Dating and Divorce

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Abstract

We introduce dating and divorce in a search and matching model without transfers. Forming a marriage allows agents to find out about each other's types, and then decide whether to incur the cost of divorce and re-enter the market. Agents can partially reveal private information to each other through communication (dating), which makes them more selective in their marriage decision. Such strategic communication improves the future prospects after divorce, which makes agents more willing to end less desirable marriages. We construct equilibria that transition to a steady state. Dating can cause both a decrease in the long-run marriage rate and an increase in the divorce rate. A lower cost of divorce can make all agents weakly worse off.

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1 Introduction

In most of the literature on search and matching, finding a partner is a time-consuming search process, but each party obtains complete information about their counterpart and the value of the match as soon as they meet (Burdett and Coles, 1997; Shimer and Smith, 2000). However, in reality often the only way to find out whether the match is right is through forming a match, even if temporarily. This paper investigates the effect of this friction in learning about match characteristics. In particular, we introduce private information about one's own type that persists at the time when the parties initially meet in the search market.

In this set-up, we investigate two issues that have been largely overlooked in the existing search and matching literature. The first issue is strategic communication. When the meeting parties possess private information about own characteristics that jointly determines the quality of their match, they can engage in cheap talk communication a la Crawford and Sobel (1982) prior to taking a decision whether or not to get married. Placed in the application of marriage search and matching, which is the main consideration of this paper, we may think of such strategic communication as "dating." The second issue is endogenous breakup. As long as cheap talk communication does not perfectly reveal all relevant private information, marriage is a gamble that may or may not work out, and so both or just one of the parties may wish to divorce. Hence, unlike in much of the existing literature on search and matching, in this paper we study an endogenous divorce decision. Specifically, this decision is made based on what the two people in a marriage have learned about each other, as well as on their future market prospects after the divorce. When there is a cost to divorce, it will also be part of the consideration.

To highlight the main issues, we construct the simplest model where strategic communication and endogenous breakup interact with each other. We assume that the parties' privately known types are "vertical," in that all market participants have the same ranking over their partners' types. Particularly, a "high" type is more attractive than a "low" type to all participants on the opposite side of the market. In the presence of private information about match types, the marriage decision is based on the pre-marital cheap talk messages only. Further, we assume that the private information about match types is revealed completely after the first period of the marriage, rather than gradually during the marriage. This implies that in equilibrium, divorce is a one-time decision made by the married couple after the first period of marriage.

Our main result is that informative pre-marriage strategic communication does not necessarily reduce the divorce rate. This is so because informative strategic communication has two opposing effects. On the one hand, to the extent that pre-marriage communication is informative about the types of the parties who have met on the market, they will be more selective in their marriage decisions. This naturally reduces the incidence of unstable marriages in which high types are married to low types, and tends to lower the divorce rate. On the other hand, precisely because pre-marriage communication is informative, high types have better prospects of matching with high types in the future. Consequently, so long as pre-marriage communication is imperfect, high types are more willing to divorce low types. We show that the second effect of imperfect but informative pre-marriage strategic communication can outweigh the first effect, resulting in a higher divorce rate in equilibrium.

In the absence of strategic communication, the high type may be unwilling to divorce a marriage partner of low type, since future matching is random and no one reveals any information about their type before the marriage. Thus, there is an equilibrium without informative communication (babbling) and no divorce.[1] Yet, for the same divorce cost and the values of other parameters, there exists another equilibrium in which both types truthfully reveal their types, supported by the high type divorcing a low-type partner after the latter lies. There is no divorce on the equilibrium path because communication is fully revealing. Most interestingly, there may exist a partially informative equilibrium. The high type truthfully reveals itself before marriage and the low type randomizes between telling the truth and lying about her type, while the high type randomizes between keeping and divorcing a low-type partner. In contrast to the babbling and fully revealing equilibria, in this partially informative equilibrium some marriages result in a costly divorce. This in turn is the reason that the low type in equilibrium does not always lie.

The increase in the divorce rate across most societies and cultures in modern times

[1]We give a complete characterization of all babbling equilibria. Some of them involve the high type divorcing the low type after marriage.

has often been attributed to the reduction in the cost of divorce. Our paper provides a counterpoint showing that an increase in the divorce rate may have instead resulted from greater ease of dating and the willingness to engage in meaningful discussion of possible marriage. Interpolating our logic a little further, our results suggest that when social norms change from arranged marriages to dating, the prospects of finding the right partner improve sufficiently to make the parties in bad marriages more willing to divorce.[2] This interpretation makes sense when privately known type involves traits other than wealth or social class that are easily observable. Examples of such private type that is more difficult to observe but is at the same time vertical include temperament, cooperativeness, empathy and health status.

In section 2, we set up the main model with two types. We impose the restriction that the agents' type-dependent strategies in any given period are symmetric and do not depend on their private histories, while allowing the strategies to vary with the time period. With additional restrictions, we show that the evolution of the type distribution depends on the strategy only through the fraction of permanent cross-type marriages.

In section 3, we characterize three classes of equilibria: babbling, fully revealing and partially informative. We provide sufficient conditions for each equilibrium to exist, requiring that the initial type distribution is sufficiently close to the steady-state type distribution. In equilibrium the type distribution either jumps to the steady state in one period or converges to the steady state.

In section 4, we focus on the steady-state partially informative equilibrium. We show that when the cost of divorce is neither too high nor too low, this equilibrium coexists with the steady-state babbling equilibrium with no divorce and with the fully-revealing equilibrium. The high type is best off in the fully revealing equilibrium, but is better off in a partially informative equilibrium than in a babbling equilibrium with no divorce, despite a positive rate of a costly divorce in the partially informative equilibrium. In contrast, the low type may be worst off in a partially informative equilibrium, even though there is a possibility of marrying a high type permanently. In a partially informative equilibrium, a decrease in the cost of divorce unambiguously lowers the high type's payoff, without necessarily making the

[2] In line with this result, some studies have documented that more marriages in India have changed from arranged marriage to love marriage in recent years (Allendorf (2013); Allendorf and Pandian (2016)). At the same time, the divorce rate in India has also been rising (Dommaraju (2016)).

low type better off.

Section 5 extends our analysis of market dynamics in section 3 to exogenous entry size and endogenous replacements, and shows that the qualitative nature of the equilibrium non-stationary dynamics in the baseline model is robust. We also extend the steady-state analysis in section 4 to horizontal differentiation, idiosyncratic shocks, and continuous types, and demonstrate that the steady-state equilibrium with dating and divorce is robust. In the continuous-type extension, we construct a steady-state equilibrium with two cheap talk messages and two "classes" defined by permanent marriages, with the lower class divided further into three subclasses who follow different strategies in their dating and marriage decisions. Unlike the partially informative equilibrium in the main model, the equilibrium with continuous types here is in pure strategies, and yet there is both a selectivity in marriage and divorce on the equilibrium path.

1.1 Literature review

Most papers in the literature on marriage search and matching assume that agents perfectly observe their potential partners' types immediately upon meeting. A non-exhaustive list includes Burdett and Coles (1997), Burdett and Wright (1998), Eeckhout (1999), Bloch and Ryder (2000), Smith (2006), Jacquet and Tan (2007) and Lauermann and Noldeke (2014). This strand of literature explores how equilibrium matching patterns depend on search technology and match payoffs. We contribute to this literature by demonstrating how private information, and the ensuing cheap talk and endogenous breakup reshape the matching pattern. In particular, unlike the "block segregation" predicted in Burdett and Coles (1997), Smith (2006) and other papers, our base model and its continuous type extension predict mixed marriages between blocks. In addition, the aforementioned search and matching models do not address the issue of divorce. With imperfect information, our model can characterize when and between whom endogenous breakup is more likely to occur.

