has no incentive to deviate to a positive campaign of any level $\tau_p < \underline{\tau}$, and symmetrically type ll has no incentive to deviate to a negative campaign of any level $\tau_n < \underline{\tau}$. Given the above out-of-equilibrium-path beliefs, the argument for this claim is identical to the corresponding case under single campaigns, because due to the perfect correlation the rival candidate runs a weak positive campaign of level $\underline{\tau}$, and so the expected posterior belief of the voter about the rival is unaffected by the deviation. Third, type hh has no incentive to deviate to a positive campaign of any level $\tau_p \in (\underline{\tau}, \overline{\tau})$, and a symmetric claim for type ll holds. This is simply because, given the above out-of-equilibrium-path beliefs, type hh cannot improve the expected posterior belief of the voter about her own quality with such a deviation. Fourth, type hl has no incentive to deviate to any campaign of level $\tau \in (\underline{\tau}, \overline{\tau})$. The reason is that the rival candidate does not run an informative campaign, so for example, the deviation payoff from a positive campaign is simply $q - q - f(\tau)$, which is strictly less than its equilibrium payoff of $q - (1 - q) - f(\overline{\tau})$.

The existence of the least costly separating equilibrium is established once we show that there exist values of $\underline{\tau}$ and $\overline{\tau}$, with $\underline{\tau} < \overline{\tau}$, such that equations (6) and (7) are satisfied. Equation (6) may be thought of as defining $\underline{\tau}$ as a best response to any $\overline{\tau}$. Let $\tau_c \in [0.5, 1]$ be uniquely defined according to equation (6) by setting $\underline{\tau} = \overline{\tau} = \tau_c$ in the equation:

$$1 - q - q = 0.5 \left(\pi \left(1 - q, 0.5; \tau_c\right) + \Pi \left(1 - q, 0.5; \tau_c, \tau_c\right)\right) - q - f(\tau_c).$$

It then follows from Lemma 1 and Lemma 3 that for any $\overline{\tau} \in [\tau_c, 1]$, there is a unique $\underline{\tau} \in [0.5, \overline{\tau}]$ that satisfies equation (6), with $\underline{\tau} = \overline{\tau}$ only if $\overline{\tau} = \tau_c$. Turning to equation (7), we can think of it as defining $\overline{\tau} \geq \underline{\tau}$ as a best response to any $\underline{\tau}$. Note that at $\underline{\tau} = \tau_c$ we have $\overline{\tau} > \tau_c$ because $\pi(1 - q, 0.5; \tau_c) > 1 - q$, while at $\underline{\tau} = 0.5$ we again have $\overline{\tau} > \tau_c$ because $\pi(1 - q, 0.5; \tau_c) < 0.5$ and $\Pi \left(1 - q, 0.5; \tau_c, \tau_c\right) < 0.5$. Thus, by continuity there exist values of $\underline{\tau}$ and $\overline{\tau}$ with $\underline{\tau} < \overline{\tau}$ that satisfy both equations, if at $\underline{\tau} = 0.5$ equation (7) implies $\overline{\tau} \leq 1$, or

$$f(1) \geq q - 0.5. \tag{8}$$

We now compare the equilibrium separating levels of campaigns $\underline{\tau}$ and $\overline{\tau}$ to the corresponding levels of campaigns $\underline{\kappa}$ and $\overline{\kappa}$ in the symmetric single campaign model, assuming that condition (8) holds.[6] The question is whether competition raises and lowers the levels of equilibrium campaigns. By equation (6), we have

$$f(\underline{\tau}) = 0.5 \left(\pi \left(1 - q, 0.5; \underline{\tau}\right) + \Pi \left(1 - q, 0.5; \underline{\tau}, \overline{\tau}\right)\right) - (1 - q)$$

[6] It is straightforward to identify additional conditions that are sufficient to imply that the symmetric equilibrium given in Proposition 2 is unique. This is will not be attempted here, as the comparisons we make below apply to each equilibrium if there are multiple of them.

in the competing campaigns case, while

$$f(\underline{\kappa}) = \pi(1-q, q; \underline{\kappa}) - (1-q)$$

in the single campaign case. We already know that

$$\Pi\left(1-q, 0.5; \underline{\tau}, \overline{\tau}\right) < \pi\left(1-q, 0.5; \underline{\tau}\right).$$

It is straightforward to verify that

$$\pi\left(1-q, 0.5; \underline{\tau}\right) < \pi(1-q, q; \underline{\tau})$$

for any $\underline{\tau}$ since $q > 0.5$, and thus

$$\Pi\left(1-q, 0.5; \underline{\tau}, \overline{\tau}\right) < \pi\left(1-q, 0.5; \underline{\tau}\right) < \pi(1-q, q; \underline{\tau}).$$

Since the function $\pi(1-q, q; \kappa)$ is decreasing in κ (part (ii) of Lemma 1), we must have $\underline{\tau} < \underline{\kappa}$.

