Informative Voting in Large Elections*

Ettore Damiano
University of Toronto

Li, Hao
University of British Columbia

Wing Suen
University of Hong Kong

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Abstract. We study two-candidate elections with multiple pivotal events. In addition to determining which candidate is elected, an individual's vote is pivotal when the winning margin exceeds some exogenously fixed levels and yields an additional positive payoff to voters of all preferences. Such events can be interpreted as a threshold margin of victory that allows voters to avoid a costly recount without changing the winner, or provides the winner a mandate for effective governance benefiting all voters. In large elections, the additional pivotal events can become the dominant consideration for rational voters, motivating a large fraction of them to vote informatively according to their private signals. In environments where elections with a single pivotal event fail to aggregate information, the addition of these pivotal events can induce asymptotically the same election outcome in equilibrium as when all private information in the election is public.

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1. Introduction

More than two centuries ago Condorcet (1875) first articulated the idea that voting groups with diverse information make a better choice the larger the group size. Economists recognize that rational voting behavior requires conditioning one's vote on the information inferred from the vote being pivotal as well as on one's own private information (Austen-Smith and Banks, 1996; Feddersen and Pesendorfer, 1996), and that the former can potentially overwhelm the latter. Despite this problem, Feddersen and Pesendorfer (1997) show that the equilibrium outcome in large elections is asymptotically the same as if private information were perfectly aggregated, an outcome they call *full information equivalence*. This result obtains because a small fraction of voters will cast their votes to reflect their own private information and preferences, and that is sufficient to bring about full information equivalence when the total number of voters is large. However, because the vast majority of voters do not vote informatively (i.e., their votes do not reflect their private information), the full information outcome can be sensitive to the model environment. For example, Feddersen and Pesendorfer (1997) point out that their result fails under "aggregate uncertainty," where there are partisan voters who randomly split their votes between the two candidates, resulting in uncertainty in realized vote shares. Another environment when full information equivalence fails involves conflicting preferences, where the same change in the public belief about a candidate can increase his appeal to some voters but lower his appeal to other voters (Bhattacharya, 2013). In both these environments, full information equivalence could obtain had all informed voters voted informatively.

In this paper, we resurrect informative voting as an equilibrium strategy in large two-candidate elections by introducing other pivotal events in addition to the standard one that determines the eventual winner. Consider an election with recount, for example. An election rule in this model is characterized by three thresholds of vote shares for a given candidate and a positive recounting cost. If the vote share for the candidate exceeds the largest threshold then that candidate is declared an outright winner; and symmetrically, if the vote share falls below the smallest threshold then the the opposing candidate is declared an outright winner. If the vote share falls between the smallest and the largest thresholds, a recount takes place after each voter incurs the recounting cost. The candidate is declared the winner if the vote share upon recount is above the middle threshold, and the opposing candidate wins otherwise. Recounting is costly not only because of the physical costs of carrying out the process, but also because of the political turmoil and confusion that it entails. We will refer to this interpretation as "costly recount."

One can also interpret our model as reflecting an election where voters obtain an additional payoff gain when either party wins with a significant winning margin. Suppose voters have different policy preferences regarding some social issues (e.g., abortion) but they all care about effective governance (e.g., bridges and roads getting built without delay or cost overrun). A party with a razor-thin winning margin (the standard pivotal event) may be tied up by political gridlock that reduces the effectiveness of governance.[1] A party with a sufficiently large winning margin, on the other hand, has the mandate to carry out the business of governing more effectively. In some legislatures, there are supermajority rules that give the majority party greater powers when the size of its majority exceeds a certain margin. The general notion that the size of the winning margin matters has been discussed extensively in the political science literature (see, for example, Fowler and Smirnov, 2009). Our model captures in a reduced-form way a political process in which each voter gets an additional payoff when the majority party wins by more than some significant margin. We will refer to this interpretation as "mandate concerns."

Our model of elections with multiple pivotal events is cast in a setting with finitely many states and conditionally independent private signals. We explicitly model aggregate uncertainty by the presence of nonstrategic voters, with the fraction of such voters voting for a given candidate remaining random even in large elections. At the same time, we leave the description of strategic voters' policy preferences as general as possible, to include the broadest set of environments including models such as that of Bhattacharya (2013), where preferences are nonmonotone in the state. The size of the additional payoff gain from a significant winning margin can depend on which of the candidates wins.

Our main result establishes that, whenever full information equivalence is feasible, it is the asymptotic outcome of a sequence of equilibria in elections with multiple pivotal events, in which every strategic agent votes informatively. In our model,

[1] Contentious agenda can crowd out relatively uncontroversial bills that are essential for government to function. A case in point is an attempt by some Republicans to ban mail delivery of abortion pills by packaging it into a crucial agriculture funding bill that must be reauthorized every five years. See "Why Congress is becoming less productive," *Reuters*, March 12, 2024.

corresponding to the middle threshold is the standard pivotal event that votes for the two candidates are tied. Costly recount or mandate concerns create two additional pivotal events. Corresponding to the largest threshold is the pivotal event when one more vote for a given candidate would make him an outright winner, and one more vote for the opponent would trigger a costly recount or reduces the effectiveness of governance but would not change the winner. Corresponding to the smallest threshold is the symmetric pivotal event. Although the probabilities of the three pivotal events conditional on the state all vanish in the limit, one of them becomes dominant because its conditional probability goes to zero at the slowest rate.[2] In our equilibrium construction, the pivotal events of winning by a significant margin always dominate the pivotal event that determines who wins the election. At these pivotal events, the desire to save recounting cost or to reduce gridlock is the only motive for a sufficiently large fraction of informed voters, whose incentives are entirely aligned. Each of them votes for one candidate or the other depending on which of the two pivotal events is more likely, as a function of his private signal. As a result, our election rule aggregates information efficiently whenever that is feasible, including in environments where a standard election rule would fail to do so. Furthermore, in the costly recount interpretation, the probability of recount in equilibrium is negligible in large elections, and thus the improvement in information aggregation is achieved at no cost. Similarly, in the mandate concerns interpretation, the probability of the winning candidate achieving the requisite margin of victory approaches one in large elections.

In environments where a standard election already achieves the full information outcome in equilibrium, having the additional pivotal events still improves the outcome by increasing the rate of convergence to full information equivalence. In other words, the same desired level of information aggregation can be obtained by an electorate of smaller size under an election rule with these additional pivotal events. The reason is that in our equilibrium construction, the dominant pivotal event does not change the identity of the winner. In equilibrium the choice between the two candidates depends not on a voter's policy preferences, but on their signals. This makes the vote count more informative compared to a standard election with a single pivotal event.

[2]This is a consequence of the theory of large deviations, which studies the limit behavior of rare events. See, for example, Dembo and Zeitouni (1998) for a textbook treatment.

In presidential elections in the United States, recounting is implemented in all 50 states and the District of Columbia. As initial counting of votes can have errors, correcting these errors may be the main reason for allowing recounting. In some states, a recount is automatically trigged whenever the initial vote count falls in a pre-specified close vote margin. In other states, there is no automatic recounting, but candidates, election officials and even voters are allowed to request a recount after the initial vote count, with states differing in whether there are requirements of a close vote margin, and how the recounting costs are allocated. Our model is one of automatic recounting, but requested recounting leads to the same equilibrium outcome. Our result is that, to the extent that recounting is costly to individual voters regardless of their preferences, full information equivalence is achieved whenever it is feasible. This highlights an information aggregation role of recounting that is independent of whether initial counting has small or no errors.

In an increasingly polarized political world of elections, voters can be "ideological" when it comes to their concerns for post-election effective governance. For example, in presidential elections in the United States, some voters care about giving a mandate to the winner when a Republican wins but not when a Democrat wins. Our main model assumes existence of pivotal events where an individual's vote changes the election outcome by conferring a mandate for the winner, and thus provides an additional payoff to voters of all preferences. These mandate payoffs can have any positive values, and can depend on the identity of the winner. When voters have ideological mandate concerns, our main result remains valid so long as some groups of voters have mandate payoffs that are independent of preferences within each group, and have signals that together are sufficient for the feasibility of full information equivalence. Broadly speaking, information aggregation occurs in large elections if enough informed voters have common interests in effective governance by the winner.

1.1. Related literature

We introduce multiple pivotal events in this paper to resurrect informative voting in large elections. The key to our equilibrium construction relies on the fact that, while the probabilities of different pivotal events are all vanishingly small in large elections, the rate at which they go to zero can be ranked. Elections with multiple pivotal events also feature in Razin (2003) and McMurray (2017) in the context of signaling policy preferences by voters, and in Bouton and Castanheira (2012) and Ahn and Oliveros (2012) in models of multi-candidate and multi-issue voting.

While the economics literature has paid relatively little attention to elections with recounts, the idea that having a mandate to govern by winning with a significant margin can change political outcomes and hence voters' payoffs has been explored previously. Castanheira (2003) and Faravelli, Man and Walsh (2015) use models of mandate to study election turnout. Hererra, Llorente-Saguer and McMurray (2019) use mandate concerns to motivate a comparison between marginal voting incentives and pivotal voting incentives in large Poisson games (Myerson, 2000).

The logic in resurrecting full information equivalence by adding new pivotal events is that they induce a desire to increase the vote share of the winning candidate that is independent of voters' policy preferences. McLennan (1998) has shown that when voters have the same preferences over candidates in each state but have different information, there is a Nash equilibrium in the election that achieves full information equivalence. In our model voters do not have the same preferences, but after new pivotal events that align the interests for a sufficiently large fraction of informed voters are introduced, they can become dominant in large elections and achieve full information equivalence whenever it is feasible.[3] The payoff gains from the additional pivotal events in our model are related to, but distinct from, the motive to vote for the winning side studied by Callander (2007; 2008). There is a single pivotal event in Callander's (2008) model, whereas there are multiple pivotal events in ours. Moreover only voters who vote for the winning side get the additional payoff gain when voters like to win. In our model, in contrast, all voters get an additional payoff when one side wins with a significant margin. This aspect of our model ensures that our equilibrium voting strategies are monotone in signals. Although there can be multiple equilibria as in Callander (2007; 2008), our model does not exhibit perverse equilibria in which the "wrong" candidate (under full information equivalence) almost surely wins.

Also relevant to the present paper is a literature that asks whether the Condorcet jury theorem continues to hold when acquiring information is costly to individual

[3]Following Feddersen and Pesendorfer (1997), we assume that signals of informed voters satisfy the monotone likelihood ratio property to characterize feasibility of full information equivalence. See Barelli, Bhattacharya and Siga (2022) for a characterization of feasibility of full information equivalence in common-value elections for general signal structures.

agents. Mukhopadhaya (2005) shows that in a symmetric mixed strategy equilibrium, as the number of committee members increases, each member chooses to collect information with a smaller probability. Koriyama and Szentes (2009) consider a model in which agents choose whether or not to acquire information in the first stage, and then the decision is made according to an ex post efficient rule in the second stage. They show that the Condorcet jury theorem fails for groups larger than a certain size. However, in a model with the quality of information as a continuous choice variable, Martinelli (2006) shows that if the marginal cost of information is near zero for nearly irrelevant information, then there will be effective information aggregation despite the fact that each individual voter will choose to be very poorly informed. Krishna and Morgan (2012) show that when participation in an election is costly but voluntary, those who choose to participate will vote informatively even in a standard election. However the fraction of participating voters is vanishingly small in a large election, rendering asymptotic information efficiency difficult to achieve if there is aggregate uncertainty in the model.

The modern version of Condorcet jury theorem with strategic voters relies on pivotal reasoning, that is, a voter casts her vote with belief conditional on the event that she is pivotal. In high-stakes settings such as supreme court decisions (Iaryczower and Shum, 2012) or shareholder voting (Maud and Rydqvist, 2009), there is evidence that voters are strategically sophisticated: they do not simply vote according to their private information and cast their votes as if they were decisive. In a laboratory setting, on the other hand, Esponda and Vespa (2014) show that a sizable fraction of subjects behave nonoptimally because they have difficulty extracting information from hypothetical (pivotal) events. Sincere voting makes it more difficult to achieve information efficiency because such voting strategy is not type-independent: a sincere voter who receives an imperfect signal in favor of one candidate may still vote for the other candidate if her preference for the latter is strong enough. However, we show that as long as the fraction of sincere voters in the electorate is not too large, information efficiency can still be achieved.

2. A Model of Elections with Multiple Pivotal Events

We study an election with a large number $n+1$ of voters to choose between two candidates: L and R. Denote the share of votes for R as V. An *election rule* consists of three thresholds v^L, v^C and v^R, satisfying $v^L < v^C < v^R$, and completely specifies all payoff-relevant outcomes of the election:

- candidate L wins the election with a significant margin of victory if $V < v^L$;
- candidate L wins with a narrow margin if $V \in [v^L, v^C]$;
- candidate R wins with a narrow margin if $V \in (v^C, v^R]$;
- candidate R wins with a significant margin if $V > v^R$.

Voters are independently drawn from a large population of potential voters. A fraction $\alpha \in [0, 1)$ of potential voters are nonstrategic voters; the rest are informed. There is a finite number $S \geq 2$ of payoff-relevant states $\mathbb{S} = \{1, \ldots, S\}$. Voters' common prior belief over $\mathbb{S}$ is described by the distribution $(\mu(1), \ldots, \mu(S))$, with $\mu(s)$ being the probability of state $s \in \mathbb{S}$. Each informed voter observes a signal j about the state from the signal space $\mathbb{J} = \{1, \ldots, J\}$, with $J \geq 2$. We assume that signals are independent across voters conditional on the state. Further,

Assumption 1. *The signal's conditional distributions, $\{\beta(\cdot|s)\}_{s \in \mathbb{S}}$, satisfy the monotone likelihood ratio property:*

$$\frac{\beta(j|s)}{\beta(j|s')} > \frac{\beta(j'|s)}{\beta(j'|s')} \quad \text{for all } s > s' \text{ and } j > j'.$$

Nonstrategic voters are introduced to preserve uncertainty about the realized vote share in each given state s in large elections. A fraction θ of them vote for candidate R and the remaining fraction $1 - \theta$ vote for L.[4] The fraction θ is a random variable distributed on $[\underline{\theta}, \overline{\theta}] \subset [0, 1]$, with a continuous and positive density function f. We refer to θ as the aggregate uncertainty state, and assume that it is independent of the payoff state s.