Two recent papers on marriage search and matching also incorporate information frictions into their models. Chade (2006) assumes that agents receive only a noisy signal about a potential partner's type before making irreversible marriage decisions, leading to an acceptance curse. Antler, Bird and Fershtman (2022) assume that agents can date to learn about marriage compatibility, which increases with the observable "pizzazz" of agents, before deciding whether to marry or separate. They illustrate the "thick market" externality caused by information acquisition. While they focus on the learning process during the dating stage, we highlight strategic communication in our model.

Another key component of our model is costly divorce. Related to our model, Chiappori and Weiss (2006) and Browning, Chiappori and Weiss (2014) capture the search externality associated with divorce and show that the prospects of remarriage could generate multiple equilibria due to a positive feedback effect. Multiple equilibria can also arise in our model due to the feedback effect between remarriage prospects and communication strategy.

Finally, most papers in the search-and-matching literature confine their analysis to steady-state equilibrium, with only a few exceptions.[3] Sandmann and Bonneton (2024) prove the existence of a non-stationary equilibrium in the canonical search-and-matching model with heterogeneous agents. Assuming non-transferable payoffs, they show that sufficient conditions for positive assortative matching in steady state are insufficient in a non-stationary environment. In a wage-posting model with random search, Moscarini and Postel-Vinay (2013) study the cyclical pattern of labor market caused by aggregate productivity shocks. Our paper constructs non-stationary equilibria and show how they transition to steady-state equilibria. From a theoretical point of view, our analysis demonstrates the robustness of steady-state equilibria, and may be used to explain how unexpected shocks can change the market outcome in the long run.

2 Model

We study an infinite-horizon dynamic matching market between men and women with no transfers. Time is discrete, starting from period 1. The two sides of the market are entirely symmetric. In period 1, there is a continuum of men on one side of the market, and an equal measure of a continuum of women on the other side. A fraction p_1 of all men, and of all women, are of type H; the remaining fraction $1 - p_1$ are of type L. All agents are risk-neutral, and have the same per-period discount factor β.

[3] In Damiano, Li and Suen (2006), the time horizon is finite, and any equilibrium is non-stationary.

In each period $t \geq 1$, the events unfold sequentially as follows.

- **Meet and date**. Newborns privately learn that their permanent type is H with probability $\phi \in (0, 1)$ and L with probability $1 - \phi$ (except for $t = 1$, when there are are no newborns). All men and women "alive" in period t are randomly paired with each other.[4] The meeting probability is equal to one for each agent. When a man and a woman meet, they do not observe each other's type. They simultaneously choose one of two messages, h and l, to send to each other, where h represents "I am a high type," and l represents "I am a low type."

- **Accept or reject**. The man and the woman in a meeting simultaneously decide whether to accept or reject their meeting partner for marriage. If at least one of them rejects their partner, the two agents each get a payoff of 0 in period t and wait for the market to open in period $t+1$. If they accept each other, each receives a match payoff $\nu_{\Theta\Theta'}$ in period t if their own type is $\Theta = H, L$ and the partner's type is $\Theta' = H, L$.

- **Keep or divorce**. The man and the woman in a marriage learn their partner's true type. They decide simultaneously whether or not to keep the marriage or divorce. If at least one of them chooses to divorce, they each incur a divorce cost $\delta > 0$ and wait for the market to open in period $t + 1$. If both of them decide to keep the marriage, they leave the market, and thereafter receive a payoff of $\nu_{\Theta\Theta'}$ in each future period if their own type is $\Theta = H, L$ and the partner's type is $\Theta' = H, L$. They are replaced by a newborn man and a new born woman in the beginning of period $t + 1$.[5]

In the main analysis, we make the following assumptions on the payoffs.

Assumption V. $\nu_{\Theta H} > \nu_{\Theta L} > 0$ *for each* $\Theta = H, L$.

Assumption V means that agents are vertically differentiated, with type H partner more attractive than type L regardless of one's own type, and matching with type L is strictly

[4]We focus on equilibria with full participation by all agents in each time period by assuming that all agents alive in period t are automatically matched at the beginning of the period. Under this assumption, agents can still effectively skip to the next period $t + 1$ by rejecting their meeting partner.

[5] In section 5.1, we show that the alternative assumption of exogenous size of newborn only affect the market size of the constructed equilibria.

better than not participating in the market. In our main analysis, this assumption helps us focus on the incentives of a single type: type L in choosing which message to send, and type H in choosing to keep or divorce a type L marriage partner.

Assumption C. $\nu_{HH} - \nu_{HL} \geq \nu_{LH} - \nu_{LL}$.

Assumption C imposes a weak payoff complementarity: type H weakly benefits more from matching with H instead of L than type L does. Unlike in most papers on dynamic search and matching (e.g., Shimer and Smith, 2000), there are no transfers between the two agents either in the dating, or divorce stage after marriage. As a result, Assumption C applies to individual match values, rather than the joint match payoffs. This assumption is convenient for us to establish sufficient conditions for equilibrium.

A special case that satisfies both Assumptions V and C is $\nu_{HH} = \nu_{LH} > \nu_{HL} = \nu_{LL}$. The match payoff to an agent is independent of their own type. This is the payoff structure in Burdett and Coles (1997). We will often use this payoff structure as an illustrative example for our main equilibrium constructions.

Assumption D. $\delta > \max\{\nu_{HL}, \nu_{LH}\}$.

Since the payoff in the current period is zero for skipping to the next period, if $\delta < \nu_{HL}$, it would be strictly dominant for type H to accept a partner regardless of the message the latter has sent. Likewise, if $\delta < \nu_{LH}$, type L would only send the message that has the greater probability of being accepted by type H. In either case, cheap talk communication at dating would not take the informative form we are interested in.

3 Equilibrium Construction

A behavior strategy of a newborn agent in period t (including all agents in period 1) maps each information set to a probability distribution over available choices. For each period $\tau \geq t$, there are three kinds of information sets:

- "dating," choosing between message h and l;
- "marriage," choosing between accepting and rejecting the meeting partner;
- "post marriage," choosing between keeping and divorcing the marriage partner.

We represent each information set by the private history of the agent that they can condition their choice on. In each period $\tau \geq t$, a "transition path" from τ to $\tau+1$ can take the following form: for each possible combinations of messages h and l sent by the agent and the meeting partner, at least one of them chooses to reject marriage; for each possible combinations of messages sent by the two, at least one of them chooses to divorce. The private history at dating in period τ consists of a transition path for each previous period $t, t+1, \ldots, \tau-1$, and the permanent type that the agent is born with; the private history at marriage consists of the messages sent by the agent and their meeting partner in period τ, and the private history at dating; the private history post marriage consists of the type of the marriage partner, and the private history at marriage.

We focus on equilibria, to be defined shortly below, where all agents alive in period $t \geq 1$, use the same strategy, regardless of their gender or their own private history. The symmetry restriction is reasonable in our model because the two sides of the market have the same payoffs $\nu_{\Theta\Theta'}$ and replacement fraction ϕ. The history-independence restriction is akin to an anonymity assumption imposed to simplify the analysis as we have an infinite horizon model. We stress the assumption of history-independence allows strategy of agents to depend on t; that is, we allow non-stationary strategies.

We impose the following additional restrictions: for each $t \geq 1$,

- at dating, type H sends message h;
- at marriage, the accept-reject decision of each type does not depend on their own message, and further, type H accepts message h and type L accepts message l;
- post marriage, the keep-divorce decision of each type does not depend on messages sent by themselves or by their marriage partners, and further, type H keeps type H and type L keeps both types.