Thus, competition reduces the least costly level of campaigns that separate type hh and type ll from the lowest type lh. There are two separate reasons for this comparison result. First, because of competition, type lh candidate has a smaller incentive to deviate because, regardless of whether she chooses the weak positive campaign or the weak negative campaign, with probability 0.5, it will be directly countered by her rival's campaign. Second, the voter is less likely to be misinformed even without being countered by the rival, because the gain for type lh, in terms of changing the voter's interim belief, is $q - 0.5$ in the competing campaign case, as opposed to $q - (1-q)$ in the single campaign case.

Finally, we compare $\overline{\tau}$ in the competing campaigns case to $\overline{\kappa}$ in the single campaign case. By equation (7), we have

$$f(\overline{\tau}) = f(\underline{\tau}) + q + \pi\left(1-q, 0.5; \underline{\tau}\right) - 1$$

in the competing campaigns model, while

$$f(\overline{\kappa}) = f(\underline{\kappa}) + 2q - 1$$

in the single campaign model. Since

$$\pi\left(1-q, 0.5; \underline{\tau}\right) < \pi\left(1-q, q; \underline{\tau}\right) < q,$$

where the second inequality follows from part (i) of Lemma 2, and since we already know that $\underline{\tau} < \underline{\kappa}$, we find that $\overline{\tau} < \overline{\kappa}$. This shows that the level of campaigns that separate the highest type hl from the middle types hh and ll is also reduced by competition. Again there are two separate reasons for this result. First, for any fixed campaign level run by types hh and ll, there is now less incentive for the two types to deviate and engage in indirect campaigning to misinform the voter. For example, for type hh, the voter's interim belief is raised from $1-q$ to just 0.5, as opposed to q in the single campaign case. Second, competition has already reduced the equilibrium campaign level for the middle types ($\underline{\tau} < \underline{\kappa}$).

5 Equilibrium Misinformation

In this section, we return to the model in which only candidate A runs an active campaign, but assume that she cares only about the winning probability. Unlike the payoff specification used in the previous sections, the value of information in this *winner-take-all* model is generally non-zero. Further, the value depends both on the type of candidate and the campaign.

5.1 Private and social values of information

The focus of this section is to show that misinformation can occur on the equilibrium path. By our definition, this is possible only if there is some pooling in equilibrium. We begin by characterizing the value of information, which corresponds to computing complete information payoffs in a typical signaling game.

Consider type lh first. Define κ_w as the level of a positive campaign that is just high enough to ensure that campaign signal $s_p = +$ leads to a win for the candidate:

$$\kappa_w \equiv \frac{qr}{qr + (1-q)(1-r)}.$$

Note that regardless of the values of q and r, the minimum level of a negative campaign that $s_n = -$ leads to a win is also κ_w, because for any $\kappa_n = \kappa_p = \kappa$,

$$\frac{r(1-\kappa)}{r(1-\kappa) + (1-r)\kappa} \leq 1 - q$$

is equivalent to

$$\frac{(1-q)\kappa}{q(1-\kappa)+(1-q)\kappa} \leq r.$$

Thus, the value of information to type lh is zero in any campaign, positive or negative, of a level lower than κ_w. The value of information is $q(1-\kappa)+(1-q)\kappa$ for any $\kappa_p \geq \kappa_w$, and $r(1-\kappa)+(1-r)\kappa$ for any $\kappa_n \geq \kappa_w$. Note that the value of information is lower for a positive campaign than for a negative campaign of the same level $\kappa \geq \kappa_w$, and the marginal value of information is negative in both kinds of campaigns. Further, if $q > r$, the value of information is higher in a negative campaign than in a positive campaign of the same level. The above is summarized as the following lemma.

Lemma 4 (Value of Information I) *Under complete information the optimal informative campaign for type lh is negative of level κ_w if $q > r$, and positive of the same level if $q < r$.*

Thus, given that the two kinds of campaign have the same cost function, the value of information to type lh systematically biases type lh's preference between positive and negative campaigns depending on the relative accuracies of her private beliefs about her own quality and that of the rival. In contrast to the expected-margin model where in equilibrium type lh runs an uninformative campaign, such complete information preference in the present winner-take-all model will affect type lh's equilibrium choice when misinformation occurs.