Potential voters who are informed are divided into $M \geq 1$ groups; the fraction of each group m is $\lambda_m > 0$, with $\sum_{m=1}^{M} \lambda_m = 1$. Voters belonging to the same group m are heterogeneous with respect to a preference type $t \in \mathbb{T}$. The distribution of preference types among the population of potential informed voters is given by a distribution function P_m over $\mathbb{T}$, and is independent of s and θ. For simplicity we

[4]Nonstrategic voters are partisan in the sense that they have preferences between the two candidates that cannot be swayed by any evidence. Otherwise, they may optimally choose to abstain from voting. See Feddersen and Pesendorfer (1996).

assume that $\mathbb{T}$ is a closed interval of $\mathbb{R}$, and P_m has a continuous density. The payoff to an informed voter of group m as a function of the vote share V of candidate R, the realized payoff relevant state s, and the voter's preference t, is given by

$$\begin{cases} w_m(L,s,t) + \delta^L & \text{if } V < v^L \\ w_m(L,s,t) & \text{if } V \in [v^L, v^C) \\ w_m(R,s,t) & \text{if } V \in [v^C, v^R] \\ w_m(R,s,t) + \delta^R & \text{if } V > v^R, \end{cases}$$

where $w_m(L,s,t)$ and $w_m(R,s,t)$ are continuous in t for all $s \in \mathbb{S}$, and $0 < \delta^L, \delta^R < \infty$. For vote shares exceeding the thresholds v^L and v^R for the winner's margin of victory, δ^L and δ^R represent the respective payoff in addition to $w_m(L,s,t)$ and $w_m(R,s,t)$. Define

$$u_m(s,t) := w_m(R,s,t) - w_m(L,s,t)$$

as the payoff *difference* when the winner switches from candidate L to R. By assumption, $u_m(s,t)$ is continuous in t.

In a benchmark model without aggregate uncertainty, that is, $\alpha = 0$, and where the realized state s is publicly observed, voters of any group $m = 1, \ldots, M$ with preference $t \in \mathbb{T}$ have a weakly dominant strategy of voting for R if and only if $u_m(s,t) > 0$. We make the following assumption to ensure that this *full information outcome* is monotone in the realized state s.

Assumption 2. *Payoff differences $\{u_m\}_{m=1,\ldots,M}$, preference distributions $\{P_m\}_{m=1,\ldots,M}$, and the corresponding fractions $\{\lambda_m\}_{m=1,\ldots,M}$ jointly satisfy*

$$\sum_{m=1}^{M} \lambda_m \int_{\{t \in \mathbb{T} | u_m(s,t) > 0\}} \mathrm{d}P_m(t) > \sum_{m=1}^{M} \lambda_m \int_{\{t \in \mathbb{T} | u_m(s',t) > 0\}} \mathrm{d}P_m(t) \quad \textit{for all } s > s'.$$

By Assumption 2, at the full information outcome the winner changing at most once as a function of the realized payoff state. Let $s^R \in \mathbb{S}$ be the lowest state for which the winner is candidate R at the full information outcome. For notational brevity, let $s^L = s^R - 1$; this is highest state for which the winner is candidate L at the full information outcome. We refer to s^R and s^L as a "critical pair of states." To make the analysis interesting, we assume that $1 \leq s^L < s^R \leq S$.

2.1. Remarks on the model

As we already mentioned in the introduction, we adopt two alternative, but not mutually exclusive, interpretations of the pivotal events $V = v^L$ and $V = v^R$. In the costly recount interpretation, a costly recount is triggered when the vote share V falls between $[v^L, v^R]$. Under this interpretation, a natural assumption is that $\delta^R = \delta^L = \delta$, but it is not necessary for our analysis. Our model only requires that the recount costs δ^R and δ^L are positive and finite, but it does not impose any restrictions on the magnitude of these costs.[5]

Under the costly recount interpretation, if the pivotal event $V = v^R$ occurs, then candidate R will win upon recount; and likewise candidate L eventually wins at the pivotal event $V = v^L$. In other words, we assume away any counting errors. If an initial vote count is subject to errors that are corrected in a recount, then regardless of whether the errors are independent across voters or systemic in the election, an election of a sufficiently large size has an equilibrium that leads to the full information outcome as long as the errors are sufficiently small. See Section 4.1 for a more thorough discussion.

Under the mandate concerns interpretation, the parameters δ^R and δ^L represent the payoff gains from more effective governance for each informed voter, regardless of which preference group they belong to. In Section 4.2 we deal with the case where mandate payoffs δ^R_m and δ^L_m vary with preference group m. Our result in the main model, that there is a sequence of equilibria that approaches the full information outcome in large elections whenever the outcome is feasible, continues to hold if the variations in δ^R_m / δ^L_m are small relative to the informativeness of signals. More generally, it is sufficient for our main result if there are enough voters with moderate values of δ^R / δ^L who possess information that can achieve the full information outcome on their own.

Although we adopt the simplifying assumption that there are three pivotal events, the model can be readily adapted to elections with more than three pivotal events. Imagine that both costly recount and mandate concerns are present. Specifically let there be five pivotal vote shares, $v_1 < v_2 < v_3 < v_4 < v_5$, where candidate R wins if

[5] We can also allow voters in some preference groups to have zero, or even negative recounting cost for one of the two candidates, because these voters want recounting after their preferred candidate loses. See Section 4.2 for details.

the vote share V exceeds v_3, recount occurs if $V \in [v_2, v_4]$, and the winning candidate wins with sufficient mandate to deliver effective governance if V is outside $[v_1, v_5]$. As will become clear in the ensuing analysis, the theory of large deviations implies that if there is a strategy profile that approaches the full information outcome in large elections, then the pivotal events that matter under such strategy profile occur when the vote share is at v_1 and v_5. The equilibrium outcome in this model with five pivotal events is the same as in our model with $v^L = v_1$, $v^C = v_3$, and $v^R = v_5$.

Our main result is robust to preference distributions. We have implicitly assumed that all M preference groups have the same support $\mathbb{T}$, but this is for notational brevity only, as we allow the distributions P_m to be different. We can also allow the distributions P_m to be discrete without affecting our main result. The only restriction we impose on preference distributions is Assumption 2. The assumption is satisfied if preferences in each group $m = 1, \dots, M$ are monotone in states, that is, $u_m(s,t) > 0$ implies $u_m(s',t) > 0$ for $s' > s$ and all $t \in \mathbb{T}$, but for our results we only need preference monotonicity in state across all groups. Non-monotonicity is allowed for any particular preference group, and Assumption 2 can be satisfied even with voters in some preference group m having "opposite" preferences ($u_m(s,t) < 0$ implies $u_m(s',t) < 0$ for all $s' > s$) so long as group m voters are in minority (see Example 2 in Section 3.1).

2.2. Strategy and equilibrium

For a given n, we consider a voting game with $n+1$ voters, Γ^n, described by common knowledge of the following:

- election rule $\{v^L, v^C, v^R\}$;
- set of payoff relevant states $\mathbb{S}$, with prior belief μ;
- fraction α of nonstrategic voters, and set of possible aggregate uncertainty states $[\underline{\theta}, \overline{\theta}]$, with density function f;
- preference type space $\mathbb{T}$, and for each preference group $m = 1, \dots, M$, payoff difference function u_m, payoff jumps δ^L and δ^R, preference distribution P_m, and fraction λ_m among potential voters;
- set of signals $\mathbb{J}$, and for each $s \in \mathbb{S}$, conditional probabilities $\beta(\cdot|s)$ over $\mathbb{J}$.

A strategy describes the probability that an informed voter of each preference group $m = 1, \ldots, M$, casts a vote in favor of candidate R as a function of both their preference type $t \in \mathbb{T}$ and the realization of their private signal $j \in \mathbb{J}$. Thus a strategy is a profile $k = (k_1, \ldots, k_M)$ of functions

$$k_m : \mathbb{T} \times \mathbb{J} \to [0, 1], \ m = 1, \ldots, M.$$

We focus on symmetric strategies in that every informed voter of the same preference group m adopts the same strategy k_m. We restrict the strategy space to profiles of functions k_m such that the integral of $k_m(\cdot, j)$ with respect to P_m is well defined.

Since preferences and information are independent from each other, we can write the probability $H(j; k)$ that a randomly drawn informed voter casts a vote for candidate R as a function of the private signal $j \in \mathbb{J}$ they observe:

$$H(j; k) = \sum_{m=1}^{M} \lambda_m \int_{\mathbb{T}} k_m(t, j) \, \mathrm{d}P_m(t).$$

For any strategy k, we define $z(s, \theta; k)$ as the probability that a randomly drawn voter casts a vote for candidate R in the payoff state s and aggregate uncertainty state θ:

$$z(s, \theta; k) = (1 - \alpha) \sum_{j=1}^{J} \beta(j|s) H(j; k) + \alpha\theta.$$

We will refer to $z(s, \theta; k)$ as the vote share for candidate R in state (s, θ) given the strategy k.[6]

Since signals are independent across voters conditional on the state, from the perspective of each informed voter, the probability of realizing a vote share V for candidate R conditional on the payoff state s and the aggregate uncertainty state θ is given by:

$$g^n(V|s, \theta; k) = \binom{n}{nV} z(s, \theta; k)^{nV} \left(1 - z(s, \theta; k)\right)^{n(1-V)}.$$

Define

$$g^n(V|s; k) = \int_{\underline{\theta}}^{\overline{\theta}} g^n(V|s, \theta; k) f(\theta) \, \mathrm{d}\theta.$$

To avoid dealing with integer problems, we have implicitly assumed in the above expressions that nv^L, nv^C and nv^R are all integers. Given our focus on the limit case

[6] When $\alpha = 0$, i.e., there is no aggregate uncertainty, we continue to use the notation $z(s; k)$ to represent the vote share for R. The same applies to the notation of g^n below.

for n large, it is equivalent to assuming that the voting rule thresholds are rational numbers.

An informed voter casts their vote assuming that their vote is pivotal. Upon observing a private signal realization $j \in \mathbb{J}$, regardless of which group m they belong to, an informed voter's private belief that the state is s becomes

$$\mu(s|j) = \frac{\mu(s)\beta(j|s)}{\sum_{s' \in \mathbb{S}} \mu(s')\beta(j|s')}.$$

By Assumption 1, the posterior distributions over $\mathbb{S}$ after observing a signal realization, $\{\mu(1|j), \ldots, \mu(S|j)\}_{j \in \mathbb{J}}$, are ordered with respect to first order stochastic dominance, so that the higher the signal observed, the more an informed voter revises their expectation about the realized state upward.

For an informed voter of group $m = 1, \ldots, M$ with preference $t \in \mathbb{T}$, at any signal $j \in \mathbb{J}$, there are three pivotal events:

- $V = v^L$: voting for candidate R instead of L deprives L a significant margin of victory, resulting in a payoff loss of δ^L in any state.
- $V = v^C$: voting for R instead of L changes the winner, resulting in a payoff change of $u_m(s,t)$ when the realized state is s.
- $V = v^R$: voting for R instead of L gives R a significant margin of victory, resulting in a payoff gain of δ^R in any state.

From now on, for simplicity we say "pivotal event v^L" instead of "pivotal event $V = v^L$", and similarly for the other two pivotal events.

Any best response k'_m to k by the voter satisfies

$$k'_m(t,j) \sum_{s \in \mathbb{S}} \mu(s|j) \left(-g^n(v^L|s;k)\delta^L + g^n(v^C|s;k)u_m(s,t) + g^n(v^R|s;k)\delta^R \right) \geq 0 \geq$$
$$(1 - k'_m(t,j)) \sum_{s \in \mathbb{S}} \mu(s|j) \left(-g^n(v^L|s;k)\delta^L + g^n(v^C|s;k)u_m(s,t) + g^n(v^R|s;k)\delta^R \right). \quad \text{(BR)}$$

Upon observing a private signal j, an informed voter of group m with preference t must cast with probability one a vote that yields a strictly larger expected payoff, where expectations are taken with respect to the probability functions $g^n(\cdot|\cdot;k)$ obtained from the primitives of the game and from strategy k, and can randomize if they are indifferent. Conditions (BR) then define the set of all best responses $k' = (k'_1, \ldots, k'_M)$ to k.

Definition 1. *An equilibrium of the Bayesian game Γ^n is a fixed point $\hat{k}^n$ of the best response correspondence given by (BR).*

The above definition allows both pure-strategy and mixed-strategy Bayesian Nash equilibrium in Γ^n.

2.3. Ranking of pivotal events

Fix a strategy k, and consider candidate R's vote share $z(s, \theta; k)$ for any $s \in \mathbb{S}$ and $\theta \in [\underline{\theta}, \overline{\theta}]$. As an election becomes larger, the probability $g^n(v|s, \theta; k)$ that the actual vote share for candidate R equals a particular value v becomes vanishingly small. A key observation of this paper is that the *rates* at which the probabilities of different pivotal events go to zero as n becomes large are different, so that in the limit some pivotal events are infinitely more likely to occur than others. Calculating the rate of convergence is therefore an important part of the analysis of large elections with multiple pivotal events.

Using Stirling's approximation formula for the binomial coefficient, we have

$$\lim_{n\to\infty} g^n(v|s, \theta; k) \frac{\sqrt{2\pi v(1-v)n}}{I\left(v; z(s, \theta; k)\right)^n} = 1,$$

where

$$I(v; z) = \left(\frac{z}{v}\right)^v \left(\frac{1-z}{1-v}\right)^{1-v}.$$

It can be readily verified that $I(v; z)$ increases in z for $z < v$ and decreases in z for $z > v$, attaining the maximum value of 1 at $z = v$. The function $-\log I(v; z)$ is known as the "rate function" or "entropy function" in the theory of large deviations, and determines the rate at which $g^n(v|s, \theta; k)$ for any $v \neq z(s, \theta; k)$ goes to 0.

In our model an informed voter does not know the aggregate uncertainty state θ. The probability that they assign to a certain pivotal event v to occur in the payoff relevant state s, $g^n(v|s; k)$, is the integral of $g^n(v|s, \theta; k)$ over all possible aggregate uncertainty states θ. The following lemma shows that in comparing the rates of convergence across different events and payoff states, only the aggregate uncertainty state θ which maximizes the function $I(v; z(s, \theta; k))$ matters. For any v, s and k, since $z(s, \theta; k)$ increases in θ, we can define

$$\theta(v, s; k) := \arg \min_{\theta\in[\underline{\theta},\overline{\theta}]} |z(s, \theta; k) - v|.$$

Given that $I(v;z)$ increases in z for $z < v$ and decreases in z for $z > v$, and since $z(s,\theta;k)$ increases in θ, we have

$$\theta(v,s;k) = \arg\max_{\theta\in[\underline{\theta},\overline{\theta}]} I(v;z(s,\theta;k)).$$

For pivotal event v in state s under strategy k, the aggregate uncertainty state that makes the vote share z closest to v maximizes the rate $I(v;z(s,\theta;k))$. For example, when $z(s,\overline{\theta};k) < v \leq v' < z(s',\underline{\theta};k)$, we have $\theta(v,s;k) = \overline{\theta}$ and $\theta(v',s';k) = \underline{\theta}$.