We stress that that all substantive restrictions on the equilibrium strategy should be thought of as equilibrium refinement. That is, we will verify in our equilibrium analysis that the restrictions are in fact incentive compatible for agents even though they could use any feasible strategy. The restriction at dating is just labeling — we are calling the message sent in equilibrium by type H message h. The restriction at marriage that the accept-reject decision of each type is independent of their own message reflects the fact that messages are cheap talk in our model, and so does the restriction post marriage that the keep-divorce decision of each type is independent of both their own and their first-marriage partner's messages. The restriction at marriage that type H accepts message h and type L accepts message l, and the restriction that type H keeps type H and type L keeps both types, are imposed to simplify how we represent an equilibrium strategy.

Strategies satisfying the above restrictions can be represented by four infinite sequences $\{m_{L,t}\}_{t=1}^{\infty}$, $\{a_{H,t}\}_{t=1}^{\infty}$, $\{a_{L,t}\}_{t=1}^{\infty}$, and $\{k_{H,t}\}_{t=1}^{\infty}$, where $m_{L,t}$ is the probability that type L sends message h at dating in period t, $a_{H,t}$ is the probability that type H accepts message l at marriage, $a_{L,t}$ is the probability that type L accepts message h at marriage, and $k_{H,t}$ is the probability that type H keeps type L post marriage. Then, for a newborn agent in any period $t \geq 1$, the strategy of the agent is the continuation $\{m_{L,\tau}, a_{H,\tau}, a_{L,\tau}, k_{H,\tau}\}_{\tau=t}^{\infty}$.

Starting with any fixed fraction p_1 of type H agents in period 1, any given strategy $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ induces an infinite sequence $\{p_t\}_{t=2}^{\infty}$ of fraction of type H in period t as follows. At any time $t \geq 1$, consider a randomly drawn agent from one side of the market. There are four profiles of the types of this agent and the meeting partner from the other side of the market. The fraction p_{t+1} of type H in period $t+1$ on the side of the agent under consideration, is equal to the total contributions over the four profiles:

$$\begin{aligned} p_{t+1} = {} & p_t^2\phi + p_t(1-p_t)(1-(m_{L,t}+(1-m_{L,t})a_{H,t})a_{L,t}k_{H,t}(1-\phi)) \\ & + (1-p_t)p_t(m_{L,t}+(1-m_{L,t})a_{H,t})a_{L,t}k_{H,t}\phi + (1-p_t)^2(m_{L,t}a_{L,t}+1-m_{L,t})^2\phi. \end{aligned} \tag{D}$$

The first and second terms in the sum are the contributions when the agent under consideration is of type H and stays on as type H in period $t+1$ unless the agent is replaced, while the third and the fourth terms are the contributions when the agent is of type L and stays on as type L unless the agent is replaced. We refer to p_t as the "state" in period t.

The dynamic system of state evolution can depend on the strategy in a complex way. However, if

$$m_{L,t}(1-a_{L,t}) = (1-m_{L,t})a_{H,t} = 0 \tag{S}$$

for all $t \geq 1$, the transition to p_{t+1} from p_t depends on period t decisions of agents at dating $m_{L,t}$, marriage $a_{H,t}$ and $a_{L,t}$, and permanent marriage $k_{H,t}$ only through

$$1 - m_{L,t}k_{H,t} \equiv z_t.$$

In all equilibria we construct below, condition (S) is satisfied. We can understand z_t as the probability in period t that, conditional on meeting type H, type L remains in the market for period $t+1$. Defining function

$$Q(p|z) = \phi + (1-2\phi)p(1-p)z,$$

we can write the transition from p_t to p_{t+1} as

$$p_{t+1} = Q(p_t|z_t). \tag{SD}$$

For notational brevity, let $\overline{\phi} \equiv \frac{1}{2}\left(\phi + \frac{1}{2}\right)$. It is straightforward to verify the following properties of the function Q:

- $\phi = \frac{1}{2}$: $Q(p|z) = \frac{1}{2}$ for all $p \in [0,1]$ and $z \in [0,1]$.
- $\phi < \frac{1}{2}$: $Q(p|z) \in [\phi, \overline{\phi}]$ for all $p \in [0,1]$ and $z \in [0,1]$; for any $z \in [0,1]$, there is a unique $\pi(z) \in [\phi, \overline{\phi}]$ such that $\pi(z) = Q(\pi(z)|z)$, with $Q(p|z)$ strictly increasing in $p \in [\phi, \overline{\phi}]$, $Q(p|z) > \pi(z)$ for $p < \pi(z)$ and $Q(p|z) < \pi(z)$ for $p > \pi(z)$; and for any $p \in [\phi, \overline{\phi}]$, $Q(p|z)$ is strictly increasing in z, with $\pi(z)$ increasing from $\pi(0) = \phi$ to $\pi(1)$.
- $\phi > \frac{1}{2}$: $Q(p|z) \in [\overline{\phi}, \phi]$ for all $p \in [0,1]$ and $z \in [0,1]$; for any $z \in [0,1]$, there is a unique $\pi(z) \in [\overline{\phi}, \phi]$ such that $\pi(z) = Q(\pi(z)|z)$, with $Q(p|z)$ strictly increasing in $p \in [\overline{\phi}, \phi]$, $Q(p|z) > \pi(z)$ for $p < \pi(z))$ and $Q(p|z) < \pi(z)$ for $p > \pi(z)$; and for any $p \in [\overline{\phi}, \phi]$, $Q(p|z)$ is strictly decreasing in z, with $\pi(z)$ decreasing from $\pi(0) = \phi$ to $\pi(1)$.

See Figure 1 for illustrations of the case of $\phi < \frac{1}{2}$ and the case of $\phi > \frac{1}{2}$ above. For any strategy $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$, starting from any $p_1 \in [0,1]$, p_2 is bounded between ϕ and $\overline{\phi}$. Thereafter, p_t for any $t \geq 3$ is bounded between ϕ, which is the fixed point $\pi(0)$ of $Q(\cdot|0)$, and $Q^{t-2}(\overline{\phi}|1)$, which converges monotonically to $\pi(1)$, the fixed point of $Q(\cdot|1)$. The main difference between $\phi < \frac{1}{2}$ and $\phi > \frac{1}{2}$ is that $\pi(0) < \pi(1)$ in the former case and $\pi(0) > \pi(1)$ in the latter case. For each $\hat{z} \in [0, 1]$, the fixed point $\pi(\hat{z})$ of the function $Q(\cdot|\hat{z})$ represents a "steady state" type distribution. Since each steady state type distribution p corresponds to a unique $z = \pi^{-1}(p)$, we refer to the steady state simply as p.