The value of information to type hl is symmetric to type lh: it is zero in any campaign if $\kappa < \kappa_w$; it is negative and equal to $q\kappa_p + (1-q)(1-\kappa_p) - 1$ in a positive campaign of any level $\kappa_p \geq \kappa_w$, with a positive marginal value; and it is equal to $r\kappa_n + (1-r)(1-\kappa_n) - 1$ in a negative campaign of any level $\kappa_n \geq \kappa_w$, which is also negative with a positive marginal value. Furthermore, if $q > r$, the value of information is higher in a positive campaign than in a negative campaign of the same level above κ_w.

For the value of information of types hh and ll, we assume without loss of generality that $q \geq r$. Define κ_ℓ as the minimum level of a positive campaign for type hh to lose after $s_p = -$, given by

$$\kappa_\ell \equiv \frac{q(1-r)}{q(1-r)+(1-q)r}.$$

Note that κ_ℓ is the same as the minimum level of a negative campaign for type hh to lose after $s_n = +$, and that $\kappa_\ell < \kappa_w$. Then, the value of information to type hh is zero in any campaign if $\kappa < \kappa_\ell$ as the candidate always wins. The value of information is negative for $\kappa \geq \kappa_\ell$, equal to $q\kappa_p + (1-q)(1-\kappa_p) - 1$ if $\kappa_p \geq \kappa_\ell$ and $r(1-\kappa_n) + (1-r)\kappa_n - 1$ if $\kappa_n \geq \kappa_\ell$.[7] The value of information is higher for positive campaigns than for negative campaigns of the same level $\kappa \geq \kappa_\ell$, and the marginal value of information is positive for positive campaigns and negative in negative campaigns.

The value of information to type ll is symmetric to type hh: the value of information is zero in any campaign if $\kappa < \kappa_\ell$; it is equal to $q(1-\kappa_p) + (1-q)\kappa_p$ if $\kappa_p \geq \kappa_\ell$, which is positive with a negative marginal value; and it is equal to $r\kappa_n + (1-r)(1-\kappa_n)$ if $\kappa_n \geq \kappa_\ell$, which has a positive marginal value and is greater than the value in a positive campaign of the same level. The above results are summarized in the following lemma.

Lemma 5 (Value of Information II) *Regardless of values of q and r, under complete information type hh weakly prefers a positive campaign to a negative campaign of the same level, while the opposite is true for type ll.*

Thus, in contrast to the expected margin model, the value of information calculations imply that there is horizontal separation between type hh and type ll under complete information. We expect such separation to persist in equilibrium even when misinformation occurs. Thus, horizontal separation in the present winner-take-all model is not driven by misinformation incentives of type lh.

For comparison with misinformation and pooling equilibria constructed below, it may be useful to derive the value of information to a planner who observes the candidate's type and can enforce both campaign choices and the ex post decision between the two candidates based on the realized campaign signal. We may think of such derivation as computing the *social value of information* to distinguish it from the *private* value of information in the two lemmas above.[8] For any type and given the kind of the campaign and the level, the efficient ex post social decision is the same as in the calculations of the private values of information. However, the social value of information is generally different from that to a given type, because the planner takes into account the value to her rival when candidate A loses.

[7] When $q = r$, we have $\kappa_\ell = 0.5$. The posterior beliefs are the same for A and B when the former runs an uninformative campaign. According to the tie-breaking rule, the payoff to A is 1, which is different from the limit of $q\kappa_\ell + (1-q)(1-\kappa_\ell)$ as q approaches r from above.

[8] In the expected-margin model of sections 3 and 4, the social value of information conditional on any type is zero, the same as the private value to the type.

For concreteness, we assume that the planner obtains a payoff of 1 either when candidate A is chosen in the state of $A = 1$ and $B = 0$, or when B is chosen in the opposite state; otherwise the payoff to the planner is 0. The only two relevant states for the calculation of social value of information are thus the state where only A is qualified and the state where only B is qualified. Although there is a degree of arbitrariness in this payoff specification, which makes it hard to compare directly with the private value of information, the payoff specification does not affect the sign of the social value and or the marginal value, or the comparison between positive and negative campaigns.