Lemma 1. *Fix any strategy k. For any v, v' and any s, s',*

$$\lim_{n\to\infty} \frac{g^n(v'|s';k)}{g^n(v|s;k)} = \frac{f(\theta(v',s';k))}{f(\theta(v,s;k))} \lim_{n\to\infty} \frac{g^n(v'|s',\theta(v',s';k);k)}{g^n(v|s,\theta(v,s;k);k)}.$$

Proof. See the appendix. ∎

Together with Sterling's approximation formula, Lemma 1 implies that for given strategy k, the ratio $g^n(v'|s';k)/g^n(v|s;k)$ for any pair of pivotal events v, v' and any pair of payoff states s, s' can have a limit different from zero or infinity only if an *equal-rate* condition is satisfied

$$I\left(v';z(s',\theta(v',s';k);k)\right) = I\left(v;z(s,\theta(v,s;k);k)\right). \tag{ER}$$

When the ratio $g^n(v'|s';k)/g^n(v|s;k)$ goes to zero, the pivotal event v in state s is "infinitely" more likely than the pivotal event v' in state s' under strategy k in an arbitrarily large election. We refer to this relation as *dominance*, and in the following definition extend it to a sequence of strategies $\{k^n\}$.

Definition 2. *Given a sequence of strategies $\{k^n\}$, a pivotal event v in state s* ***dominates*** *another pivotal event v' in state s', if*

$$\lim_{n\to\infty} \frac{g^n(v'|s';k^n)}{g^n(v|s;k^n)} = 0.$$

From Lemma 1, if for example $z(s,\overline{\theta};k^n) < v < v' < z(s',\underline{\theta};k^n)$ for all n sufficiently large, then v dominates v' in state s, while v' dominates v in state s'.

3. Full Information Equivalence

The central question of this paper is whether pivotal events with additional preference-independent payoffs can help achieve the same outcome as in an election with just informed voters and common knowledge of all the private signals about the payoff-relevant states. We have defined the full information outcome as electing candidate R in state s^R and higher, and candidate L in state s^L and lower. The following definition, adapted from Feddersen and Pesendorfer (1997), reflects the presence of aggregate uncertainty in our model, and requires that the election outcome is not affected by the aggregate uncertainty state realization.

Definition 3. *A sequence of strategies $\{k^n\}$ achieves **full information equivalence** if for all $\epsilon > 0$, there is an N such that for all $n > N$ and for any realization of the uncertainty state θ, candidate L wins the election with probability greater than $1 - \epsilon$ when the payoff relevant state is $s \leq s^L$, and candidate R wins with probability greater than $1 - \epsilon$ if $s \geq s^R$.*

We say that full information equivalence is *feasible* if it is achieved by some $\{k^n\}$. In the presence of aggregate uncertainty, full information equivalence may not be feasible. While the aggregate information available to informed voters would always be sufficient to identify the payoff relevant state in a large election, the noise introduced in the voting outcome by nonstrategic voters may be so large enough that no sequence of strategies ever satisfies the conditions in Definition 3. The following simple result, which immediately follows from the weak law of large numbers, provides a characterization of feasibility of full information equivalence.

Lemma 2. *Full information equivalence is feasible if and only if there exists a strategy k such that*

$$z(s, \overline{\theta}; k) < v^C < z(s', \underline{\theta}; k) \quad \textit{for all } s \leq s^L \textit{ and } s' \geq s^R. \tag{FE}$$

Full information equivalence requires that there is a voting strategy k by informed voters that generates a sufficiently large spread of R's vote share between state s^R and higher, and state s^L and lower, so that the election outcome is determined by informed voters only and not by the aggregate uncertainty state. Whether there exists a strategy k that satisfies (FE) depends jointly on the outcome-determining threshold v^C together with the informativeness of the voters' signals and the distribution of aggregate uncertainty. It does not depend on whether the election has a single pivotal event v^C, or two other pivotal events represented by thresholds v^L and v^R.

3.1. Illustrating examples

With a single pivotal event v^C, full information equivalence may fail even if Assumption 2 is satisfied. There are strategies that induce a large enough vote share spread to separate s^R and higher states from s^L and lower states, but they are not part of any equilibrium. We use separate examples to illustrate two underlying reasons, a sizable presence of aggregate uncertainty and a significant level of aggregate non-monotone preferences in state. Following each example, we preview our main result by constructing a sequence of equilibria that achieves full information equivalence in elections with two additional pivotal events v^L and v^R.

Example 1: Aggregate uncertainty

Consider the following binary-state, binary-signal election with a single pivotal event. Let $\mathbb{S} = \{1, 2\}$, with equal prior $\mu(1) = \mu(2) = 1/2$; $\mathbb{J} = \{1, 2\}$, with symmetric conditional probabilities $\beta(j = 1|s = 1) = \beta(j = 2|s = 2) = q \in (1/2, 1)$; and $v^C = 1/2$. As in Feddersen and Pesendorfer (1997), there is an aggregate uncertainty state θ, which we assume is distributed on $[0, 1]$ with $f(\theta) = 1$ for all $\theta \in [0, 1]$. There is a single preference group, with $M = 1$ and a payoff difference function (we drop the subscript m):

$$u(s, t) = \begin{cases} -t & \text{if } s = 1 \\ 1 - t & \text{if } s = 2, \end{cases}$$

with t uniformly distributed on $[0, 1]$.

The full information outcome is electing candidate L when $s = 1$, and candidate R when $s = 2$. Assumption 2 is satisfied, and full information equivalence is feasible, whenever the strategy of $\hat{k}$ of "vote-your-signal," i.e., vote for L at $j = 1$ and R at $j = 2$, satisfies (FE):

$$z(s = 1, \theta = 1; \hat{k}) = (1 - \alpha)(1 - q) + \alpha < 1/2 < (1 - \alpha)q = z(s = 2, \theta = 0; \hat{k}).$$

The above inequalities reduce to $(1 - \alpha)q > 1/2$.

Fix any n. For any value of α, the following strategy $\hat{k}$ for an informed voter is an equilibrium: at signal $j = 1$, choose candidate L if $t > 1 - q$ candidate R if $t \leq 1 - q$; at signal $j = 2$, choose L if $t > q$ and R if $t \leq q$. To see this, note that given the assumed strategy profile, the vote share functions are:

$$z(s=1,\theta;\hat{k}) = 2(1-\alpha)q(1-q) + \alpha\theta,$$
$$z(s=2,\theta;\hat{k}) = (1-\alpha)\left(q^2 + (1-q)^2\right) + \alpha\theta.$$

By symmetry, the probability that candidate R receives exactly half of the votes of n randomly selected voters in state $s = 2$ satisfies

$$\begin{aligned} g^n\left(v^C \middle| s=2;\hat{k}\right) &= \binom{n}{nv^C} \int_0^1 \left(z(2,\theta;\hat{k})(1-z(2,\theta;\hat{k}))\right)^{nv^C} \,\mathrm{d}\theta \\ &= \binom{n}{nv^C} \int_0^1 \left(z(1,\tilde{\theta};\hat{k})(1-z(1,\tilde{\theta};\hat{k}))\right)^{nv^C} \,\mathrm{d}\tilde{\theta} = g^n\left(v^C \middle| s=1;\hat{k}\right), \end{aligned}$$

where the second equality follows from the substitution $z(2,\theta;\hat{k}) = 1 - z(1, 1-\theta;\hat{k})$ and a change of variable $\tilde{\theta} = 1-\theta$. It follows immediately from (BR) that every informed voter is best-responding.

The above strategy $\hat{k}$ is essentially the unique equilibrium of the game Γ^n with a single pivotal event $v^C = 1/2$. To see this, consider any strategy k^n that satisfies $g^n(v^C|2;k^n) > g^n(v^C|1;k^n)$. From (BR), with δ^L and δ^R both set to 0, the best response $\tilde{k}^n$ to k^n satisfies $H(j=2;\tilde{k}^n) > q$ and $H(j=1;\tilde{k}^n) > 1-q$. But this in turn implies that $g^n(v^C|2;\tilde{k}^n) < g^n(v^C|1;\tilde{k}^n)$, and therefore k^n cannot be an equilibrium strategy.[7] Similarly, any k^n such that $g^n(v^C|2;k^n) < g^n(v^C|1;k^n)$ cannot be an equilibrium strategy either.

For values of α such that $z(2,0;\hat{k}) = (1-\alpha)\left(q^2+(1-q)^2\right) < 1/2$, which is equivalent to $z(1,1;\hat{k}) > 1/2$, the unique equilibrium does not achieve full information equivalence. For example, if $\alpha = 1/5$ and $q = 2/3$, for n sufficiently large and with probability arbitrarily close to 1, candidate L is elected in state R when the uncertainty state satisfies $\theta < 5/18$, and candidate R is elected in state L when $\theta > 13/18$. More generally, full information equivalence is feasible and yet unachievable in any equilibrium with a single pivotal event $v^C = 1/2$ if

$$2(1-\alpha)(q^2+(1-q)^2) < 1 < 2(1-\alpha)q.$$

[7] Since $H(2;\tilde{k}^n) > q$ and $H(1;\tilde{k}^n) > 1-q$, we have $z(2,\theta;\tilde{k}^n) > 1 - z(1,1-\theta;\tilde{k}^n)$. It is straightforward to show that $z(2,\theta;\tilde{k}^n) > 1 - z(2,\theta;\tilde{k}^n)$ if and only if $1 - z(1,1-\theta;\tilde{k}^n) > z(1,1-\theta;\tilde{k}^n)$. As a result, $z(2,\theta;\tilde{k}^n)(1-z(2,\theta;\tilde{k}^n)) < (1-z(1,1-\theta;\tilde{k}^n))z(1,1-\theta;\tilde{k}^n)$. It then follows that $g^n(v^C|2;\tilde{k}^n) < g^n(v^C|1;\tilde{k}^n)$.

For q close to $1/2$, the above holds for α close to 0. Thus, when the signals are very inaccurate, a tiny fraction of nonstrategic voters can make an election with a single pivotal event go wrong, even though full information equivalence is feasible. $\square$

Our main result, Proposition 1 below, shows by construction that in any large election with two additional pivotal events represented by *any* vote share thresholds v^L and v^R sufficiently close to v^C, there is an equilibrium that achieves full information equivalence whenever it is feasible. In this example, for $v^L = 1/2 - \nu$ and $v^R = 1/2 + \nu$ with any $\nu \in (0, (1-\alpha)q - 1/2)$, the preference-independent strategy $\hat{k}$ of vote-your-signal is an equilibrium for moderate values of δ^R/δ^L. To see this, note that by construction we have

$$z(s = 1, \theta = 1; \hat{k}) < v^L < v^C < v^R < z(s = 2, \theta = 0; \hat{k}).$$

By Lemma 1, the pivotal event v^L dominates v^C and v^R in state $s = 1$, and the pivotal event v^R dominates v^C and v^L in state $s = 2$. Further, using the same argument by symmetry and a change of variables as above, we have

$$g^n(v^L | s = 1; \hat{k}) = g^n(v^R | s = 2; \hat{k})$$

for all n. From (BR), $\hat{k}$ is an equilibrium in Γ^n for large n if at $j = 1$,

$$-\mu(1|1)\delta^L + \mu(2|1)\delta^R \leq 0,$$

and at $j = 2$,

$$-\mu(1|2)\delta^L + \mu(2|2)\delta^R \geq 0.$$

The above two conditions are satisfied for $\delta^R/\delta^L \in [(1-q)/q, q/(1-q)]$.

Example 2: Non-monotone preferences

Consider the same binary-state, binary-signal election model with a single pivotal event v^C as in Example 1, with $\mathbb{S} = \{1, 2\}$ and $\mu(1) = \mu(2) = 1/2$; $\mathbb{J} = \{1, 2\}$ and $\beta(j = 1 | s = 1) = \beta(j = 2 | s = 2) = q \in (1/2, 1)$; and $v^C = 1/2$. However, there are two preference groups, with $M = 2$ and $\lambda_1 > \lambda_2$. Preferences of group $m = 1$, referred to as the "majority," are the same as in Example 1, with the same payoff difference function $u_1(s, t) = u(s, t)$ and uniformly distributed t on $[0, 1]$. As in Bhattacharya (2013), preferences of group $m = 2$, referred to as the "minority," are opposite of group $m = 1$, with payoff difference function $u_2(s,t)$ given by

$$u_2(s,t) = \begin{cases} t & \text{if } s = 1 \\ -(1-t) & \text{if } s = 2. \end{cases}$$

The distribution of t in group $m = 2$ is given by $P_2(t)$ on $[0,1]$. There is no aggregate uncertainty, with $\alpha = 0$.

Assumption 2 is satisfied because $\lambda_1 > \lambda_2$. The full information outcome remains the same as in Example 1: electing candidate L when $s = 1$ and R when $s = 2$. Further, the same vote-your-signal strategy $\hat{k}$ for both preference groups achieves the full information outcome in a large enough election, as the vote share of candidate L in state $s = 1$ and the vote share of R in state $s = 2$ are both equal to $q = 2/3$, which is greater than $v^C = 1/2$.