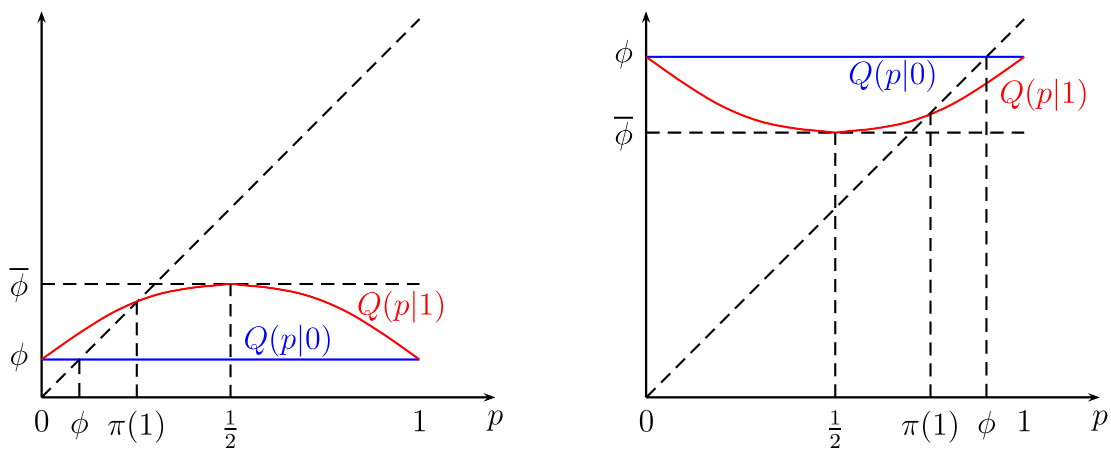

Figure 1. Families of $Q(p|z)$: left panel, $\phi < \frac{1}{2}$; right panel, $\phi > \frac{1}{2}$.

Now we are ready to define the class of equilibria we are interested in. We refer to them as "perfect Bayesian equilibria" because newborn agents in each period $t \geq 1$ have correct beliefs about the sequence of fractions of type H agent $\{p_\tau\}_{\tau=t}^{\infty}$, and the entire sequence $\{p_{t+1}\}_{t=1}^{\infty}$ is generated from the given initial value p_1 from the equilibrium strategy $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$.

Definition 1 A **perfect Bayesian equilibrium** for a given p_1 is a collection of infinite sequences $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$, such that no newborn agent in any period $t \geq 1$ has a profitable unilateral deviation from $\{m_{L,\tau}, a_{H,\tau}, a_{L,\tau}, k_{H,\tau}\}_{\tau=t}^{\infty}$ given $\{p_\tau\}_{\tau=t}^{\infty}$ for any $\tau \geq t$, and $\{p_{t+1}\}_{t=1}^{\infty}$ satisfies condition (D).

From now on, we often refer to a perfect Bayesian equilibrium simply as an equilibrium. When the equilibrium strategy $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ satisfies condition (S) for all $t \geq 1$, then condition (SD) replaces condition (D) in the above definition.

We say that an equilibrium $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$ for a given p_τ is "stationary" if the strategy $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ does not depend on t. In a stationary equilibrium, the market dynamics represented by $\{p_t\}_{t=1}^{\infty}$ may not be in a steady state. We say that a continuation equilibrium $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$ for a given p_1 is a "steady-state equilibrium" if the strategy $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ is stationary and the market dynamics is in a steady state.

We view a stationary equilibrium as a prediction of long run behavior of agents in the matching market. When the market dynamics also reaches a steady state, we can then compare market performance and agent welfare in the long run across different steady-state equilibria. Our analytical perspective thus focuses our attention on construction of different stationary equilibria. Further, we want to incorporate market dynamics into our construction. This means that we do not assume that the market dynamics is already in the steady state in the first period.

In our following constructions of equilibria, we will make use of the one-deviation principle. To do so, we denote $U_{\Theta,t}$ as the equilibrium payoff of type $\Theta = H, L$ in period t of an equilibrium $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$.

3.1 Babbling equilibria

A perfect Bayesian equilibrium $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$ is referred to as a "babbling equilibrium" if $m_{L,t} = 1$ for all $t \geq 1$. We have restricted type H to send message h. This means that message l, and hence the choice at marriage for each type $\Theta = H, L$ when the meeting partner has sent message l, is off the equilibrium path. As we have also restricted type H to accept message h and type L to accept message l, we require $a_{H,t} = a_{L,t} = 1$ for all $t \geq 1$ to capture the idea that in a babbling equilibrium the accept-reject decision at marriage is independent of the message sent by the meeting partner.[6] Condition (S) is then satisfied and $z_t = 1 - k_{H,t}$ for all $t \geq 1$.

With $m_{L,t} = a_{H,t} = a_{L,t} = 1$ for all $t \geq 1$, for either type there is no profitable one-shot deviation at dating in any time period t, because the choices at marriage and post

[6] The classic cheap talk game (Crawford and Sobel, 1982) is one-sided with a sender and a receiver, and babbling is when the sender's message contains no information about their payoff-relevant type and thus the receiver's action is independent of the sender's message. We have a two-sided setup for strategic communication between the two meeting partners. Although the message of their meeting partner has no information content, the two types can make the accept-reject choice at marriage conditional on their own message. We do not consider this possibility.

marriage are independent of messages sent by agents themselves, their meeting partner, and their marriage partner. For the accept-reject decision at marriage in any period $t \geq 1$, the equilibrium condition is that no type $\Theta = H, L$ wishes to skip to the next period $t+1$ by rejecting their meeting partner:

$$U_{\Theta,t} \geq \beta U_{\Theta,t+1}. \tag{A}$$

For the keep-divorce decision post marriage in any period $t \geq 1$, for an agent of type $\Theta = H, L$, it is optimal to keep a first-marriage partner of type $\Theta' = H, L$ if

$$\beta \frac{\nu_{\Theta\Theta'}}{1-\delta} \geq -\delta + \beta U_{\Theta,t+1}. \tag{K}$$

For convenience, define

$$U_H^* = \frac{\delta}{\beta} + \frac{\nu_{HL}}{1-\beta}.$$

Type H is indifferent between keeping and divorce type L post marriage in period t if $U_{H,t+1} = U_H^*$.

We first consider "babbling equilibria with no divorce," where $k_{H,t} = 1$ for all $t \geq 1$. By construction, such an equilibrium is stationary. In any such equilibrium, $z_t = 0$ and $p_{t+1} = Q(p_t|0) = \phi$ for all $t \geq 1$, starting from any p_1. All agents are replaced at the end of each period. The market dynamics jumps to the steady state ϕ in period 2, starting from any p_1. For each time period $t \geq 1$ and each type $\Theta = H$, the equilibrium payoff $U_{\Theta,t}^N$ is given by

$$U_{\Theta,t}^N = p_t \frac{\nu_{\Theta H}}{1-\beta} + (1-p_t)\frac{\nu_{\Theta L}}{1-\beta}.$$

Since p_t jumps from p_1 to ϕ in period 2, $U_{\Theta,t}^N$ jumps from $U_{\Theta,1}^N$ to its steady state value $\widehat{U}_\Theta^N$, given by

$$\widehat{U}_\Theta^N = \phi \frac{\nu_{\Theta H}}{1-\beta} + (1-\phi)\frac{\nu_{\Theta L}}{1-\beta}.$$

The characterization of babbling equilibria with no divorce follows.[7]

Proposition 1. *(i) A babbling equilibrium with no divorce exists only if $\widehat{U}_H^N \leq U_H^*$. (ii) If $\widehat{U}_H^N \leq U_H^*$, a babbling equilibrium with no divorce exists for any p_1 sufficiently close to ϕ.*

[7] All omitted proofs can be found in the appendix.

In a babbling equilibrium with no divorce, the market dynamics jumps from the initial state to the steady state in one period. The necessary condition of $\widehat{U}_H^N \leq U_H^*$ ensures that in the steady state there is no incentive to divorce the low type; this is also sufficient for there to exist a steady-state babbling babbling equilibrium with no divorce. The initial condition on p_1 ensures no agent, regardless of their type, wishes to skip the first period, anticipating that the market dynamics jumps to the steady in the second period. Proposition 1 is stated for p_1 sufficiently close to ϕ for clear comparisons with later results, but a babbling equilibrium with no divorce exists for all p_1 such that $U_{\Theta,1}^N \geq \beta \widehat{U}_\Theta^N$ for each $\Theta = H, L$. In particular, by Assumption V, we just need $p_1 \geq \beta\phi$.