First, consider type lh. The conditional probability is qr for the state of $A = 0$ and $B = 1$, and $(1-q)(1-r)$ for the state of $A = 1$ and $B = 0$. If the planner runs an uninformative campaign, B will be chosen with probability one, and the expected payoff is qr. By the definition of κ_w, if the planner runs either a positive or a negative campaign of level lower than κ_w, the expected payoff remains the same. The value of information to the planner is zero, as is the case to type lh. If the planner runs either a positive campaign of level $\kappa \geq \kappa_w$ or a negative campaign of level of the same level, the expected payoff is $(qr + (1-q)(1-r))\kappa$. Thus, unlike the value to type lh, the marginal value of information to the planner is always positive, and is independent of the kind of campaign. The case of type hl is symmetric. The value of information to the planner conditional on type hl is zero for any campaign, positive or negative, of level lower than κ_w. If the level is $\kappa \geq \kappa_w$, regardless of whether it is positive or negative, the value of information to the planner is given by $(qr + (1-q)(1-r))\kappa - qr$, which has a positive marginal value, as is true for the value of information to type hl, but is the same for positive and negative campaigns, which is different from the value to type hl.

For the remaining two types, hh and ll, assume as before that $q \geq r$. Then, the value of information to the planner conditional on either type is zero for any campaign, positive or negative, of level lower than κ_ℓ, as is true for the value to the candidate of either type. If the level is $\kappa \geq \kappa_\ell$, regardless of whether it is positive or negative and regardless of whether the planner is conditioning on type hh or ll, the value of information to the planner is given by $(q(1-r) + (1-q)r)\kappa - q(1-r)$, which is always positive with a positive marginal value and is independent of the kind of campaign. These properties contrast sharply with the values of information to type hh and type ll.

5.2 Pooling equilibria

We first now establish a few necessary conditions for type lh to pool with some types in equilibrium. The first condition is that under complete information, type lh has an incentive to run an informative campaign. This requires the value of information in a negative campaign of level κ_w to be greater than the cost, or

$$r(1-\kappa_w)+(1-r)\kappa_w > f(\kappa_w). \tag{9}$$

If the above condition is not satisfied, under complete information type lh prefers to run an uninformative campaign. The analysis in this case would be similar to that in section 3 where type lh is separated from other types in equilibrium. Note that in any equilibrium in which type lh is pooling with some type, the level of campaign cannot exceed κ_w.

The second condition for some pooling in equilibrium is that under complete information, for at least one of the two types, hh and ll, the preferred level of campaign above κ_ℓ is below κ_w. Since by Lemma 5 the value of information is higher in positive campaigns than in negative campaigns above the level of κ_ℓ for type hh, the above condition is satisfied if the marginal value of a positive campaign is lower than the marginal cost at $\kappa_p = \kappa_w$, or

$$f'(\hat{\kappa}_p) \equiv 2q-1 < f'(\kappa_w), \tag{10}$$

together with the standard assumption that f is convex. For type ll, the value of information is higher in negative campaigns, so the corresponding condition is

$$f'(\hat{\kappa}_n) \equiv 2r-1 < f'(\kappa_w). \tag{11}$$

If the preferred level of campaign is above κ_w for both type hh and type ll, then again there will not be pooling equilibria involving type lh. Note that if $\hat{\kappa}_p < \kappa_w$ then the former is the lowest level of equilibrium pooling involving types hh and lh, and similarly $\hat{\kappa}_n$ is the lowest level for types ll and lh. For example, if types hh and lh both run a positive campaign of level $\kappa_p < \hat{\kappa}_p < \kappa_w$ with positive probabilities, then the interim beliefs after any deviation to a positive campaign of a level between κ_p and $\hat{\kappa}_p$ is observed must be such that the candidate loses even if the realized campaign signal is $s_p = +$. However, such beliefs cannot satisfy the standard belief refinement, as type lh would not benefit from such a deviation even if the candidate wins with $s_p = +$ while type hh would.

Assume that if $q \geq r$. Then, $\hat{\kappa}_p \geq \hat{\kappa}_n$, and the necessary conditions for some pooling in equilibrium are (9) and (11). We claim that these two conditions are also sufficient when the difference between q and r is small enough, in that there exist pooling equilibria both type lh and type ll choose the same negative campaign with positive probabilities. More precisely, fix any $\kappa_n' \in [\hat{\kappa}_n, \kappa_w]$. Let κ_p' be the level of positive campaign such that type lh is indifferent between this campaign, which leads to a win if and only if the realized campaign signal is $s_p = +$, and the equilibrium campaign of level κ_n'. That is, define κ_p' such that

$$q(1-\kappa_p') + (1-q)\kappa_p' - f(\kappa_p') = r(1-\kappa_n') + (1-r)\kappa_n' - f(\kappa_n'). \tag{12}$$

Note that $\kappa_p' \leq \kappa_n' \leq \kappa_w$ by Lemma 4. We have the following result about pooling equilibria.