Fix any n. For any strategy $k = (k_1, k_2)$, define

$$\gamma^n = \frac{g^n(v^C|s=2;k)}{g^n(v^C|s=1;k) + g^n(v^C|s=2;k)}.$$

Given that $\mu(1) = \mu(2) = 1/2$, γ^n is the probability of $s = 2$ conditional on the pivotal event v^C under k. The best response to k as given by (BR), where we set $\delta^L = \delta^R = 0$, depends on k only through γ^n. Specifically, it is given by a threshold preference type t_j^n for each signal $j = 1,2$ such that, upon observing signal j, a majority voter (from group $m = 1$) with t votes for R if and only if $t \leq t_j^n$ while a minority voter (from group $m = 2$) with t votes R if and only if $t \geq t_j^n$, where

$$t_1^n = \frac{(1-q)\gamma^n}{(1-q)\gamma^n + q(1-\gamma^n)}; \quad t_2^n = \frac{q\gamma^n}{q\gamma^n + (1-q)(1-\gamma^n)}.$$

Under the above best response function, which we denote as $k(\gamma^n)$, the probability $z(s;k(\gamma^n))$ that a randomly drawn voter chooses R in state $s = 1,2$ is given by:

$$z(1;k(\gamma^n)) = q(\lambda_1 t_1^n + \lambda_2(1 - P_2(t_1^n))) + (1-q)(\lambda_1 t_2^n + \lambda_2(1 - P_2(t_2^n))),$$
$$z(2;k(\gamma^n)) = q(\lambda_1 t_2^n + \lambda_2(1 - P_2(t_2^n))) + (1-q)(\lambda_1 t_1^n + \lambda_2(1 - P_2(t_1^n))).$$

Any limit point $\hat{\gamma}$ of a sequence of equilibrium values $\{\hat{\gamma}^n\}$ must satisfy the following version of condition (ER):

$$\left| z(1;k(\hat{\gamma})) - v^C \right| = \left| z(2;k(\hat{\gamma})) - v^C \right|.$$

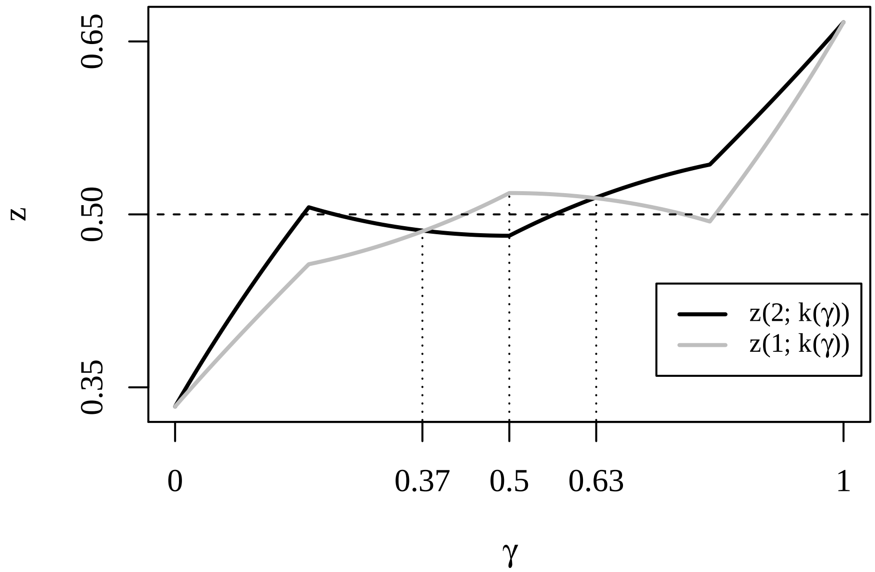

Figure 1. *The limit of equilibrium outcomes all fail full information equivalence. In the limit equilibrium with* $\hat{\gamma} = 0.37$*, candidate L wins in both states. In the limit equilibrium with* $\hat{\gamma} = 0.63$*, candidate R wins in both states. In the limit equilibrium with* $\hat{\gamma} = 1/2$*, candidate L wins in state* $s = 2$ *and candidate R wins in state* $s = 1$.

Otherwise, if $z(1; k(\hat{\gamma}))$ is closer to $v^C = 1/2$ than $z(2; k(\hat{\gamma}))$ is, Lemma 1 implies that the pivotal event v^C in state $s = 1$ dominates the same pivotal event v^C in state $s = 2$. As a result, $\{\hat{\gamma}^n\}$ converges to $\hat{\gamma} = 0$. But then both t_1^n and t_2^n converge to 0, and as a result $z(1; k(\hat{\gamma}^n))$ and $z(2; k(\hat{\gamma}^n))$ converge to the same value of λ_2, contradicting the assumption that $z(1; k(\hat{\gamma}))$ is closer to $v^C = 1/2$ than $z(2; k(\hat{\gamma}))$ is. Symmetrically, $z(2; k(\hat{\gamma}))$ cannot be closer to $v^C = 1/2$ than is $z(1; k(\hat{\gamma}))$.

Figure 1 illustrates how condition (ER) determines the limit equilibrium outcomes by plotting $z(1; k(\gamma))$ and $z(2; k(\gamma))$. Here we assume that $q = 2/3$, $\lambda_2 = 1/3$, and the distribution of t for minority voters is uniform on $t \in [1/3, 2/3]$.[8] The three

[8]This technically violates the common support assumption. The diagram does not change qualitatively if we set $P_2(t)$ to $3\epsilon t$ for $t \in [0, 1/3]$, $\epsilon + (3t-1)(1-2\epsilon)$ for $t \in [1/3, 2/3]$ and $1 - \epsilon + (3t-2)\epsilon$ for $t \in [2/3, 1]$ for some sufficiently small and positive ϵ.

interior values of $\hat{\gamma}$ that satisfy the equal-rate condition are all limits of equilibria. All three fail to achieve full information equivalence: for sufficiently large n, with probability close 1, either the same candidate is elected in both states, or the candidate favored by minority voters is elected. In particular, for $\hat{\gamma} = 1$, we have $t_1 = 1/3$ and $t_2 = 2/3$, which means all minority voters vote for candidate R at signal $j = 1$ and L at signal $j = 2$. As a result, $z(1; k(\hat{\gamma})) = 14/27$ and $z(2; k(\hat{\gamma})) = 13/27$, satisfying condition (ER) but electing R in state 1 and L in state 2.

Due to Assumption 1, for any value of γ^n, we have $t_1^n < t_2^n$. Since majority voters have monotone preferences, we have $z(1; k(\gamma^n)) < z(2; k(\gamma^n))$ for all γ^n if λ_2 is sufficiently small, and the outcome of electing the "wrong" candidate cannot occur. On the other hand, for q close to $1/2$, the preference thresholds t_1^n and t_2^n are close to each other for any γ^n. If the preference types t of minority voters are concentrated on an arbitrarily small interval around $1/2$, even a small fraction λ_2 of minority voters can get the wrong candidate elected in the limit. To see this, suppose that preference types t of minority voters are uniformly distributed on $[1-q, q]$, which brackets $1/2$ and shrinks as q gets closer to $1/2$. For $\hat{\gamma} = 1/2$, we have $t_1 = 1 - q$ and $t_2 = q$, and thus $z(1; k(\hat{\gamma}))$ and $z(2; k(\hat{\gamma}))$ satisfy condition (ER) for all λ_2. However, $z(1; k(\hat{\gamma})) > 1/2 > z(2; k(\hat{\gamma}))$ as long as $\lambda_2 > 1 - 1/(2q)$. As q becomes closer to $1/2$, an arbitrarily small fraction of minority voters gets the wrong candidate elected. $\square$

With two additional pivotal events $v^L = 1/2 - \nu$ and $v^R = 1/2 + \nu$, the strategy $\hat{k}$ of vote-your-signal for both the majority and the minority voters is an equilibrium for sufficiently small $\nu > 0$ and sufficiently large n. With no aggregate uncertainty, the induced vote shares are $z(2; \hat{k}) = q$ and $z(1; \hat{k}) = 1 - q$. For any $\nu \in (1/2, q)$, the pivotal event v^L dominates the other two pivotal events in state $s = 1$, while the pivotal event v^R dominates in state $s = 2$; further, $g^n(v^L|1; \hat{k}) = g^n(v^R|2; \hat{k})$ for all n. The rest of the argument is the same as in Example 1: $\hat{k}$ is an equilibrium and achieves full information equivalence in the limit if $\delta^R/\delta^L \in ((1-q)/q, q/(1-q))$.

In both Example 1 and Example 2, we are able to directly verify that vote-your-signal is an equilibrium strategy without using approximations for large elections, assuming that δ^R/δ^L is bounded by the likelihood ratios $(1-q)/q$ and $q/(1-q)$. We exploit symmetries of the model parameters in these examples to show that the vote-your-signal strategy $\hat{k}$ satisfies $g^n(v^L|1; \hat{k})/g^n(v^R|2; \hat{k}) = 1$ for all n, which allows us to evaluate (BR) without using any equal-rate condition.[9] The vote-your-signal strategy does not depend on a voter's preference or which preference group they belong to, or the size of the election. The construction for our main result Proposition 1 "almost" has the same features, without restricting the ratio δ^R/δ^L or assuming any symmetry. Specifically, except at a single signal, equilibrium voting depends not on a voter's preference type, the group they belong to or the size of the election, but on their signals. Preference-dependent voting at the single signal is such that when the election is sufficiently large, the pivotal events of v^L in state s^L and v^R in state s^R dominate, and the equilibrium vote shares in the critical states s^L and s^R converge and satisfy condition (ER). Full information equivalence is achieved as in the two examples.

3.2. Main result

We first introduce a class of strategies that are both monotone in signals and independent of preferences. We show that whether a monotone preference-independent strategy can distinguish between the critical pair of states s^R and s^L is a sufficient test for the feasibility of full information equivalence.

Definition 4. *A strategy $k = (k_1, \ldots, k_M)$ is **monotone in signals** if for almost all $t \in \mathbb{T}$ and for all $m = 1, \ldots, M$*

$$k_m(t,j) > 0 \implies k_m(t,j') = 1 \quad \text{for all } j' > j$$
$$k_m(t,j) < 1 \implies k_m(t,j') = 0 \quad \text{for all } j' < j,$$

*and is **preference-independent** if $k_m(t,j)$ does not depend on t or m for all $j \in \mathbb{J}$.*

A monotone strategy allows voting by at least some group m to depend on their preference types t. A preference-independent strategy allows voting to be nonmonotone in signals. A monotone preference-independent strategy allows neither. The combination of monotonicity and preference-independence means that any such strategy k can be represented by a mixing signal $j \in \mathbb{J}$ and a mixing probability

[9] Approximations for large elections are not used, but we still need to use Stirling's formula to argue that the pivotal event v^L dominates v^C and v^R in state $s = 1$, and the pivotal event v^R dominates v^C and v^L in state $s = 2$.

$y \in [0,1]$, such that for all $t \in \mathbb{T}$ and all $m = 1, \ldots, M$,

$$k_m(t, j') = \begin{cases} 0 & \text{if } j' \leq j - 1 \\ y & \text{if } j' = j \\ 1 & \text{if } j' \geq j + 1. \end{cases}$$

Denote the above strategy as $\kappa(y; j)$. The class of all monotone preference-independent strategies forms a continuous path from $\kappa(0; J)$ to $\kappa(1; 1)$, as j varies from J to 1 and y varies from 0 to 1. Along the path, the vote share $z(s, \theta; \kappa(y; j))$ of candidate R for any $s \in \mathbb{S}$ and $\theta \in [\underline{\theta}, \overline{\theta}]$ continuously increases.

The following result uses Assumption 1 to show that any strategy that satisfies (FE) can be replaced by a monotone preference-independent strategy that increases the spread of vote shares for candidate R under the critical pair of states s^L and s^R and hence also satisfies (FE).[10]

Lemma 3. *Full information equivalence is feasible if and only if there exists a monotone preference-independent strategy k such that*

$$z(s^L, \overline{\theta}; k) < v^C < z(s^R, \underline{\theta}; k).$$

Proof. See the appendix. ∎

Our main result, Proposition 1 below, shows by construction that, with two additional pivotal events v^L and v^R that bracket v^C, whenever full information equivalence is feasible, it is achieved by a sequence of equilibria so long as v^L and v^R are close enough to v^C. We provide a heuristic argument here, leaving the formal proof to the appendix. By Lemma 3, we can focus our construction on monotone preference-independent strategies. As suggested by the two examples in Section 3.1, we look for a monotone preference-independence strategy $\hat{k}$ that satisfies two further conditions. First, under $\hat{k}$, the maximum vote share for candidate R in the critical state s^L and the minimum vote share for R in the critical state s^R fall outside of $[v^L, v^R]$,

$$z(s^L, \overline{\theta}; \hat{k}) < v^L < v^C < v^R < z(s^R, \underline{\theta}; \hat{k}), \tag{DOM}$$

[10] The proof of the lemma shows that the increase in the spread of vote shares is strict, unless the strategy that satisfies (FE) is already monotone preference-independent (the first case), or is almost monotone preference-independent as defined later in this section (the second case).

so that the pivotal event v^L in state s^L dominates v^C and v^R in any state $s \leq s^L$, and the pivotal event v^R in state s^R dominates v^C and v^L in any state $s \geq s^R$. Second, under $\hat{k}$, a version of condition (ER) is satisfied,

$$I(v^L; z(s^L, \overline{\theta}; \hat{k})) = I(v^R; z(s^R, \underline{\theta}; \hat{k})), \tag{CON}$$

so that the ratio $g^n(v^L|s^L;\hat{k})/g^n(v^R|s^R;\hat{k})$ converges to a positive and finite limit as n goes to infinity.

For given v^L and v^R, there is a unique monotone preference-independent strategy $\hat{k} = \kappa(\hat{y};\hat{j})$ that satisfies (DOM) and (CON). When $\hat{y}$ is strictly between 0 and 1, the strategy $\hat{k} = \kappa(\hat{y};\hat{j})$ calls for mixing between the two candidates at signal $\hat{j}$ by all informed voters of all preference types t in all groups m. Unlike in the two examples of Section 3.1, $\hat{k}$ itself is not an equilibrium for generic values of v^L and v^R, because the best response to it, although monotone, is not preference-independent. For any fixed n, (BR) implies that at signal $j \in \mathbb{J}$, the best response to $\hat{k}$ by any informed voter of group m, $m = 1, \ldots, M$, with preference $t \in \mathbb{T}$, is to vote for candidate R if and only if

$$\sum_{s \in \mathbb{S}} \mu(s|j) \left(g^n(v^R|s;\hat{k})\delta^R + g^n(v^C|s;\hat{k}) u_m(s,t) - g^n(v^L|s;\hat{k})\delta^L \right) \geq 0.$$

Rewrite the above as

$$\begin{aligned} &\sum_{s \geq s^R} \mu(s|j) \left(\frac{g^n(v^R|s;\hat{k})}{g^n(v^R|s^R;\hat{k})}\delta^R + \frac{g^n(v^C|s;\hat{k})}{g^n(v^R|s^R;\hat{k})} u_m(s,t) - \frac{g^n(v^L|s;\hat{k})}{g^n(v^R|s^R;\hat{k})}\delta^L \right) g^n(v^R|s^R;\hat{k}) \\ \geq &\sum_{s \leq s^L} \mu(s|j) \left(-\frac{g^n(v^R|s;\hat{k})}{g^n(v^L|s^L;\hat{k})}\delta^R - \frac{g^n(v^C|s;\hat{k})}{g^n(v^L|s^L;\hat{k})} u_m(s,t) + \frac{g^n(v^L|s;\hat{k})}{g^n(v^L|s^L;\hat{k})}\delta^L \right) g^n(v^L|s^L;\hat{k}). \end{aligned}$$

Both the left-hand side and the right-hand side depend on the voter's preference t and the group m they belong to.