Next, we consider "babbling equilibria with divorce," where $k_{H,t} = 0$ for all $t \geq 1$. By construction, such equilibrium is stationary. In any such equilibrium, $z_t = 1$, and $p_{t+1} = Q(p_t|1)$. By the properties of $Q(\cdot|1)$, starting from any p_1 we have p_2 lies between ϕ and $\overline{\phi}$. Further, $\{p_{t+1}\}_{t=1}^\infty$ is an increasing sequence if $p_2 < \pi(1)$, and a decreasing sequence if $p_2 > \pi(1)$, converging to $\pi(1)$ as t goes to infinity in both cases. The equilibrium payoffs of type H and type L in period t are respectively derived from the following iterations:[8]

$$U_{H,t}^D = p_t \frac{\nu_{HH}}{1-\beta} + (1-p_t)(\nu_{HL} - \delta + \beta U_{H,t+1}^D),$$
$$U_{L,t}^D = (1-p_t) \frac{\nu_{LL}}{1-\beta} + p_t(\nu_{LH} - \delta + \beta U_{L,t+1}^D).$$

In the babbling equilibrium with divorce constructed in Proposition 2 below, the market dynamics converges to the steady state $\pi(1)$. In the steady state, the payoffs are

$$\widehat{U}_H^D = \frac{1}{1-(1-\pi(1))\beta} \left(\pi(1) \frac{\nu_{HH}}{1-\beta} - (1-\pi(1))(\delta - \nu_{HL}) \right);$$
$$\widehat{U}_L^D = \frac{1}{1-\pi(1)\beta} \left((1-\pi(1)) \frac{\nu_{LL}}{1-\beta} - \pi(1)(\delta - \nu_{LH}) \right).$$

Proposition 2. *(i) A babbling equilibrium with divorce exists only if* $\widehat{U}_H^D \geq U_H^*$ *and* $\widehat{U}_L^D \geq 0$. *(ii) If* $\widehat{U}_H^D > U_H^*$ *and* $\widehat{U}_L^D > 0$, *a babbling equilibrium with divorce exists for* p_1 *sufficiently close to* $\pi(1)$.

[8] Lemma A in the appendix establishes that the above are well-defined for all $t \geq 1$ given $p_{t+1} = Q(p_t|1)$.

The two necessary conditions in Proposition 2 ensure that in the steady state type H prefers to keep type L post marriage, and type L prefers to accept a random partner at marriage. They are also sufficient for a steady-state babbling equilibrium with divorce. As in Proposition 1, the equilibrium strategies of both types are stationary, but unlike in Proposition 1, the market dynamics does not jump to the steady state in period 2. Instead, in a babbling equilibrium with divorce, $\{p_t\}_{t=1}^{\infty}$ converges monotonically to the steady state $\pi(1)$. As a result, the equilibrium payoffs $\{U_{\Theta,t}^D\}_{t=1}^{\infty}$ of each type $\Theta = H, L$ converges to the steady state value of $\widehat{U}_{\Theta}^D$. The necessary condition for a babbling equilibrium with divorce, $\widehat{U}_H^D \geq U_H^*$, only ensures that in the steady state type H wants to divorce type L. We need p_1 to be sufficiently close to the steady fraction $\pi(1)$ to ensure type H is willing to divorce type L even if $\{U_{H,t}^D\}_{t=1}^{\infty}$ is increasing. The initial condition on p_1 is also needed, together with $\widehat{U}_L^D > 0$, to ensure that no type Θ skips any period t as $\{U_{\Theta,t}^D\}_{t=1}^{\infty}$ converges to $\widehat{U}_{\Theta}^D$.

Finally, we consider "babbling equilibria with randomized divorce," where $k_{H,t} \in (0,1)$ for all $t \geq 1$. In any such equilibrium, since type H is indifferent between keeping and divorcing a type-L marriage partner, the equilibrium payoff of type H in any period $t \geq 1$ is pinned down:

$$U_{H,t+1}^R = U_H^*$$

for all $t \geq 1$. This in turn requires p_{t+1} to be constant, denoted as p^R, strictly between $\pi(0) = \phi$ and $\pi(1)$, and uniquely determined by type H's value function iteration:

$$U_H^* = p^R \frac{\nu_{HH}}{1-\beta} + (1-p^R)\frac{\nu_{HL}}{1-\beta}. \tag{R}$$

In the case of $\phi = \frac{1}{2}$, we have $p^R = \frac{1}{2}$, and condition (R) cannot be satisfied generically. Otherwise, there is a steady state p^R with $z^R = \pi^{-1}(p^R) \in (0,1)$, such that the market dynamics jumps to the steady state in period 2, with $k_{H,t} = 1 - z^R$ for all $t \geq 2$. The steady state payoffs for types H and L are given by

$$\widehat{U}_H^R = U_H^*,$$
$$\widehat{U}_L^R = \frac{1}{1 - p^R \pi^{-1}(p^R)\beta}\left((1-p^R)\frac{\nu_{LL}}{1-\beta} + p^R\left((1-\pi^{-1}(p^R))\frac{\nu_{LH}}{1-\beta} - \pi^{-1}(p^R)(\delta - \nu_{LH})\right)\right).$$

The following proposition gives the necessary and sufficient conditions for this construction of a babbling equilibrium with randomized divorce.

Proposition 3. *(i) A babbling equilibrium with randomized divorce exists only if* $\widehat{U}_H^N < U_H^* < \widehat{U}_H^D$ *when* $\phi < \frac{1}{2}$*, and* $\widehat{U}_H^D < U_H^* < \widehat{U}_H^N$ *when* $\phi > \frac{1}{2}$*, and only if* $\widehat{U}_L^R \geq 0$*. (ii) If* $\widehat{U}_H^N < U_H^* < \widehat{U}_H^D$ *when* $\phi < \frac{1}{2}$*, and* $\widehat{U}_H^D < U_H^* < \widehat{U}_H^N$ *when* $\phi > \frac{1}{2}$*, and if* $\widehat{U}_L^R > 0$*, there is a unique* $p^R \in (\min\{\phi, \pi(1)\}, \max\{\phi, \pi(1)\})$ *such that a babbling equilibrium with randomized divorce exists for* p_1 *sufficiently close to* p^R*.*

The necessary condition in Proposition 3 ensures that there is a unique probability $k_H \in (0, 1)$ in the steady state of type H keeping type L post marriage. As in Propositions 1 and 2, the other necessary condition ensures that type L wants to accept a random partner at marriage. These two conditions together are sufficient for a steady-state babbling equilibrium with randomized divorce.

As in an equilibrium with no divorce, and unlike in an equilibrium with divorce, the market dynamics jumps from the initial state to the steady state in one period in an equilibrium with randomized divorce. Correspondingly, the equilibrium payoff for each type $\Theta = H, L$ jumps from $U_\Theta^R(p_1)$ to the steady level of $\widehat{U}_\Theta^R$. The condition on the initial fraction p_1, that it is sufficiently close to the steady state fraction p^R, together with $\widehat{U}_L^R > 0$, ensures that in the jump from the initial state to the steady state, neither type wishes to skip the initial period. Unlike in both Propositions 1 and 2, Proposition 3 establishes that the strategy in an equilibrium with randomized divorce is not stationary. In particular, in the initial period $t = 1$, the probability $k_{H,1}$ of type H keeping type L in a permanent marriage needs to take the initial fraction p_1 to the steady state p^R, while for all $t \geq 2$ the corresponding probability $k_{H,t}$ is pinned down by the steady state p^R, given by $1 - \pi^{-1}(p^R)$.