Proposition 3 *Suppose that $q \geq r$, conditions (9) and (11) hold, and f is convex. For any $\kappa_n' \in [\hat{\kappa}_n, \kappa_w]$, if κ_p' given by equation (12) is greater than or equal to $\hat{\kappa}_p$, then there exists an equilibrium in which type lh and type ll run a negative campaign of level κ_n' with positive probabilities.*

In equilibrium, type lh runs the negative campaign of level κ_n' with probability β and the positive campaign of level κ_p' with the remaining probability. Type ll runs the negative campaign of level κ_n' with probability one, while types hh and hl run the positive campaign of level κ_p' with probability one. The out-of-equilibrium interim beliefs are: if either a negative campaign of level $\kappa_n < \kappa_n'$ or a positive campaign of level $\kappa_p < \kappa_p'$ is observed, the candidate's type is lh with probability 1; if a negative campaign of level $\kappa_n > \kappa_n'$ is observed, the candidate's type is ll with probability one; and if a positive campaign of level $\kappa_p > \kappa_p'$ is observed, the candidate's type is hh with probability one.

To verify that the equilibrium constructed above, first observe that since type lh is indifferent between κ_n' and κ_p', type hl weakly prefers the latter to the former. This is because $q \geq r$ and equation (12) imply that

$$q\kappa_p' + (1-q)(1-\kappa_p') - f(\kappa_p') \geq r\kappa_n' + (1-r)(1-\kappa_n') - f(\kappa_n'),$$

and this inequality is strict if $q > r$. Next, consider the interim beliefs on the equilibrium path when $\beta = 0$. If the negative campaign of level κ_n' is observed, the beliefs are that the candidate is qualified with probability $1-q$, and her rival is qualified with probability $1-r$. Since $\kappa_n' \geq \kappa_\ell$, the candidate wins if and only if the campaign signal resulting from κ_n' is $s_n = -$. If the positive campaign of level κ_p' is observed, the beliefs are that the candidate is qualified with probability

$$\frac{\Pr(A=1|t=lh)+\Pr(A=1|t=hh)+\Pr(A=1|t=hl)}{1-\Pr(t=ll)} = \frac{1}{3}(1+q),$$

and her rival is qualified with probability

$$\frac{\Pr(B=1|t=lh)+\Pr(B=1|t=hh)+\Pr(B=1|t=hl)}{1-\Pr(t=ll)} = \frac{1}{3}(1+r).$$

Recall that $\kappa_p' \geq \kappa_\ell$ implies that the candidate wins if and only if the campaign signal resulting from κ_p' is $s_p = +$, if the interim belief is that the candidate's type is hh. Since $q \geq r$ implies

$$\frac{1+q}{1+r} \leq \frac{q}{r},$$

and since

$$\frac{1}{3}(1+q)(1-\kappa_p') + \left(1 - \frac{1}{3}(1+q)\right)\kappa_p' > q(1-\kappa_p') + (1-q)\kappa_p',$$

on the equilibrium path after κ_p' is observed, the candidate wins if and only if the campaign signal resulting from κ_p' is $s_p = +$. When $q > r$, by continuity there exist values of β strictly positive and sufficiently close to 0, such that under the resulting interim beliefs the candidate wins with κ_n' if and only if $s_n = -$, and with κ_p' if and only if $s_p = +$. Given the above analysis, it is now straightforward to verify the equilibrium.

If $q = r$ and thus $\hat{\kappa}_n = \hat{\kappa}_p = \hat{\kappa}$, the condition in Proposition 3 that κ_p' as given by equation (12) is greater than or equal to $\hat{\kappa}_p$ is trivially satisfied for any $\kappa_n' \in [\hat{\kappa}, \kappa_w]$. In this case, we can construct a pooling equilibrium in which both type lh and type hl randomize between the negative campaign of level κ_n' and the positive campaign of the same level, with equal probabilities.