The proof of our main result relies on the fact that the above best response condition to $\hat{k}$ approaches one that is independent of voter preference t and group m when the size of election is arbitrarily large. As we will make precise in the proof, since $\hat{k}$ satisfies (DOM), for n sufficiently large, the dominant term on the left-hand side of the best response condition is $\mu(s^R|j)\delta^R g^n(v^R|s^R;\hat{k})$, and the dominant term on the right-hand side is $\mu(s^L|j)\delta^L g^n(v^L|s^L;\hat{k})$. Thus, we have the following *approximate* best response to $\hat{k}$: for any informed voter of group m, $m = 1, \ldots, M$, with preference

$t \in \mathbb{T}$, choose R at signal j if and only if

$$\frac{\delta^R}{\delta^L}\frac{\mu(s^R|j)}{\mu(s^L|j)} \geq \frac{g^n(v^L|s^L;\hat{k})}{g^n(v^R|s^R;\hat{k})}.$$

The above no longer depends on t or m. To find an equilibrium of Γ^n for a fixed sufficiently large n, we consider strategies k^n that are close to $\hat{k}$ and are *almost* preference-independent, in that all preferences types t from any preference group $m = 1, \ldots, M$ choose R at any signal $j > \hat{j}$ and L at any signal $j < \hat{j}$, and only voting at signal $\hat{j}$ depends on preference type t and preference group m. Then, the best response against any such strategy k^n is the same as that against a monotone preference-independent strategy $\kappa(y;\hat{j})$ for some mixing probability y. Since $\hat{k}$ satisfies (CON), the ratio $g^n(v^L|s^L;\kappa(y;\hat{j}))/g^n(v^R|s^R;\kappa(y;\hat{j}))$ varies from arbitrarily small for y just below $\hat{y}$, which makes all informed voters choose R, to arbitrarily large for y just above $\hat{y}$, which makes all vote for L.[11] Thus, we can find a strategy $\hat{k}^n$ with preference-dependent voting only at signal $\hat{j}$, to match $\hat{y}$ and to have the requisite fixed point property to be an equilibrium of Γ^n. The vote shares in equilibrium converge:

$$\lim_{n\to\infty} z(s^L,\overline{\theta};\hat{k}^n) = z(s^L,\overline{\theta};\hat{k}); \quad \lim_{n\to\infty} z(s^R,\underline{\theta};\hat{k}^n) = z(s^R,\underline{\theta};\hat{k}).$$

Since $\hat{k}$ satisfies (DOM), $\{\hat{k}^n\}$ achieves full information equivalence. Under the costly recount interpretation, the probability of a recount in equilibrium is vanishingly small for large elections; under the mandate concerns interpretation, with probability converging to one a large election is won with a significant margin.

Proposition 1. *Suppose that full information equivalence is feasible. Then, for all v^L and v^R sufficiently close to v^C, there exists a sequence of strategies $\{\hat{k}^n\}$ such that $\hat{k}^n$ is an equilibrium of Γ_n for all n sufficiently large, and $\{\hat{k}^n\}$ achieves full information equivalence.*

[11] As a function of y, the limit of $g^n(v^L|s^L;\kappa(y;\hat{j}))/g^n(v^R|s^R;\kappa(y;\hat{j}))$ is discontinuous at $y = \hat{y}$. Since $\hat{k}$ satisfies (CON), by Lemma 1 and Stirling's formula we have

$$\lim_{n\to\infty}\frac{g^n(v^L|s^L;\hat{k})}{g^n(v^R|s^R;\hat{k})} = \frac{f(\overline{\theta})}{f(\underline{\theta})}\lim_{n\to\infty}\frac{g^n(v^L|s^L,\overline{\theta};\hat{k})}{g^n(v^R|s^R,\underline{\theta};\hat{k})} = \frac{f(\overline{\theta})}{f(\underline{\theta})}\frac{\sqrt{v^R(1-v^R)}}{\sqrt{v^L(1-v^L)}}.$$

The above is the limit of the right-hand side of the approximate best response to $\hat{k}$. Due to the discontinuity, it plays no role in determining the limit of the equilibrium ratio $g^n(v^L|s^L;\hat{k}^n)/g^n(v^R|s^R;k^n)$, which is given by the left-hand side of the approximate best response to $\hat{k}$.

Proof. See the appendix. ∎

For any fixed v^C, Proposition 1 holds as long as v^L and v^R are close enough to v^C. How close they need to be depends on how "difficult" it is to achieve full information equivalence. More precisely, by Lemma 3, full information equivalence is feasible only if there is a monotone preference-independent strategy k that successfully spreads the vote shares $z(s^R, \underline{\theta}; k)$ and $z(s^L, \overline{\theta}; k)$ around v^C under the least favorable aggregate uncertainty states. Proposition 1 holds for any

$$v^L \in (z(s^L, \overline{\theta}; k), v^C) \quad \text{and} \quad v^R \in (v^C, z(s^R, \underline{\theta}; k)).$$

If $z(s^R, \underline{\theta}; k) - z(s^L, \overline{\theta}; k)$ is large, the range of values of v^L and v^R for which our main result applies is correspondingly large. An implication is that, under the interpretation of mandate concerns, our result can accommodate scenarios where the winner of an election has to garner a supermajority of the votes to meet institutionalized requirements for effective governance. We state the above observation as a corollary.

Corollary 1. *Suppose that (FE) is satisfied by a monotone preference-independent strategy k. Then, for any $v^L \in (z(s^L, \overline{\theta}; k), v^C)$ and $v^R \in (v^C, z(s^R, \underline{\theta}; k))$, there exists a sequence of strategies $\{\hat{k}^n\}$ such that $\hat{k}^n$ is an equilibrium of Γ_n for all n sufficiently large, and $\{\hat{k}^n\}$ achieves full information equivalence.*

Our main result does not require an explicit characterization of $\{\hat{k}^n\}$. There may be multiple equilibria in Γ^n, but for any selection the limit of $\hat{k}^n$ as n goes to infinity is uniquely pinned down. Unlike the two examples in Section 3.1, this limit, denoted as $\hat{k}_\infty$, generally differs from $\hat{k} = \kappa(\hat{y}; \hat{j})$. It is monotone and preference-independent except at $\hat{j}$, and yields the same vote shares in the critical states s^L and s^R as $\hat{k}$. As is true with $\hat{k}$, the limit $\hat{k}_\infty$ does not depend on the preference distributions.

When the payoff difference function $u_m(s, t)$ is monotone in t for all s and all m, the limit strategy $\hat{k}_\infty$ has a threshold structure. This can be illustrated by generalizing the two examples of Section 3.1, where we assume $\hat{j} = 1$ and $\hat{y} \in (0, 1)$ are determined by any fixed v^L and v^R close to v^C according the respective condition (CON). In both examples, $\hat{k}_\infty$ requires all informed voters to choose candidate R at signal $j = 2$. In Example 1, with a payoff difference function $u(s, t)$ decreasing in t for a single preference group and any distribution $P(t)$, the limit strategy $\hat{k}_\infty$ is given by $\hat{t}$ satisfying

$$P(\hat{t}) = \hat{y},$$

so that the fraction of informed voters choosing R at signal $\hat{j} = 1$ is equal to $\hat{y}$. In Example 2, with $u_1(s,t)$ decreasing in t for the majority preference group and $u_2(s,t)$ increasing in t for the minority, $\hat{k}_\infty$ is given by a threshold $\hat{t}_1$ for the majority and $\hat{t}_2$ for the minority. If they are interior, the thresholds satisfy

$$\begin{aligned}\lambda_1 P_1(\hat{t}_1) + \lambda_2(1 - P_2(\hat{t}_2)) &= \hat{y} \\ u_1(\hat{s}, \hat{t}_1) &= u_2(\hat{s}, \hat{t}_2),\end{aligned}$$

where $\hat{s}$ is such that the pivotal event v^C in $\hat{s}$ dominates v^C in the other state under strategy $\hat{k}$; that is, $\hat{s} = 1$ if $z(1, \overline{\theta}; \hat{k})$ is closer to v^C than $z(2, \underline{\theta}; \hat{k})$ is, and $\hat{s} = 2$ otherwise.[12] The first condition above ensures that $\hat{k}_\infty$ yields the same vote shares as $\hat{k}$; the second arises because (BR) depends on preference type t and group m only at pivotal event v^C.[13]

Proposition 1 focuses on constructing a sequence of equilibria $\{\hat{k}^n\}$ that achieves full information equivalence of separating s^L and lower states from s^R and higher states. The pivotal events v^L and v^R do not determine the winner of the election, and full information equivalence is used only in so far as it is feasible (Lemma 2). As a result, there may be other equilibria that do not achieve the full information outcome in a given game Γ^n. For example, suppose for some state $\tilde{s}$, there exists a strategy $\tilde{k}$ such that

$$z(\tilde{s}, \overline{\theta}; \tilde{k}) < v^L < v^R < z(\tilde{s}+1, \underline{\theta}; \tilde{k}).$$

Then, the same construction as in the proof of Proposition 1 shows that for large enough n, there exists an equilibrium $\tilde{k}^n$ such that candidate L is elected in state $\tilde{s}$ and lower and candidate R is elected in all other states. If $\tilde{s} < s^L$, the "wrong" candidate is elected in states $\tilde{s}+1, \ldots, s^L$. However, since the equilibrium $\tilde{k}^n$ is necessarily monotone, the "correct" candidate is still elected in all other states.

[12] In the original Example 2, $u_1(1,t) < 0 < u_2(1,t')$ and $u_1(2,t) > 0 > u_2(2,t')$ for all t, t'. As a result, when $\hat{s} = 1$, only $\hat{t}_2$ is interior and satisfies $\lambda_2(1 - P(\hat{t}_2)) = \hat{y}$ if $\hat{y} < \lambda_2$, and $\hat{t}_1$ is interior and satisfies $\lambda_1 P_1(\hat{t}_1) + \lambda_2 = \hat{y}$ if $\hat{y} > \lambda_2$. When $\hat{s} = 2$, the limit strategy $\hat{k}_\infty$ has a symmetric threshold structure with only one interior threshold.

[13] We can obtain a threshold structure of $\hat{k}_\infty$ on a transformed type space without the monotonicity of $u_m(s,t)$ in t for each m. For any given $\hat{k}$, let $\hat{s} = s^L$ if $I(v^C; z(s^L, \overline{\theta}; \hat{k})) > I(v^C; z(s^R, \underline{\theta}; \hat{k}))$ and let $\hat{s} = s^R$ otherwise. Since $\hat{k}$ satisfies (DOM), pivotal event v^C in state s^L dominates v^C in all $s \neq s^L$ if $\hat{s} = s^L$, and likewise v^C in state s^R dominates v^C in all $s \neq s^R$ if $\hat{s} = s^R$. For each $m = 1, \ldots, M$, we can always transform the preferences types from t to $\tilde{t}$, and correspondingly the distribution from $P_m(t)$ to $\tilde{P}_m(\tilde{t})$, such that $u_m(\hat{s}, \tilde{t})$ is monotone in $\tilde{t}$. The threshold structure of $\hat{k}_\infty$ can be then defined on the transformed distributions $\tilde{P}_1, \ldots, \tilde{P}_M$.

3.3. Informativeness of equilibrium voting

Our main result establishes that introducing pivotal events v^L and v^R can restore full information equivalence when such outcome is not an equilibrium in a standard election with a single pivotal event v^C. But even when full information equivalence is achieved in standard elections, multiple pivotal events can still improve on the equilibrium outcome. Here, we show that introducing multiple pivotal events increases the "informativeness" of the equilibrium strategy profile by generating a larger spread of the vote shares for R across the two critical pair of states s^R and s^L. This is consistent with our main result that full information equivalence is more robust to aggregate uncertainty with the additional pivotal events v^L and v^R.

The key to understand why introducing pivotal events v^L and v^R improves the informativeness of the equilibrium is that these pivotal events dominate the pivotal event v^C, but they do not change the identity of the winner. Since the additional payoffs δ^L and δ^R are by assumption independent of preference types t, the incentives to vote conditional on the pivotal events v^L and v^R are preference-independent. This allows us to construct an equilibrium where all preference types vote informatively according to their signals, except at a single signal where their vote depends on their preferences. In contrast, in a standard election, at the only pivotal event v^C the vote determines the election outcome. Thus a voter's belief over payoff-relevant states and their preference type matter for their voting decision. If their beliefs are not significantly changed by their private signals conditional on the pivotal event v^C, their votes may be uninformative. We introduce the following assumption.

Assumption 3. *For any probability distribution γ over $\mathbb{S}$, there exists $m = 1, \dots, M$ with a subset of preference types $\mathbb{T}_m \subseteq \mathbb{T}$, such that $\int_{t \in \mathbb{T}_m} \mathrm{d}P_m > 0$ and, for all $t \in \mathbb{T}_m$,*

$$\left(\sum_{s \in \mathbb{S}} \gamma(s) \beta(j|s) u_m(s,t) \right) \left(\sum_{s \in \mathbb{S}} \gamma(s) \beta(j'|s) u_m(s,t) \right) > 0 \quad \textit{for all } j, j' \in \mathbb{J}.$$

If γ is the posterior belief conditional on the pivotal event v^C, then all preference types in $\mathbb{T}_m$ vote uninformatively in any equilibrium of a standard election, as their expected payoff difference between voting for R and voting for L does not change sign with their private information. Assumption 3 says that there is always a positive mass of such types. This requires preferences types to vary sufficiently and private signals not to be too informative. Assumption 3 is satisfied in most strategic voting models, including Feddersen and Pesendorfer (1997) and Bhattacharya (2013).[14]

Proposition 2. *Suppose that the conditions in Proposition 1 hold, and let $\{\hat{k}^n\}$ be the sequence of equilibria in $\{\Gamma^n\}$ that achieves full information equivalence. If a sequence of equilibria of $\{\hat{k}_0^n\}$ in standard elections $\{\Gamma_0^n\}$ with a single pivotal event v^C also achieves full information equivalence, then*

$$\lim_{n\to\infty} z(s^L,\overline{\theta};\hat{k}^n) \leq \lim_{n\to\infty} z(s^L,\overline{\theta};\hat{k}_0^n) < \lim_{n\to\infty} z(s^R,\underline{\theta};\hat{k}_0^n) \leq \lim_{n\to\infty} z(s^R,\underline{\theta};\hat{k}^n),$$

with all strict inequalities if Assumption 3 holds.