3.2 Informative equilibria

A perfect Bayesian equilibrium $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$ is referred to as a "fully revealing equilibrium" if $m_{L,t} = 0$ for all $t \geq 1$. Since we have restricted type H to send message h and accept message h, we require $a_{H,t} = 0$ for all $t \geq 1$ to capture the idea that in a fully revealing equilibrium the accept-reject decision at marriage is entirely determined by the message sent by the meeting partner. Given that $a_{H,t} = 0$, type L's accept-reject decision with respect to message h does not affect their payoff. Post marriage in any period $t \geq 1$, the keep-divorce decision $k_{H,t}$ of type H with respect to a type-L marriage partner is off the equilibrium path: type H only marries type H in a marriage as $m_{L,t} = a_{H,t} = 0$. We will construct a fully revealing equilibrium with $k_{H,t} = 0$. Given this restriction, in equilibrium we have $a_{L,t} = 0$.

Since $m_{L,t} = a_{H,t} = 0$ for all $t \geq 1$, condition (S) is satisfied. We have $z_t = 1$ for all $t \geq 1$, and the market dynamics is given by $p_{t+1} = Q^t(p_1|1)$. By the properties of $Q(\cdot|1)$, starting from any p_1 we have p_2 lies between ϕ and $\overline{\phi}$. Further, $\{p_{t+1}\}_{t=1}^{\infty}$ is an increasing sequence if $p_2 < \pi(1)$, and a decreasing sequence if $p_2 > \pi(1)$, converging to $\pi(1)$ as t goes to infinity in both cases. The equilibrium payoffs of type H and type L satisfy:

$$U_{H,t}^F = p_t \frac{\nu_{HH}}{1-\beta} + (1-p_t)\beta U_{H,t+1}^F,$$
$$U_{L,t}^F = (1-p_t)\frac{\nu_{LL}}{1-\beta} + p_t \beta U_{L,t+1}^F.$$

The market dynamics of the equilibrium we construct below converges to the steady state $\pi(1)$. In the steady state, the payoffs are given by

$$\widehat{U}_H^F = \frac{\pi(1)}{1-(1-\pi(1))\beta} \cdot \frac{\nu_{HH}}{1-\beta};$$
$$\widehat{U}_L^F = \frac{1-\pi(1)}{1-\pi(1)\beta} \cdot \frac{\nu_{LL}}{1-\beta}.$$

Since the keep-divorce decision post marriage is made after partners observe each other's true type, the equilibrium conditions the decision are governed by condition (K), just as in babbling equilibria. In contrast, the accept-reject decision at marriage is made after dating, which is perfectly informative of type in a fully revealing equilibrium. Due to the equilibrium restrictions, the message decision at dating coincides with condition (A) in a babbling equilibrium: for each type $\Theta = H, L$, it is optimal to truthfully reveal their type if and only if

$$U_{\Theta,t} \geq \beta U_{\Theta,t+1} \tag{T}$$

for each $t \geq 1$. The following proposition characterizes necessary and sufficient conditions for a fully revealing equilibrium.

Proposition 4. *(i) A fully revealing equilibrium exists only if $\widehat{U}_H^F \geq U_H^*$. (ii) If $\widehat{U}_H^F > U_H^*$, a fully revealing equilibrium exists for p_1 sufficiently close to $\pi(1)$.*

A fully revealing equilibrium constructed in Proposition 4 is a stationary equilibrium with market dynamics that converges to a steady state. The necessary condition ensures that in the steady state, type H weakly prefers to divorce a type-L partner who has deviated by pretending to be type H post marriage. It is also sufficient for a steady-state fully revealing equilibrium. The condition on the initial fraction p_1 is to ensure the same incentives for type H with regards to a deviating type L as the equilibrium market dynamics converges to the steady state, even if the equilibrium payoff sequence $\{U_{H,t+1}^F\}_{t=1}^{\infty}$ is increasing.

A perfect Bayesian equilibrium $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$ is referred to as a "partially informative equilibrium" if $m_{L,t} \in (0,1)$ for all $t \geq 1$. Since we have restricted type H to send message h and accept message h, and type L to accept message l, we require $a_{H,t} = 0$ and $a_{L,t} = 1$ for all $t \geq 1$ to capture the trade-off for type L in a partially informative equilibrium between sending message h to have a chance at a permanent marriage with type H and sending message l to completely avoid a costly divorce. Condition (S) is then satisfied and $z_t = 1 - m_{L,t}k_{H,t}$ for all $t \geq 1$. Given that $a_{H,t} = 0$ and $a_{L,t} = 1$ for all $t \geq 1$, in any partially informative equilibrium we have $k_{H,t} \in (0,1)$: for if $k_{H,t} = 1$ then type L would strictly prefer message h to message l by Assumption V; and if $k_{H,t} = 0$ then type L would strictly prefer message l to message h by Assumption D. In either case, a partially informative equilibrium with $m_{L,t} \in (0,1)$ is impossible.

Given that $k_{H,t} \in (0,1)$ for all $t \geq 1$, condition (K) holds with equality for type H with respect to type L. Thus, the equilibrium payoff for type H in any $t \geq 1$ satisfies:

$$
\begin{aligned}
U_{H,t+1}^I &= U_H^*; \\
U_{H,t}^I &= p_t \frac{\nu_{HH}}{1-\beta} + (1-p_t)\left(m_{L,t}\frac{\nu_{HL}}{1-\beta} + (1-m_{L,t})\beta U_{H,t+1}^I\right),
\end{aligned}
$$

where the first equation comes from type H's indifference to randomize between keeping and divorcing type L, and the second is type H's value function iteration assuming that type H

keeps type L. Given that $m_{L,t} \in (0,1)$ for all $t \geq 1$, type L is indifferent between messages h and l. Thus, the equilibrium payoff for type L in any period $t \geq 1$ satisfies:

$$\beta U^I_{L,t+1} = k_{H,t}\frac{\nu_{LH}}{1-\beta} + (1-k_{H,t})(\nu_{LH} - \delta + \beta U^I_{L,t+1});$$
$$U^I_{L,t} = (1-p_t)\frac{\nu_{LL}}{1-\beta} + p_t\beta U^I_{L,t+1},$$

where the first equation comes from type L's indifference to randomize messages l and h, and the second is type L's value function iteration assuming that type L sends message l.

We will construct a partially informative equilibrium where the market dynamics jumps to a steady state p^I in period 2 from p_1, with $z^I = \pi^{-1}(p^I)$. In the steady state p^I, i.e., in period $t \geq 2$, we have $m_{L,t} = m^I_L \in (0,1)$ and $k_{H,t} = k^I_H \in (0,1)$, with $z^I = 1 - m^I_L k^I_H$. The equilibrium payoffs $\widehat{U}^I_\Theta$ for each type $\Theta = H, L$, and m^I_L and k^I_H satisfy the steady state versions for type H,

$$\widehat{U}^I_H = U^*_H; \tag{R$_H$}$$
$$\widehat{U}^I_H = p^I\frac{\nu_{HH}}{1-\beta} + (1-p^I)\left(m^I_L\frac{\nu_{HL}}{1-\beta} + (1-m^I_L)\beta\widehat{U}^I_H\right), \tag{K$_H$}$$

and for type L

$$\beta\widehat{U}^I_L = \frac{\nu_{LH}}{1-\beta} - \frac{1-k^I_H}{k^I_H}(\delta - \nu_{LH}); \tag{R$_L$}$$
$$\widehat{U}^I_L = (1-p^I)\frac{\nu_{LL}}{1-\beta} + p^I\beta\widehat{U}^I_L. \tag{T$_L$}$$