The consequences of equilibrium misinformation are especially easy to see in the symmetric case of $q = r$, with symmetric randomizations. For any pooling level κ' strictly between $\hat{\kappa}$ and κ_w, type lh in equilibrium wins with probability $q(1-\kappa') + (1-q)\kappa'$, either with $s_p = +$ in a positive campaign, or with $s_n = -$ in a negative campaign. When this occurs, an ex post inefficient choice is made between the two candidates because $\kappa' < \kappa_w$. Similarly, at the same pooling level κ', type hl in equilibrium wins with probability $q\kappa' + (1-q)(1-\kappa')$, either with $s_p = +$ in a positive campaign, or with $s_n = -$ in a negative campaign. In this case, again an ex post inefficient choice is made between the two candidates because $\kappa' < \kappa_w$.

The existence of pooling equilibria is robust. When the difference between q and r is large, we can still construct pooling equilibria involving types lh and ll running the same negative campaign so long as the necessary conditions (9) and (11) are satisfied. Observe that $\kappa_w > q$. We claim that for any $\kappa'_n \in [q, \kappa_w]$, there is an equilibrium in which type lh and type ll run a negative campaign of level κ'_n with positive probabilities. On the equilibrium path, types lh and ll run the negative campaign of level κ'_n with probability one; and types hh and hl run a positive campaign of level equal to $\max\{\kappa'_p, \hat{\kappa}_p\}$ with probability one, where κ'_p is given by equation (12). Given this strategy profile, and given that $\kappa'_n \geq q$, the interim beliefs after the equilibrium campaign κ'_n are such that the candidate wins if and only if the realized signal is $s_n = -$. The out-of-equilibrium interim beliefs are given in the same way as in the equilibria constructed for Proposition 3, with $\max\{\kappa'_p, \hat{\kappa}_p\}$ replacing κ'_p. To verify this equilibrium, first recall that type hl prefers κ'_p to κ'_n when $\kappa'_p > \hat{\kappa}_p$. Conditional on running a positive campaign, so long as the candidate wins if and only if the realized campaign signal is $s_p = +$, type hl has the same expected payoff as type hh, and for both types the preferred level above κ_ℓ is $\hat{\kappa}_p$. As a result, type hl prefers the equilibrium positive campaign to the equilibrium negative campaign regardless of whether or not $\kappa'_p > \hat{\kappa}_p$. This completes verifying the construction.

The pooling equilibria we have constructed are also robust in the sense that the specifications of the out-of-equilibrium interim beliefs pass the standard refinement test. This is due to the fact that in the present winner-take-all model, the candidate cares only about increasing the probability of winning. For example, for any positive campaign of level between $\hat{\kappa}_p$ and κ_w, type lh benefit from deviating to this campaign for some out-of-equilibrium interim beliefs if and only if the deviation increases her probability of producing a campaign signal $s_p = +$, which is the same condition for type hh to benefit from it under the same beliefs. In the last section, we briefly describe the issue of equilibrium selection when the candidate cares also about the winning margin.

6 Discussion

6.1 Simultaneous campaigning

We have assumed throughout the paper that the candidates must choose either a positive or a negative campaign. This assumption helps us frame the answer one of our central questions, namely whether the candidate with different types of private news about her own quality and the rival's quality should go positive or negative. Such assumption is natural in applications of our framework where running both kinds of campaigns simultaneously is prohibitively costly. In the opposite environment, we can imagine that the cost function is additively separable, so that the total cost of simultaneously running a positive campaign of level κ_p and a negative campaign of level κ_n is simply $f(\kappa_p) + f(\kappa_n)$. For simplicity we consider only the expected-margin payoff specification with a single active campaign.

In the least costly separating equilibrium, type lh runs an uninformative campaign. Type hh runs an informative positive campaign only, at the level $\underline{\kappa}_p$ that makes type lh indifferent between running an uninformative campaign and imitating type hh. The level $\underline{\kappa}_p$ satisfies the same equation (1) as in the case where one kind of campaign is allowed. Symmetrically, type ll in equilibrium runs only an informative negative campaign of level $\underline{\kappa}_n$ that satisfies (2). Type hl runs both a positive campaign of level $\underline{\kappa}_p$ and a negative campaign of level $\overline{\kappa}_n$. The out-of-equilibrium interim beliefs for any deviation to simultaneous campaigning of any levels κ_p and κ_n are separable: the candidate is qualified with probability $1 - q$ if $\kappa_p < \underline{\kappa}_p$ and q if $\kappa_p \geq \underline{\kappa}_p$ regardless of κ_n; and the rival is qualified with probability r if $\kappa_n < \underline{\kappa}_n$ and $1 - r$ if $\kappa_n \geq \underline{\kappa}_n$ regardless of κ_p.