Proof. See the appendix. ■

Proposition 2 establishes that, under Assumption 3, the probability of a "mistake," meaning an election outcome different from the full information outcome, is smaller in both state s^L and in state s^R, when the additional two pivotal events v^L and v^R are introduced to a standard election with a single pivotal event v^C. Specifically, the probabilities of electing candidate R in state s^L in an election with v^L and v^R and without satisfy

$$\lim_{n\to\infty} \frac{\int_{v^C}^1 g^n(v|s^L;\hat{k}^n)\,\mathrm{d}v}{\int_{v^C}^1 g^n(v|s^L;\hat{k}_0^n)\,\mathrm{d}v} = \lim_{n\to\infty} \frac{g^n(v^C|s^L;\hat{k}^n)}{g^n(v^C|s^L;\hat{k}_0^n)} = 0,$$

where the first equality follows from a similar argument as in Lemma 1, and the second from $I(v^C, z(s^L,\overline{\theta};\hat{k}^n)) < I(v^C, z(s^L,\overline{\theta};\hat{k}_0^n))$. The same comparison holds for the probabilities of electing L in state s^R. Furthermore, if the vote share function $z(\cdot,\overline{\theta};\hat{k}_0^n)$ is increasing, which is true if all informed voters of any group m have state-monotone preferences in that $u_m(\cdot,t)$ is weakly increasing for all t, we have the same ranking of total probabilities of electing the wrong candidate R in state lower than or equal to s^L in an election with v^L and v^R and without:

$$\lim_{n\to\infty} \frac{\sum_{s\leq s^L}\int_{v^C}^1 g^n(v|s;\hat{k}^n)\,\mathrm{d}v}{\sum_{s\leq s^L}\int_{v^C}^1 g^n(v|s;\hat{k}_0^n)\,\mathrm{d}v} = \lim_{n\to\infty} \frac{g^n(v^C|s^L;\hat{k}^n)}{g^n(v^C|s^L;\hat{k}_0^n)} = 0.$$

Thus, whenever a standard electoral rule with a single pivotal event achieves full information equivalence, introducing two additional pivotal events is still beneficial by providing a faster rate of convergence to the same full information outcome.

[14]In the two examples of Section 3.1, Assumption 3 is satisfied because t is distributed on $[0,1]$ and $q<1$. The proof of Proposition 2 below makes it clear that the role of Assumption 3 is to ensure that no equilibrium strategy of a standard election game is almost preference-independent and monotone.

4. Extensions

In this section we offer two extensions to our main result of Proposition 1. They provide separate robustness checks that are specific to the two alternative interpretations of the model, costly recount and mandate considerations.

4.1. Counting errors

Under the interpretation of costly recount, our model does not allow for counting errors, so that the vote count in the initial stage is identical to the vote count in the recount stage. There are two natural ways to introduce counting errors, depending on whether the errors are independent or correlated.

In the first version of a model with counting error, we assume that each vote for candidate R has an independent probability $\zeta < 1/2$ of being miscounted as a vote for candidate L, and likewise each vote for L has an independent probability ζ of being miscounted as a vote for R. Miscounting occurs to both strategic and nonstrategic voters. We further assume that if there is a recount, all the counting errors are corrected. Since counting errors are independent across voters, when the "true" vote share for candidate R is $z(s,\theta;k)$ in state (s,θ) under strategy k, in the initial count the vote share for candidate R is instead

$$z_\zeta(s,\theta;k) = (1-\zeta)z(s,\theta;k) + \zeta(1 - z(s,\theta;k)).$$

Due to regression to the mean, $z_\zeta(s,\theta;k) > z(s,\theta;k)$ if and only if $z(s,\theta;k) < 1/2$. For any n, the probability distribution $g^n(V|s,\theta;k)$ of the realized vote share V in the initial count is determined by $z_\zeta(s,\theta;k)$ rather than $z(s,\theta;k)$. Counting errors may lead to the election of a wrong candidate not intended by voters, but since all counting errors are corrected in the recount, if V falls into $[v^L, v^R]$, for sufficiently large n the winner is determined by the true vote share $z(s,\theta;k)$.

With counting errors, whether full information equivalence is feasible now depends on the specifics of the electoral rule as well as the probability of miscounting. Specifically, full information equivalence is feasible if there exists a strategy k such that for all $s \leq s^L$ and $s' \geq s^R$, either

$$z_\zeta(s,\overline{\theta};k), z_\zeta(s',\underline{\theta};k) \in (v^L, v^R) \quad \text{and} \quad z(s,\overline{\theta};k) < v^C < z(s',\underline{\theta};k),$$

so that recounting always takes place and the full information outcome is achieved after correcting the initial counting errors, or

$$z_\zeta(s,\overline{\theta};k) < v^L < v^C < v^R < z_\zeta(s',\underline{\theta};k),$$

so that the correct candidate is elected in spite of counting errors in the initial vote count, with no recounting taking place.

Comparing the above two scenarios with Lemma 2, we see that both of them are a tightening of condition (FE). However, only the second scenario is consistent with the equilibrium construction of Proposition 1. This is because the construction requires some monotone preference-independent strategy $\hat{k}$ to ensure that the pivotal event v^L in state s^L dominates all other pivotal events including v^C in all states $s \leq s^L$, and symmetrically for v^R in state s^R, but counting errors cause the realized vote share to diverge from the true share. Further, Proposition 1 holds for any v^L and v^R so long as they are sufficiently close to v^C. With counting errors, this is no longer possible, as recounting thresholds v^L and v^R that are too tight may make the wrong candidate the winner of the election. More precisely, if condition (FE) is satisfied by some strategy k that creates only a narrow spread around v^C between $z_\zeta(s,\overline{\theta};k)$ for $s \leq s^L$ and $z(s',\underline{\theta};k)$ for $s' \geq s^R$, then for any fixed counting error ζ, either $z_\zeta(s,\overline{\theta};k) > v^C$ for some $s \leq s^L$ or $z(s',\underline{\theta};k) < v^C$ for some $s' \geq s^R$. If v^L and v^R are too close to v^C, mistakes would not be corrected.

However, for any pair (v^L, v^R), if the counting error ζ is sufficiently small, then for any strategy k and any state (s,θ), the initial count $z_\zeta(s,\theta;k)$ of vote share for R is close enough to the true vote share $z(s,\theta;k)$, and the same is true with $z_\zeta(s,\theta;k)$ and $z(s,\theta;k)$. If condition (FE) holds so that full information equivalence is feasible with no counting errors, then the same equilibrium construction of Proposition 1 can be replicated. We summarize the discussion so far in the following statement.

Proposition 3. *Suppose full information equivalence is feasible without counting errors. For all v^L, v^R sufficiently close to v^C there is a $\overline{\zeta}(v^L,v^R) > 0$ such that, for all miscounting probabilities $\zeta < \overline{\zeta}(v^L,v^R)$, there exists a sequence of monotone equilibria $\{k^n\}$ of Γ^n that achieves full information equivalence.*

Our second model of counting errors assumes they are systemic instead of independent mistakes in counting each ballot. For example, such correlated errors may occur when a certain counting protocol (how to deal with hanging chads, etc.) is not properly followed, so that all the votes in the same polling station or even the entire election are miscounted is a specific way. To model these errors, we assume that when the "true" vote share for candidate R is $z(s,\theta;k)$ in state (s,θ) under strategy k, given any n and the probability distribution $g^n(V|s,\theta;k)$ of the realized true vote share V determined by $z(s,\theta;k)$, in the initial count the realized vote share is

$$V_\xi = \begin{cases} 1 & \text{if } V+\xi > 1, \\ 0 & \text{if } V+\xi < 0, \\ V+\xi & \text{otherwise.} \end{cases}$$

In the above, ξ is a random variable with positive and continuous density on the support $[\underline{\xi},\overline{\xi}]$. Upon recounting, all errors are detected so that the election outcome is based on the true vote share V.

The effect of the systemic counting error ξ is similar to the effect of aggregate uncertainty θ, except that ξ only influences the initial vote share but not the final tally. Specifically, if $z(s,\underline{\theta};k)+\underline{\xi} > v^R$, then in state s the pivotal event $V_\xi = v^R$ dominates the other pivotal events $V_\xi = v^C$, and $V_\xi = v^L$ for sufficiently large n. Proposition 1 continues to hold if there exists a strategy k such that

$$z(s^L,\overline{\theta};k)+\overline{\xi} < v^C < z(s^R,\underline{\theta};k)+\underline{\xi}.$$

While the above is stronger than condition (FE) that full information equivalence is feasible, it is implied by the latter whenever the distribution of the systemic error is sufficiently concentrated, i.e., $\overline{\xi}-\underline{\xi}$ is sufficiently small. Thus, similarly to the first model of counting errors, the result of Proposition 1 is robust to the introduction of small counting errors.

4.2. Heterogenous mandate concerns

Under the mandate concerns interpretation, the main model allows for different groups $m = 1,\ldots,M$ to differ in their ideal candidates in each state, so long as an average condition given in Assumption 2 is satisfied. However, all preference groups are assumed to share the same concerns for effective governance. In this subsection, we allow groups to have different mandate concerns. Let δ_m^R and δ_m^L be the respective payoff boost for informed voters in group $m = 1,\ldots,M$, when candidate R wins with a share of at least v^R and when L wins with a share of at least $1-v^L$. For simplicity, we continue to assume that δ_m^R and δ_m^L are strictly positive and finite. Without loss we assume δ^R_m/δ^L_m is weakly increasing in m.[15]

When δ^R_m/δ^L_m varies with m, the construction in the proof of Proposition 1 still requires identifying a monotone preference-independent strategy $\hat{k}$ that satisfies (DOM) and (CON), but with mixing between candidates R and L occurring for a single group m (or all groups sharing the same ratio δ^R_m/δ^L_m) as well as at a single signal j. This is because the approximate best response to $\hat{k}$ for sufficiently large n is: for any informed voter of group m with preference t, choose R at signal j if and only if

$$\frac{\delta^R_m}{\delta^L_m}\frac{\mu(s^R|j)}{\mu(s^L|j)} \geq \frac{g^n(v^L|s^L;\hat{k})}{g^n(v^R|s^R;\hat{k})}.$$

The left-hand side above now depends on m as well as on j.

Define a class of strategies $\kappa(y;m,j)$ for any $y \in [0,1]$, $m = 1,\ldots,M$ and $j \in \mathbb{J}$

$$k_{m'}(t,j') = \begin{cases} 0 & \text{if } (\delta^R_{m'}/\delta^L_{m'})(\mu(s^R|j')/\mu(s^L|j')) < (\delta^R_m/\delta^L_m)(\mu(s^R|j)/\mu(s^L|j)) \\ y & \text{if } (\delta^R_{m'}/\delta^L_{m'})(\mu(s^R|j')/\mu(s^L|j')) = (\delta^R_m/\delta^L_m)(\mu(s^R|j)/\mu(s^L|j)) \\ 1 & \text{if } (\delta^R_{m'}/\delta^L_{m'})(\mu(s^R|j')/\mu(s^L|j')) > (\delta^R_m/\delta^L_m)(\mu(s^R|j)/\mu(s^L|j)), \end{cases}$$

for all $m' = 1,\ldots,M$, $j' \in \mathbb{J}$ and $t \in \mathbb{T}$. These strategies are monotone in signal and independent of preference types t for each group m. Consider the path formed by $\kappa(y;m,j)$, where the pairs (m,j) are in *decreasing* order of the product $(\delta^R_m/\delta^L_m)(\mu(s^R|j)/\mu(s^L|j))$, and from (m,j) to the next pair (m',j'), y *increases* from 0 to 1.[16] The path begins at $\kappa(0;M,J)$, under which all informed voters of all preference groups vote for candidate L, and ends at $\kappa(1;1,1)$, under which all vote for R. It is straightforward to verify that for any state (s,θ), along the path the vote share $z(s,\theta;\kappa(y;m,j))$ for candidate R increases continuously.

Once we identify $\hat{k} = \kappa(\hat{y};\hat{m},\hat{j})$ that satisfies (DOM) and (CON), we can establish the following counterpart of Proposition 1 using a similar equilibrium construction.

[15]It is straightforward to incorporate preference groups with voters who get zero or even negative mandate payoffs for one of the two candidates. For example, we can gather all preference groups m such that $\delta^L_m > 0$ and $\delta^R_m \leq 0$ to form a super-group. By the approximate best response condition, in any equilibrium of the sequence constructed in Proposition 4 below to achieve full information equivalence, all voters in this supergroup vote for candidate L at every signal. We only need to modify the voter share function under any κ in stating the condition for Proposition 4, and the rest goes through without change.

[16]For simplicity, we assume that the model is generic in that two pairs (m,j) and (m',j') have the same product only when $\delta^R_m/\delta^L_m = \delta^R_{m'}/\delta^L_{m'}$ and $j = j'$.

By the approximate best response condition, only voting by informed voters of group $\hat{m}$ at signal $\hat{j}$ can depend on their preference type. All other informed voters of group $m = 1, \ldots, M$ at any signal $j \in \mathbb{J}$ vote for R if the pair (m, j) has a greater product $(\delta^R_m/\delta^L_m)(\mu(s^R|j)/\mu(s^L|j))$ than $(\delta^R_{\hat{m}}/\delta^L_{\hat{m}})(\mu(s^R|\hat{j})/\mu(s^L|j))$, and vote for L if the pair (m, j) has a smaller product.

Proposition 4. *Suppose that there exists $\kappa(\hat{y}; \hat{m}, \hat{j})$ that satisfies (FE). Then, for any $v^L \in (z(s^L, \overline{\theta}; \kappa(\hat{y}; \hat{m}, \hat{j})), v^C)$ and $v^R \in (v^C, z(s^R, \underline{\theta}; \kappa(\hat{y}; \hat{m}, \hat{j})))$, there exists a sequence of equilibria $\{\hat{k}^n\}$ of Γ_n for all n sufficiently large that achieves full information equivalence.*

Proposition 4 includes Proposition 1 as a special case. With the same ratio δ^R_m/δ^L_m for all $m = 1, \ldots, M$, the class of strategies $\kappa(y; m, j)$ is the same as the class of monotone preference-independent strategies in Definition 4. As a result, the condition for Proposition 4 is the same as that for Proposition 1, which is the feasibility condition (FE) of full information equivalence.