For notational convenience, define

$$\widehat{U}^f_H = \frac{\phi}{1-(1-\phi)\beta} \cdot \frac{\nu_{HH}}{1-\beta}.$$

Proposition 5. *(i) A partially informative equilibrium exists only if $\widehat{U}^N_H < U^*_H \leq \widehat{U}^F_H$ when $\phi < \frac{1}{2}$ or $\widehat{U}^D_H < U^*_H \leq \widehat{U}^f_H$ when $\phi > \frac{1}{2}$. (ii) If $\max\{\widehat{U}^N_H, \widehat{U}^D_H\} < U^*_H < \min\{\widehat{U}^F_H, \widehat{U}^f_H\}$, there exists some $p^I \in (\min\{\phi, \pi(1)\}, \max\{\phi, \pi(1)\})$ such that a partially informative equilibrium exists for p_1 sufficiently close to p^I.*

The necessary conditions in Proposition 5 depend on whether $\phi < \frac{1}{2}$ or $\phi > \frac{1}{2}$ because the bounds on the market dynamics are reversed in these two cases. In a partially informative equilibrium, we need type L to randomize between the two messages at dating in order to keep type H indifferent between keeping and divorcing a type-L partner post marriage. When $\phi < \frac{1}{2}$, the worst continuation payoff for type H after a divorce occurs when the fraction of type H in the market is ϕ and the best is when the fraction is $\pi(1)$. The opposite order of the worst and the best is true when $\phi > \frac{1}{2}$.

Unlike in previous equilibrium constructions, from Proposition 1 to Proposition 4, the necessary conditions are not sufficient for there to be a steady-state partially informative equilibrium. The steady-state fraction p^I satisfies $\phi < p^I < \pi(1)$ when $\phi < \frac{1}{2}$, and $\pi(1) < p^I < \phi$ when $\phi > \frac{1}{2}$. In either case, we prove the existence of a steady-state fraction using a fixed point argument in p^I. For k_H^I uniquely determined by (R_L) and (T_L) for given p^I, we need $m_L^I \in (0,1)$ such that conditions (R_H) and (K_H) are satisfied, with $p^I = \pi(z^I)$ and $z^I = 1 - m_L^I k_H^I$. Since p^I lies between the minimum of ϕ and $\pi(1)$ and the maximum of the two, the fixed-point argument goes through when both sets of the necessary conditions hold.

For the special case of $\phi = \frac{1}{2}$, we have $p_t = \phi$ for all $t \geq 2$. In this case, $U_H^* > \widehat{U}_H^N$ is equivalent to $U_H^* > \widehat{U}_H^D$, and $\widehat{U}_H^F = \widehat{U}_H^f$. Since the market dynamics is degenerate for $t \geq 2$, the conditions of $\widehat{U}_H^N < U_H^* \leq \widehat{U}_H^F$ are both necessary and sufficient for there to exist a partially informative equilibrium.

If the initial fraction p_1 is sufficiently close to the steady state fraction p^I, there exists $m_{L,1} \in (0,1)$ such that, together with $k_{H,1} = k_H^I$, the market dynamics jumps from p_1 to p^I in one period. We also need the initial fraction p_1 to be sufficiently close to p^I to ensure that in the initial period type H prefers prefers to send message h at dating and accept message h at marriage. As in the babbling equilibrium with randomized divorce, the partially informative equilibrium is not stationary. Indeed, even though there are two variables, $m_{L,t}$ and $k_{H,t}$, that equilibrate the conditions for a partially informative equilibrium, generically there are no equilibria where the market dynamics takes more than one period to reach the steady state. To see this, suppose that for some $T > 2$ and for some initial fraction p_1 of type H, we have a partially informative equilibrium $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$ with $m_{L,t} = m_L^I$, $k_{H,t} = k_H^I$ and $p_t = p^I$ for all $t \geq T$. In period $T-1$, since type L's continuation payoff is $U_{L,T}^T = \widehat{U}_L^I$, from type L's indifference condition between messages h and l we have $k_{H,T-1} = k_H^I$. Since at the end of period $T-2$ type H is indifferent between keeping and divorcing type L, we have $U_{H,T-1} = U_H^*$. From type H's value iteration in period $T-1$ we then have

$$U_H^* = p_{T-1}\frac{\nu_{HH}}{1-\beta} + (1-p_{T-1})\left(m_{L,T-1}\frac{\nu_{HL}}{1-\beta} + (1-m_{L,T-1})\beta U_H^*\right).$$

At the same time, since $p_T = p^I$, by condition (SD) we have

$$p^I = Q(p_{T-1}|1 - m_{L,T-1}k_H^I).$$

Generically, $p_{T-1} = p^I$ and $m_{L,T-1} = m_L^I$ are the unique pair that satisfy the above two equations.[9] As a result, for $T > 2$, period $T-1$ is already in the steady state.

4 Steady-state Equilibria

It is natural to focus on steady state when we are mostly interested in performance of the market and welfare of the participants in the long run. Further, the analysis in the previous section establishes that steady-state equilibria are "stable" in that if the initial distribution of types is sufficiently close to the steady state, then the equilibrium either jumps to the steady state in one period or converges to it. We are especially interested in the steady-state partially informative equilibrium. In such an equilibrium, cross-type marriages – type H marrying type L – occur with a positive probability in permanent marriages, and at the same time, some of them end up in a divorce.

4.1 Co-existence

Part (i) in each proposition from Proposition 1 to Proposition 4, establishes the necessary and sufficient conditions for the corresponding steady-state equilibrium. Combining with Proposition 5, we immediately have the following result.

[9] When $T = 2$, period $T-1$ is the initial period and the first equation does not hold; that is, $U_{H,1}^I$ is generally not equal to U_H^*. This explains why we can construct a partially informative equilibrium with the market dynamics that jumps to the steady state in a one period.

Corollary 1. *A steady-state partially informative equilibrium coexists with a unique steady-state babbling equilibrium with no divorce and a unique steady-state fully revealing equilibrium, with no other steady-state equilibria, if*

$$\max\{\phi, \pi(1)\}(\nu_{HH} - \nu_{HL}) < \frac{1-\beta}{\beta}\delta < \frac{\min\{\phi, \pi(1)\}}{1-(1-\min\{\phi, \pi(1)\})\beta}\nu_{HH} - \nu_{HL}.$$

In a steady-state partially informative equilibrium, the fraction of type H is p^I, which lies strictly between ϕ and $\pi(1)$. The first inequality on in Corollary 1 requires the divorce cost δ to be sufficiently high that type H does not wish to divorce type L, given that after the divorce type H expects to randomly meet and marry an agent from the other side for any fraction of type H between ϕ and $\pi(1)$. This therefore implies the existence of a unique steady-state babbling equilibrium with no divorce. The second inequality requires the divorce cost δ to be sufficiently low that type H prefers to divorce type L, if the divorce allows type H to marry only type H from the other side. This implies the existence of a unique steady-state fully revealing equilibrium. The two inequalities involve only type H's incentives to randomize between keeping and divorcing a type L-partner after the first period of marriage. The reason for the absence of conditions on type L's match values is that type L's equilibrium condition to mix between the high message and the low message can always be satisfied for some probability k of type H keeping the low type. Broadly speaking, type L and type H play different roles in the construction of the equilibrium.