The result of horizontal separation between type hh and type ll remains intact in this simultaneous campaigning model. The payoff to type hh of deviating to running the equilibrium campaign of type ll is

$$1 - q - \pi(r, 1 - r; \underline{\kappa}_n) - f(\underline{\kappa}_n),$$

which is the same as that in section 3 when type hh can run only one campaign. Furthermore, given that the cost function is additively separable and the out-of-equilibrium beliefs are separably specified, there is no incentive for type hh to add to her equilibrium positive campaign of level $\underline{\kappa}_p$ a negative campaign of any level. For example, if type hh runs the equilibrium campaign of type ll as well her own equilibrium positive campaign, her payoff is

$$q - \pi(r, 1-r; \underline{\kappa}_n) - f(\underline{\kappa}_p) - f(\underline{\kappa}_n),$$

which, by the definition of $\underline{\kappa}_n$ (equation 2), is equal to her equilibrium payoff $q - r - f(\underline{\kappa}_p)$ from running only the positive campaign of level $\underline{\kappa}_p$.

One immediate consequence of the above analysis is that, just as in the model in which the candidate must choose one kind of campaign, in the above least costly separating equilibrium constructed there remains no vertical separation between type hl from types hh and ll. More precisely, both types hh and ll are indifferent between running simultaneous campaigns as type hl does and running their respective single equilibrium campaign, and so is type hl. In particular, the fact that type hl may use both kinds of campaigns simultaneously to signal her favorable private information does not reduce the cost of separation in equilibrium.

6.2 Exogenous new information

In section 4 we have introduced competition in information campaigns and shown that it results in vertical separation of type hl from types hh and ll. It may appear that the absence of such vertical separation in the single-campaign model in section 3 is that the voter does not have a source of information other than the candidate's campaign choices and the realized campaign signals. Now we extend the single-campaign model in section 3 to illustrate the key to the vertical separation is that the information from the rival's campaign is correlated with the candidate's campaign choices. We ask if the absence of vertical separation is due to the assumption in our model that the voter does not have independent information about the candidate other than the campaign signal to form the posterior beliefs. This turns out not to be the case if the information that the voter receives is independent of the candidate's campaign choices.

Without loss of generality, we consider only signals about the candidate that is actively campaigning. Suppose that with probability v the voter receives a perfect signal about candidate A's quality. This reduces type lh's incentive to misinform the voters by running a positive campaign of any level κ_p, as the value of such misinformation becomes $(1-v)(\pi(1-q, q; \underline{\kappa}_p) - (1-q))$. Type hh's incentive to deviate from running an informative positive campaign to not running a campaign also increased, because the loss in terms of a lower expected posterior belief of the voter about her quality becomes $(1-v)(q-(1-q))$ due to the voter's independent opportunity of learning her quality. Thus, the least costly level $\underline{\kappa}_p$ of positive campaign to prevent type lh from deviating is still defined by the indifference condition of type lh:

$$(1-v)(\pi(1-q, q; \underline{\kappa}_p) - (1-q)) = f(\underline{\kappa}_p).$$

It follows from part (ii) of Lemma 1 that an increase in v decreases $\underline{\kappa}_p$. On the other hand, since type lh does not misinform the voter about herself in a negative campaign, the effectiveness of such a campaign is unaffected, and so the least costly level $\underline{\kappa}_n$ of a direct negative campaign does not depend on v. For indirect campaigning, the possible public signal makes any such misinformation targeting the candidate herself less effective, reducing the gain to $(1-v)(q-(1-q))$, while leaving any indirect campaigning that targets the rival unaffected. Either way, vertical separation remains absent in this extension, because the loss to type hl from deviating to running a weak negative campaign of level $\underline{\kappa}_n$, or a weak positive level $\underline{\kappa}_p$, is still equal to the value of misinformation to type ll by running a strong negative campaign, or type hh by running a strong positive campaign.

The presence of exogenous new information has implications to the equilibrium campaign choice of type hl. The value of misinformation through indirect campaigning relative to direct campaigning is now given by $(1-v)(q-\pi(1-q, q; \underline{\kappa}_p))$ when the target is the candidate herself, and remains $r-\pi(1-r, r; \underline{\kappa}_n))$ when the target is the rival. Since an increase in v reduces the least costly separating level of direct positive campaign $\underline{\kappa}_p$, there is now a smaller relative value of misinformation about the candidate, even though an increase in v decreases the effectiveness of both indirect campaigning and direct campaigning. This makes it less likely that type hl of the candidate uses a strong positive campaign to inform the voter of her good news about her own quality and bad news about the rival's quality.