More generally, the condition in Proposition 4 can be viewed as a qualification of feasibility of full information equivalence when preference groups differ in mandate concerns. By Lemma 3, condition (FE) is satisfied if and only if it holds for some monotone preference-independent strategy. The class of strategies $\kappa(y; m, j)$ used in Proposition 4 to satisfy (FE) is monotone in signals and independent of t for each m, but is not preference-independent because of the dependence on m. A sufficient condition for the qualification of feasibility of full information equivalence is that fractions of preference groups with "moderate" mandate concerns are large enough. More precisely, suppose that for two preference groups $\underline{m} < \overline{m}$ we have

$$\frac{\delta^R_{\underline{m}}}{\delta^L_{\underline{m}}}\frac{\mu(s^R|j)}{\mu(s^L|j)} > \frac{\delta^R_{\overline{m}}}{\delta^L_{\overline{m}}}\frac{\mu(s^R|j-1)}{\mu(s^L|j-1)}$$

for all $j = 2, \ldots, J$. Under the above condition, variations in δ^R_m/δ^L_m, with $\underline{m} \le m \le \overline{m}$, across all preference groups with moderate mandate concerns are small relative to informativeness of signals. For any $\kappa(\hat{y}; \hat{m}, \hat{j})$ with $\underline{m} \le \hat{m} \le \overline{m}$, compared to a corresponding monotone preference-independent strategy,[17] the spread of the vote shares $z(s^R, \underline{\theta}; \kappa(\hat{y}; \hat{m}, \hat{j}))$ and $z(s^L, \overline{\theta}; \kappa(\hat{y}; \hat{m}, \hat{j}))$ under $\kappa(\hat{y}; \hat{m}, \hat{j})$ is reduced by at most

$$(1-\alpha)\left(\sum_{j>\hat{j}}\beta(j|s^R)\sum_{m<\underline{m}}\lambda_m + \sum_{j<\hat{j}}\beta(j|s^L)\sum_{m>\overline{m}}\lambda_m\right).$$

[17] This is uniquely given by $\kappa(y; \hat{j})$ such that $H(\hat{j}; \kappa(\hat{y}; \hat{m}, \hat{j})) = H(\hat{j}; \kappa(y; \hat{j}))$.

Thus, when the fractions of preference groups with extreme mandate concerns are sufficiently small, the condition in Proposition 4 is satisfied whenever (FE) holds.

Full information equivalence may fail even when it is feasible, if variations in δ_m^R/δ_m^L across all m are too large relative to informativeness of signals. Consider the extreme case where

$$\frac{\delta_m^R}{\delta_m^L}\frac{\mu(s^R|1)}{\mu(s^L|1)} > \frac{\delta_{m-1}^R}{\delta_{m-1}^L}\frac{\mu(s^R|J)}{\mu(s^L|J)}$$

for all $m = 2, \ldots, M$. Under any strategy $\kappa(\hat{y}; \hat{m}, \hat{j})$, only informed voters in group $\hat{m}$ can generate the necessary spread in vote shares, as all voters in any group $m < \hat{m}$ vote L and those in group $m > \hat{m}$ vote R. If the fraction $\lambda_{\hat{m}}$ is too small, the spread may not be large enough to overcome the aggregate uncertainty. For example, in a three-group polarized electorate where a left-leaning preference group who have little mandate concerns for candidate R and hence a ratio δ_1^R/δ_1^L close to 0, and an opposite group with a ratio δ_3^R/δ_3^L sufficiently large, the full information outcome outcome is not achievable under significant aggregate uncertainty if not enough informed voters belong to the middle group.

5. Concluding Remarks

In models of two-candidate elections as games of strategic aggregation of information about a payoff-relevant state, rational voters make their choices as if they are pivotal. When there is a single pivotal event that determines the outcome of the election, voters choose uninformatively if their signals do not sufficiently change their belief about the state conditional on being pivotal relative to their preferences over the outcome. Achieving the full information outcome in equilibrium relies on a large enough fraction of "independent" voters with moderate preferences voting informatively at the likelihood ratio conditional on being pivotal. This creates vulnerabilities to uncertainty about participation by nonstrategic voters and local concentration of voters with preferences not aligned with the full information outcome.

In this paper we have presented a channel through which equilibrium voting in large elections is made almost preference-independent. We introduce two new pivotal events, each corresponding to a threshold winning margin for a candidate. All informed voters have the same interests at these pivotal events, either to avoid costly recounting or to confer a mandate to the winner. In elections of a sufficiently large size, equilibrium voting achieves the full information outcome if the new pivotal events are infinitely more likely than the old one, and have a balanced likelihood ratio across them. Because at the new pivotal events voters of all preferences over the outcome have the same interest, the equilibrium conditions are satisfied by having all informed voters choose according to their signals regardless of their preferences over the outcome, except at a single signal in order to generate the required likelihood ratio. Preference-independent voting at all but one signal maximizes the informativeness of vote shares. This is not affected by the preference-dependent voting at the single signal, and so changes in the distribution of preferences have no impact. Almost preference-independent voting with the new pivotal events thus overcomes the vulnerabilities of preference-dependent voting with a single pivotal event.

That almost preference-independent voting achieves the full information outcome is robust but is not fool-proof. When the new pivotal events arise from costly recounting, there can be counting errors in the initial round but the errors cannot be too large. When they arise from mandate concerns, voters can have small "ideological" differences in how much they care about mandates for their own candidate versus the other one, but not too many of them can have extreme differences.

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Appendix

Proof of Lemma 1

Since $z(s,\cdot;k)$ is strictly increasing, $I(v;z(s,\theta;k))$ is increasing in θ for $\theta < \theta(v,s;k)$ and decreasing in θ for $\theta > \theta(v,s;k)$. Let $B_\epsilon(v,s) \subset [\underline{\theta},\overline{\theta}]$ be a small interval of width ϵ that contains $\theta(v,s;k)$. Specifically, if $\theta(v,s;k) = \underline{\theta}$, choose $B_\epsilon(v,s) = [\underline{\theta},\overline{b})$ where $\overline{b} = \underline{\theta} + \epsilon$; and if $\theta(v,s;k) = \overline{\theta}$, choose $B_\epsilon(v,s) = (\underline{b},\overline{\theta}]$ where $\underline{b} = \overline{\theta} - \epsilon$. If $\theta(v,s;k)$ is interior, choose $B_\epsilon(v,s) = (\underline{b},\overline{b})$ such that $\overline{b} - \underline{b} = \epsilon$ and $I(v;z(s,\underline{b};k)) = I(v;z(s,\overline{b};k))$. Denote $B_\epsilon^c(v,s) = [\underline{\theta},\overline{\theta}]\backslash B_\epsilon(v,s)$ to be the complement of $B_\epsilon(v,s)$. Note that $I(v;z(s,\theta;k)) > I(v;z(s,\theta';k))$ for any $\theta \in B_\epsilon(v,s)$ and $\theta' \in B_\epsilon^c(v,s)$.

By Stirling's approximation formula, when n is sufficiently large, for any pivotal event v and any state s, we have

$$\int_{B_\epsilon^c(v,s)} g^n(v|s,\theta;k) f(\theta)\,\mathrm{d}\theta < g^n(v|s,\theta'^n;k)\Pr[\theta \in B_\epsilon^c(v,s)],$$

where θ'^n is equal to $\underline{b}$ or $\overline{b}$. This inequality follows because $g^n(v|s,\theta;k)$ is single-peaked in θ for $\theta \in [\underline{b},\overline{b}]$. Continuity of $g^n(v|s,\cdot;k)$ also implies that there is a $\hat{\theta}^n \in B_\epsilon(v,s)$ such that

$$\int_{B_\epsilon(v,s)} g^n(v|s,\theta;k) f(\theta)\,\mathrm{d}\theta = g^n(v|s,\hat{\theta}^n;k)\Pr[\theta \in B_\epsilon(v,s)].$$

We further claim that $\lim_{n\to\infty}\hat{\theta}^n = \theta(v,s;k)$. To see this, note that by definition

$$\lim_{n\to\infty}\int_{B_\epsilon(v,s)} \frac{g^n(v|s,\theta)}{g^n(v|s,\hat{\theta}^n)} f(\theta)\,\mathrm{d}\theta = \Pr[\theta \in B_\epsilon(v,s)],$$

which is only possible if $\hat{\theta}^n$ converges to $\theta(v,s;k)$, because from the fact that $\theta(v,s;k)$ maximizes $I(v;z(s,\theta;k))$, we must have $\lim_{n\to\infty} g^n(v|s,\theta;k)/g^n(v|s,\theta(v,s;k)) = 0$ for all $\theta \neq \theta(v,s;k)$.

From the two conditions above, we obtain that for any ϵ positive,

$$\lim_{n\to\infty}\frac{\int_{B_\epsilon^c(v,S)} g^n(v|s,\theta;k) f(\theta)\,\mathrm{d}\theta}{\int_{B_\epsilon(v,s)} g^n(v|s,\theta;k) f(\theta)\,\mathrm{d}\theta} \le \lim_{n\to\infty}\frac{g^n(v|s,\theta'^n;k)\Pr[\theta \in B_\epsilon^c(v,s)])}{g^n(v|s,\hat{\theta}^n;k)\Pr[\theta \in B_\epsilon(v,s)]} = 0,$$

where the equality follows because $\lim_{n\to\infty} g^n(v|s,\theta';k)/g^n(v|s,\theta;k) = 0$ whenever $\theta' \in B_\epsilon^c(v,s)$ and $\theta \in B_\epsilon(v,s)$, and because $\hat{\theta}^n$ is bounded away from θ'^n. As a result, for any v, v' and s, s',

$$\lim_{n\to\infty}\frac{g^n(v|s;k)}{g^n(v'|s';k)} = \lim_{n\to\infty}\frac{\int_{\underline{\theta}}^{\overline{\theta}} g^n(v|s,\theta;k)f(\theta)\,\mathrm{d}\theta}{\int_{\underline{\theta}}^{\overline{\theta}} g^n(v'|s',\theta;k)f(\theta)\,\mathrm{d}\theta} = \lim_{n\to\infty}\frac{\int_{B_\epsilon(v,s)} g^n(v|s,\theta;k)f(\theta)\,\mathrm{d}\theta}{\int_{B_\epsilon(v',s')} g^n(v'|s',\theta;k)f(\theta)\,\mathrm{d}\theta}.$$

The above holds for any ϵ positive and thus

$$\lim_{n\to\infty}\frac{g^n(v|s;k)}{g^n(v'|s';k)} = \lim_{\epsilon\to 0}\lim_{n\to\infty}\frac{\int_{B_\epsilon(v,s)} g^n(v|s,\theta;k)f(\theta)\,\mathrm{d}\theta}{\int_{B_\epsilon(v',s')} g^n(v'|s',\theta;k)f(\theta)\,\mathrm{d}\theta}.$$

Reversing the limit order and calculating the inner limit using l'Hopital's rule, we obtain the desired result.

Proof of Lemma 3

The "if" part of the statement follows from Assumption 1 and Lemma 2, because $z(s,\theta;k)$ is increasing in state s for every monotone strategy k.

For the "only if" part, fix any $k = (k_1, \ldots, k_M)$. It suffices to establish the claim that there is a monotone preference-independent strategy $\kappa(\tilde{y};\tilde{j})$ such that

$$\beta(\tilde{j}|s^R)\tilde{y} + \sum_{j>\tilde{j}}\beta(j|s^R) = \sum_{j=1}^{J}\beta(j|s^R)H(j;k),$$

$$\beta(\tilde{j}|s^L)\tilde{y} + \sum_{j>\tilde{j}}\beta(j|s^L) \le \sum_{j=1}^{J}\beta(j|s^L)H(j;k).$$

Let $\underline{j} = \max\{j\in\mathbb{J} : H(j';k) = 0\ \forall j' < j\}$ and $\bar{j} = \min\{j\in\mathbb{J} : H(j';k) = 1\ \forall j' > j\}$. Then, for all $t\in\mathbb{T}$ and all $m = 1, \ldots, M$, we have $k_m(t,j) = 0$ for $j = 1, \ldots, \underline{j}-1$ and $k_m(t,j) = 1$ for all $j = \bar{j}+1, \ldots, J$. There are three cases.

In the first case, $\underline{j} = \bar{j}+1$. Since k_m is already monotone preference-independent, the claim follows by setting $\tilde{y} = 0$ and $\tilde{j} = \bar{j}$, or equivalently, $\tilde{y} = 1$ and $\tilde{j} = \underline{j}$.

In the second case, $\underline{j} = \bar{j} = j'$. We have $H(j';k) \in (0,1)$. The claim follows immediately by letting $\tilde{j} = j'$, and $\tilde{y} = H(j';k)$.

In the third case, $\underline{j} < \bar{j}$. Construct a new strategy $k' = (k'_1, \ldots, k'_M)$ by modifying k as follows. For all $t' \in \mathbb{T}$ and $m' = 1, \ldots, M$, let $k'_{m'}(t',j) = k_{m'}(t',j)$ for all $j \neq \underline{j}, \bar{j}$; if

$$\beta(\underline{j}|s^R)H(\underline{j};k) < \beta(\bar{j}|s^R)(1 - H(\bar{j};k)),$$

let $k'_{m'}(t', \underline{j}) = 0$ and

$$k'_{m'}(t', \bar{j}) = H(\bar{j}; k) + \frac{\beta(\underline{j}|s^R)}{\beta(\bar{j}|s^R)} H(\underline{j}; k);$$

otherwise, let $k'_{m'}(t', \bar{j}) = 1$ and

$$k'_{m'}(t', \underline{j}) = H(\underline{j}; k) - \frac{\beta(\bar{j}|s^R)}{\beta(\underline{j}|s^R)} (1 - H(\bar{j}; k)).$$

By construction,

$$\sum_{j=1}^{J} \beta(j|s^R) H(j; k') = \sum_{j=1}^{J} \beta(j|s^R) H(j; k).$$

By Assumption 1,

$$\sum_{j=1}^{J} \beta(j|s^L) H(j; k') \leq \sum_{j=1}^{J} \beta(j|s^L) H(j; k).$$

Compared with k, the number of signals j for which $k'_m(t, j)$ is not 0 or 1 for all t and all m is reduced by 1. The claim then follows by repeating these modifications until either the first case or the second case applies to the resulting strategy.