6.3 The winning-margin model

In section 5 we have shown that a continuum of pooling equilibria exists in the winner-take-all model. We have argued that these equilibria are robust to the standard belief refinement due to the nature of the payoff specification. When candidates also care about the margin of winning when they win, however, there are payoff complementarity between interim beliefs and private beliefs that can be used in the belief refinement exercise. For simplicity, we assume that $q = r$. Note that in this case the tie-breaking rule does not matter to the analysis.

The values of $\hat{\kappa}$ and κ_w are still defined as the optimal campaign level under complete information corresponding to type hh and type lh respectively, can be derived in the same way as in section 5. We assume that the counterpart of condition (9) and the counterpart of (11) hold. As in Proposition 3, there is a set of pooling equilibria indexed by some $\kappa' \in [\hat{\kappa}, \kappa_w]$, in each of which both type lh and type hl randomize between the negative campaign of level κ' and the positive campaign of the same level, with equal probabilities, while type hh chooses the positive campaign and type ll chooses the negative campaign with probability one.

Fix a pooling equilibrium associated with some κ' strictly less than κ_w. We will show that under any interim belief that the candidate is qualified with some probability χ and the rival is qualified with probability $\tilde{\chi}$, if type lh weakly prefers a positive campaign of level κ_p to the equilibrium positive campaign of level κ', with $\kappa_p > \kappa'$, then type hh strictly prefers the former to the latter. In the symmetric pooling equilibrium with κ', the interim belief after the equilibrium positive campaign is chosen is that both the candidate and the rival are qualified with probability 0.5. After both the deviation campaign and the equilibrium campaign, the candidate wins if and only if the realized campaign signal is $s_p = +$. For type lh to weakly prefer the higher level κ_p to the equilibrium level κ', we need

$$((1-q)\kappa_p+q(1-\kappa_p))\left(\frac{\chi\kappa_p}{\chi\kappa_p+(1-\chi)(1-\kappa_p)}-\tilde{\chi}\right)-f(\kappa_p) \geq ((1-q)\kappa'+q(1-\kappa'))(\kappa'-0.5)-f(\kappa').$$

Since $\kappa_p > \kappa'$, and since the winning probability is decreasing and the cost is increasing in the level of campaign, the above implies that the winning margin must be greater in deviation than in equilibrium:

$$\frac{\chi\kappa_p}{\chi\kappa_p+(1-\chi)(1-\kappa_p)}-\tilde{\chi} > \kappa' - 0.5.$$

For type hh, however, the winning probability is increasing in the level of campaign. As a result, type hh strictly prefers to deviate under the same interim beliefs:

$$(q\kappa_p+(1-q)(1-\kappa_p))\left(\frac{\chi\kappa_p}{\chi\kappa_p+(1-\chi)(1-\kappa_p)}-\tilde{\chi}\right)-f(\kappa_p) \geq (q\kappa'+(1-q)(1-\kappa'))(\kappa'-0.5)-f(\kappa').$$

It thus follows that only the pooling equilibrium with the highest level κ_w survives the standard belief refinement argument such as D1 of Banks and Sobel (1987).

7 Conclusion

In our model of information campaigns, both the kind and the level of a campaign reveal information about a candidate's private information about herself and her rival. The candidate may have an incentive to misinform the voter by affecting the interim beliefs that the voter uses to evaluate the public campaign signals. A crucial element of the model is that the candidate does not control the realized campaign signal by making the campaign choices. That is, our novel framework is a signaling game in which the signal is an information structure. This allows us to distinguish interim beliefs from ex post beliefs, in order to compare the value of information and the value of misinformation. A related idea in principal-agent models is that the private type of the agent is a signal structure. This appears in the price discrimination model of Courty and Li (2000), in which a consumer knows only the distribution of his valuation for the good and has to report the private realized valuation later in a sequential mechanism by the seller. Perhaps more related, in the optimal auction model of Bergemann and Pesendorfer (2007), a bidder's private type is a signal structure that is optimally designed together with the auction. See also related models in an industrial organization context, for example in monopoly pricing by Ottaviani and Moscarini (2001) and price competition by Damiano and Li (2007). Unlike these models, in our paper the realized campaign signal is publicly observed. Indeed, all we need in our model is that the campaign signal is verifiable; the standard unraveling argument implies that the voter can infer the signal in equilibrium given the candidate's disclosure policy. How to generalize this to richer type and signal space is a topic for further research.

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