Proof of Proposition 1

By Assumption 1, there exists $\epsilon > 0$ such that for $j > j'$,

$$\left(\frac{\delta^R}{\delta^L} - \epsilon \right) \frac{\mu(s^R|j)}{\mu(s^L|j)} > \left(\frac{\delta^R}{\delta^L} + \epsilon \right) \frac{\mu(s^R|j')}{\mu(s^L|j')}.$$

By Lemma 3, since full information equivalence is feasible, there is a monotone preference-independent strategy k satisfying (DOM) for any $v^L \in (z(s^L, \overline{\theta}; k), v^C)$ and $v^R \in (v^C, z(s^R, \underline{\theta}; k))$. For any such fixed v^L and v^R, there is a unique $\hat{k} = \kappa(\hat{y}; \hat{j})$ that also satisfies (CON). For simplicity, we assume that $\hat{y} \in (0, 1)$. Since $\hat{k}$ satisfies condition (DOM) with strictly inequalities, and $z(s, \theta; \kappa(y; \hat{j}))$ is strictly increasing in y, we can find $\eta > 0$ sufficiently small, such that condition (DOM) continues to hold for all $\kappa(y; \hat{j})$ with $y \in [\hat{y} - \eta, \hat{y} + \eta]$. Then, since $\hat{k}$ satisfies (CON), there exists N_g such that for all $n \geq N_g$,

$$\frac{g^n(v^L|s^L; \kappa(\hat{y} - \eta; \hat{j}))}{g^n(v^R|s^R; \kappa(\hat{y} - \eta; \hat{j}))} < \left(\frac{\delta^R}{\delta^L} - \epsilon \right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})};$$

$$\frac{g^n(v^L|s^L; \kappa(\hat{y} + \eta; \hat{j}))}{g^n(v^R|s^R; \kappa(\hat{y} + \eta; \hat{j}))} > \left(\frac{\delta^R}{\delta^L} + \epsilon \right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})}.$$

Fix any n, and consider any $k = \kappa(y;\hat{j})$ for $y \in [\hat{y}-\eta, \hat{y}+\eta]$. For each $j \in \mathbb{J}$, $t \in \mathbb{T}$ and $m = 1, \ldots, M$, let $\rho^n_m(j,t;k)$ be such that

$$\begin{aligned}
&\sum_{s \geq s^R} \frac{\mu(s|j)}{\mu(s^R|j)} \left(\frac{g^n(v^R|s;k)}{g^n(v^R|s^R;k)}\delta^R + \frac{g^n(v^C|s;k)}{g^n(v^R|s^R;k)}u_m(s,t) - \frac{g^n(v^L|s;k)}{g^n(v^R|s^R;k)}\delta^L \right) \\
&= \rho^n_m(j,t;k) \sum_{s \leq s^L} \frac{\mu(s|j)}{\mu(s^L|j)} \left(-\frac{g^n(v^R|s;k)}{g^n(v^L|s^L;k)}\delta^R - \frac{g^n(v^C|s;k)}{g^n(v^L|s^L;k)}u_m(s,t) + \frac{g^n(v^L|s;k)}{g^n(v^L|s^L;k)}\delta^L \right).
\end{aligned}$$

Since k satisfies (DOM), by Assumption 1,

$$z(s,\theta;k) \geq z(s^R,\underline{\theta};k) > v^R$$

for all $s \geq s^R$ and all θ. This implies that the pivotal event v^R in state s^R dominates any pivotal event v in any state $s \geq s^R$. Symmetrically, the pivotal event v^L in state s^L dominates any pivotal event v in any state $s \leq s^L$. As a result, for all $j \in \mathbb{J}$, all $t \in \mathbb{T}$ and all $m = 1, \ldots, M$,

$$\lim_{n\to\infty} \rho^n_m(j,t;k) = \frac{\delta^R}{\delta^L}.$$

Since by definition $\rho^n_m(j,\cdot;\kappa(\cdot;\hat{j}))$ is continuous on closed and bounded intervals $\mathbb{T}$ and $[\hat{y}-\eta, \hat{y}+\eta]$, there exists N_ρ such that for all $n \geq N_\rho$,

$$\frac{\delta^R}{\delta^L} - \epsilon < \rho^n_m(j,t;k) < \frac{\delta^R}{\delta^L} + \epsilon$$

for all $j \in \mathbb{J}$, $t \in \mathbb{T}$ and $m = 1, \ldots, M$, and $y \in [\hat{y}-\eta, \hat{y}+\eta]$.

Fix $n \geq \max\{N_g, N_\rho\}$. For any $y \in [\hat{y}-\eta, \hat{y}+\eta]$ and for each $m = 1, \ldots, M$, let $A^n_m(y)$ be the set of informed voters of preference group m with preference types $t \in \mathbb{T}$ that weakly prefer candidate R at the critical signal $\hat{j}$ against $\kappa(y;\hat{j})$

$$A^n_m(y) = \left\{ t \in \mathbb{T} : \rho^n_m\left(\hat{j},t;\kappa(y;\hat{j})\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} \geq \frac{g^n(v^L|s^L;\kappa(y;\hat{j}))}{g^n(v^R|s^R;\kappa(y;\hat{j}))} \right\},$$

and let

$$B^n(y) = \sum_{m=1}^{M} \lambda_m \int_{t\in A^n_m(y)} \mathrm{d}P_m(t) - y.$$

Since $n \geq \max\{N_g, N_\rho\}$, for any $t \in \mathbb{T}$ and any $m = 1, \ldots, M$, we have

$$\begin{aligned}
\rho^n_m\left(\hat{j},t;\kappa(\hat{y}-\eta;\hat{j})\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} &> \left(\frac{\delta^R}{\delta^L} - \epsilon\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} > \frac{g^n(v^L|s^L;\kappa(\hat{y}-\eta;\hat{j}))}{g^n(v^R|s^R;\kappa(\hat{y}-\eta;\hat{j}))}; \\
\rho^n_m\left(\hat{j},t;\kappa(\hat{y}+\eta;\hat{j})\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} &< \left(\frac{\delta^R}{\delta^L} + \epsilon\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} < \frac{g^n(v^L|s^L;\kappa(\hat{y}+\eta;\hat{j}))}{g^n(v^R|s^R;\kappa(\hat{y}+\eta;\hat{j}))}.
\end{aligned}$$

As a result, $B^n(\hat{y}-\eta) = 1-(\hat{y}-\eta) > 0$ and $B^n(\hat{y}+\eta) = 0-(\hat{y}+\eta) < 0$. It then follows from the Intermediate Value Theorem that there exists some $\hat{y}^n \in (\hat{y}-\eta, \hat{y}+\eta)$ such that $B^n(\hat{y}^n) = 0$. Further, we can assume that $B^n(y)$ is not locally constant at $y = \hat{y}^n$, and thus there exist $\hat{t}^n \in \mathbb{T}$ and some $\hat{m} = 1, \ldots, M$ such that

$$\rho^n_{\hat{m}}\left(\hat{j}, \hat{t}^n; \kappa(\hat{y}^n; \hat{j})\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} = \frac{g^n(v^L|s^L; \kappa(\hat{y}^n; \hat{j}))}{g^n(v^R|s^R; \kappa(\hat{y}^n; \hat{j}))}.$$

Consider the strategy $\hat{k}^n$ that differs from the monotone preference-independent strategy $\kappa(\hat{y}^n; \hat{j})$ only in that, for each preference group $m = 1, \ldots, M$, at signal $\hat{j}$, all preference types $t \in A^n_m(\hat{y}^n)$ vote for candidate R, and all other types $t \notin A^n_m(\hat{y}^n)$ vote for candidate L. We claim that $\hat{k}^n$ is an equilibrium.

First, $\hat{k}^n$ and $\kappa(\hat{y}^n; \hat{j})$ have the same set of best responses. The two strategies coincide except at signal $\hat{j}$, and by construction

$$\sum_{m=1}^{M} \lambda_m \int_{t \in A^n_m(\hat{y}^n)} \mathrm{d}P_m(t) = \hat{y}^n.$$

Thus, a randomly chosen informed voter votes candidate R with the same probability under $\hat{k}^n$ as under $\kappa(\hat{y}^n; \hat{j})$. Second, by construction $\hat{k}^n$ is optimal against $\kappa(\hat{y}^n; \hat{j})$ for each $t \in \mathbb{T}$ and each preference group m at signal $\hat{j}$. Third, $\hat{k}^n$ is optimal against $\kappa(\hat{y}^n; \hat{j})$ for each $t \in \mathbb{T}$ and each preference group m at all signals $j \neq \hat{j}$. This is because, for any $j > \hat{j}$,

$$\begin{aligned}
\rho^n_m\left(j, t; \kappa(\hat{y}^n; \hat{j})\right) \frac{\mu(s^R|j)}{\mu(s^L|j)} &> \left(\frac{\delta^R}{\delta^L} - \epsilon\right) \frac{\mu(s^R|j)}{\mu(s^L|j)} \\
&> \left(\frac{\delta^R}{\delta^L} + \epsilon\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} \\
&> \rho^n_{\hat{m}}\left(\hat{j}, \hat{t}^n; \kappa(\hat{y}^n; \hat{j})\right) \frac{\mu(s^R|\hat{j})}{\mu(s^L|\hat{j})} \\
&= \frac{g^n(v^L|s^L; \kappa(\hat{y}^n; \hat{j}))}{g^n(v^R|s^R; \kappa(\hat{y}^n; \hat{j}))},
\end{aligned}$$

where the first inequality comes from $n \geq N_\rho$, the second from $j > \hat{j}$ and the choice of ϵ, the third from $n \geq N_\rho$, and the last from the definition of $\hat{t}^n$. Thus, it is optimal to choose candidate R. The argument for $j < \hat{j}$ is symmetric.

The equilibrium sequence $\{\hat{k}^n\}$ achieves full information equivalence. To see this, note that the monotone preference-independent strategy $\kappa(\hat{y}^n; \hat{j})$ associated with $\hat{k}^n$ satisfies

$$\lim_{n\to\infty} \hat{y}^n = \hat{y}.$$

Otherwise, since $\hat{k}$ satisfies (CON), for any N there is some $n > N$ such that the ratio $g^n(v^L|s^L, \kappa(\hat{y}^n; \hat{j}))/g^n(v^R|s^R, \kappa(\hat{y}^n; \hat{j}))$ is either arbitrarily close to 0, which implies that all informed voters prefer candidate R at $\hat{j}$, or arbitrarily large, which implies that they all prefer L, contradicting that $\hat{k}^n$ is a best response to itself. Thus, the equilibrium vote shares converge:

$$\lim_{n\to\infty} z(s^L, \overline{\theta}; \hat{k}^n) = z(s^L, \overline{\theta}; \hat{k}); \ \lim_{n\to\infty} z(s^R, \underline{\theta}; \hat{k}^n) = z(s^R, \underline{\theta}; \hat{k}).$$

Since $\hat{k}$ satisfies (DOM), by Lemma 2, the equilibrium sequence $\{\hat{k}^n\}$ achieves full information equivalence.

5.1. Proof of Proposition 2

First, we show that along the sequence of equilibria $\{k_0^n\}$ of $\{\Gamma_0^n\}$ the equilibrium vote shares $z(s^L, \overline{\theta}; k_0^n)$ and $z(s^R, \underline{\theta}; k_0^n)$ converge, and the limits satisfy an equal-rate condition

$$I\left(v^C; \lim_{n\to\infty} z(s^L, \overline{\theta}; k_0^n)\right) = I\left(v^C; \lim_{n\to\infty} z(s^R, \underline{\theta}; k_0^n)\right).$$

Otherwise, by Lemma 1, for any N there is some $n > N$ such that either the ratio $g^n(v^C|s; k_0^n)/g^n(v^C|s^R; k_0^n)$ is arbitrarily close to 0 for all $s \neq s^R$, or the ratio $g^n(v^C|s; k_0^n)/g^n(v^C|s^L; k_0^n)$ is arbitrarily close to 0 for all $s \neq s^L$. In either case, the best response to k_0^n of every preference type t in every group m is independent of their signal, as conditional on the pivotal event v^C the state is known. This contradicts the hypothesis that the $\{k_0^n\}$ achieves full information equivalence.

Since by assumption the sequence of equilibria $\{\hat{k}^n\}$ of $\{\Gamma^n\}$ also satisfy the equal-rate condition (CON), we have

$$I\left(v^L; \lim_{n\to\infty} z(s^L, \overline{\theta}; \hat{k}^n)\right) = I\left(v^R; \lim_{n\to\infty} z(s^R, \underline{\theta}; \hat{k}^n)\right).$$

By Lemma 1, for n sufficiently large, if $z(s^R, \underline{\theta}; k_0^n) > z(s^R, \underline{\theta}; \hat{k}^n)$, then the above two equal-rate conditions can be satisfied only if $z(s^L, \overline{\theta}; k_0^n) < z(s^L, \overline{\theta}; \hat{k}^n)$. To prove the proposition it is then sufficient to establish the following claim:

$$z(s^R, \underline{\theta}; k_0^n) \geq z(s^R, \underline{\theta}; \hat{k}^n) \implies z(s^L, \overline{\theta}; k_0^n) \geq z(s^L, \overline{\theta}; \hat{k}^n),$$

with the second inequality being strict if Assumption 3 holds.

Suppose that $z(s^R, \underline{\theta}; k_0^n) \geq z(s^R, \underline{\theta}; \hat{k}^n)$. In the proof of Lemma 3, we have shown that given k_0^n there is a monotone preference-independent monotone strategy $\kappa(y_0; j_0)$ such that $z(s^R, \underline{\theta}; \kappa(y_0; j_0)) = z(s^R, \underline{\theta}; k_0^n)$ and $z(s^L, \overline{\theta}; \kappa(y_0; j_0)) \leq z(s^L, \overline{\theta}; k_0^n)$. The latter inequality holds strictly if k_0^n is not almost type-independent and monotone, that is, if there is no $j' \in \mathbb{J}$ such that under k_0^n all informed voters vote for L at any signal $j < j'$ and R at any $j > j'$, regardless of their preference type t and their group m. In the proof of Proposition 1, we have shown that there is a monotone preference-independent strategy $\kappa(\hat{y}^n; \hat{j})$ such that $z(s, \theta; k^n) = z(s, \theta; \kappa(\hat{y}^n; \hat{j}))$ for all s, θ. It follows that

$$z(s^R, \underline{\theta}; \kappa(y_0; j_0)) \geq z(s^R, \underline{\theta}; \kappa(\hat{y}^n; \hat{j})).$$

Since both $\kappa(y_0; j_0)$ and $\kappa(\hat{y}^n; \hat{j})$ are monotone preference-independent, the above implies that $j_0 \leq \hat{j}$ with $y_0 \geq \hat{y}^n$ if $j_0 = \hat{j}$. As a result, we have

$$z(s^L, \overline{\theta}; \kappa(y_0; j_0)) \geq z(s^L, \overline{\theta}; \kappa(\hat{y}^n; \hat{j})),$$

and therefore

$$z(s^L, \overline{\theta}; k_0^n) \geq z(s^L, \overline{\theta}; \kappa(y_0; j_0)) \geq z(s^L, \overline{\theta}; \kappa(\hat{y}^n; \hat{j})) = z(s^L, \overline{\theta}; \hat{k}^n),$$

where the first inequality is strict if k_0^n is not almost preference-independent and monotone. When Assumption 3 holds, the equilibrium k_0^n of Γ_0^n involves a positive mass of informed voters whose votes are uninformative. Since by assumption $\{k_0^n\}$ achieves full information equivalence, a positive mass of informed voters cast informative votes. Thus, k_0^n is not almost preference-independent. The claim follows immediately.