The sufficient conditions for the coexistence of the three steady equilibria of babbling are satisfied if the cost of divorce is neither too high or too low. If it is too high, satisfying

$$\frac{1-\beta}{\beta}\delta > \frac{\min\{\phi, \pi(1)\}}{1-(1-\min\{\phi, \pi(1)\})\beta}\nu_{HH} - \nu_{HL}$$

the continuation payoff of the high type is not high enough for them to divorce the low type post marriage even when dating is fully revealing. The low type cannot be incentivized to communicate truthfully at dating, and we have the babbling equilibrium with no divorce. At the other extreme, if the cost of divorce is too low, satisfying

$$\frac{1-\beta}{\beta}\delta < \max\{\phi, \pi(1)\}(\nu_{HH} - \nu_{HL}),$$

the continuation payoff payoff of the high type is sufficiently high for them to divorce the low type even when dating is uninformative. Since by Assumption D the cost of divorce still exceeds the one-period match payoffs of a cross-type marriage, the low type will be truthful at dating and we have the fulling revealing equilibrium.[10]

To see how the conditions for co-existence can be satisfied, we assume away any impact from market dynamics, with $\phi = \frac{1}{2}$ and thus $\pi(1) = \frac{1}{2}$. Recall that Assumption V requires $\nu_{HH} > \nu_{HL}$, and Assumption D requires $\delta > \nu_{HL}$. The inequalities in Corollary 1 become

$$\frac{1}{2}\left(\frac{\nu_{HH}}{\nu_{HL}} - 1\right) < \frac{1-\beta}{\beta}\frac{\delta}{\nu_{HL}} < \frac{1}{2-\beta}\frac{\nu_{HH}}{\nu_{HL}} - 1.$$

Fix any $\beta \in (0,1)$. For any $\nu_{HL} > 0$ and any $\nu_{HH} > \nu_{HL}(2-\beta)/\beta$, we have

$$\max\left\{\frac{1}{2}\left(\frac{\nu_{HH}}{\nu_{HL}} - 1\right), \frac{1-\beta}{\beta}\right\} < \frac{1}{2-\beta}\frac{\nu_{HH}}{\nu_{HL}} - 1.$$

Thus, there exist values of $\delta > \nu_{HL}$ such that the conditions in Corollary 1 are satisfied.

The coexistence of the steady-state equilibria, partially informative and babbling equilibrium with no divorce, is especially interesting. The latter equilibrium may be interpreted as the outcome of "traditional" market, where there are no opportunities for market participants to exchange cheap talk messages before marriage. When such opportunities arise in a "modern" market, without any changes to match payoffs or the divorce cost, participants can "switch" to the steady-state partially informative equilibrium, where informative communication at dating results in both cross-type permanent marriages and divorce.[11]

[10] After lying at dating, type L will reject type H for marriage because type L anticipates a costly divorce after marrying a truthful type H, and will be rejected by other type-L agents for the same reason.

[11] Such switching can even occur as part of equilibrium. More precisely, we can construct an equilibrium in which for any given period $\tau > 1$, all agents babble at dating with no divorce post marriage until period $\tau - 1$, anticipating the continuation equilibrium starting from τ to be a partially informative equilibrium constructed in Proposition 5. To see this, suppose that $\phi = \frac{1}{2}$. Following the proof of Proposition 1, we can show that $\widehat{U}_\Theta^N \geq \beta \widehat{U}_\Theta^I$ for each $\Theta = H, L$, then in period $\tau - 1$ type Θ will not want to skip the last period of babbling with no divorce for the steady-state partially informative equilibrium starting in the next period of τ. From Corollary 2 in the next subsection, this condition is always satisfied for type L, and we make β small enough for it to hold type H. If in addition $\beta\nu_{\Theta L}/(1-\beta) \geq -\delta + \beta\widehat{U}_\Theta^I$, for each $\Theta = H, L$, then type Θ will not want to divorce type L for the steady-state partially informative equilibrium starting in period τ. For type H, the above condition is satisfied with equality because $U_H^I = U_H^*$. For type L, it is also satisfied as shown in the proof of Proposition 5.

4.2 Welfare comparisons

In a steady-state partially informative equilibrium, type H rejects the fraction of type L agents who choose the low message so the former's marriage decision is selective. However, the cheap talk communication is imperfect, as the remaining fraction of type L agents send the high message and marry type H agents. Type H nonetheless ends up divorcing a fraction of type L in these cross-type marriages. Thus, costly divorce impacts both type H and type L in this steady-state equilibrium. We compare welfare across the three steady-state equilibria.

Corollary 2. *When a steady-state partially informative equilibrium coexists with a babbling equilibrium with no divorce,* $\widehat{U}_H^I \geq \widehat{U}_H^N$ *and* $\widehat{U}_L^I < \widehat{U}_L^N$*; when it coexists with a fully revealing equilibrium,* $\widehat{U}_H^I \leq \widehat{U}_H^F$*, and* $\widehat{U}_L^I > \widehat{U}_L^F$ *if* $\phi < \frac{1}{2}$ *and* $\widehat{U}_L^I < \widehat{U}_L^F$ *if* $\phi > \frac{1}{2}$.

For type H, the comparisons derive immediately from the necessary conditions for the existence of the equilibria. When the conditions in Corollary 1 are satisfied, so that all three steady-state equilibria exist, the comparisons are strict:

$$\widehat{U}_H^N < \widehat{U}_H^I < \widehat{U}_H^F.$$

For type L, the comparisons follow from condition (T_L): the payoff of type L in the steady-state partially informative equilibrium can be computed by assuming that type L sends message l, and therefore satisfies

$$\widehat{U}_L^I = \frac{1-p^I}{(1-p^I\beta)(1-\beta)}\nu_{LL} < \frac{1}{1-\beta}\nu_{LL}.$$

In a steady-state partially informative equilibrium, as in the steady-state fully revealing equilibrium, the best that can happen to a type-L agent is that they meet another type-L agent from the other side of the market. Whether type L is better off in a steady-state partially informative equilibrium or in the steady-state fully revealing equilibrium therefore depends on the comparison between p^I and ϕ, and hence the comparison between ϕ and $\frac{1}{2}$. In contrast, in the steady-state babbling equilibrium with no divorce, type L has positive probability of marrying type H permanently. By Assumption V, we have $\widehat{U}_L^I < \widehat{U}_L^N$.

By Assumption D, the divorce cost is higher than the one-period match value from a temporary cross-type marriage for both types H and L. However, the fact that in a steady-state partially informative equilibrium it is type H, rather than type L, who is indifferent when making the keep-divorce decision in a cross-type marriage implies that type H is better off in such equilibrium than in a steady-state babbling equilibrium with no divorce. The gain in type H's equilibrium payoff arises from a more selective marriage. At the same time, type L is in expectation worse off in a steady-state partially informative equilibrium, even though type L sometimes has a permanent marriage with type H. This is so because for the cheap talk to be informative, type L has to be indifferent between masquerading as type H and self-identifying as type L, but in the latter case type L loses one-period payoff after being rejected by type H.

Costly divorce has different welfare impacts when we compare a steady-state partially informative equilibrium to the steady-state fully revealing equilibrium. As in the steady-state babbling equilibrium, there is no divorce in the steady-state fully revealing equilibrium, but the reason is that type H never enters a marriage with type L in the first place. Indeed, in the steady-state fully revealing equilibrium type H stands ready to divorce any type L who pretends to be type H at dating. As result, costly divorce makes type H unambiguously worse off in a steady-state partially informative equilibrium than in the steady-state fully revealing equilibrium. In contrast, type L can be better off in a steady-state partially informative equilibrium in spite of a positive probability of costly divorce. This happens whenever $\phi < \frac{1}{2}$, so that $p^I < \pi(1)$ and type L has a greater chance of meeting type L in a steady-state partially informative equilibrium than in the steady-state fully revealing equilibrium.

4.3 Comparative statics

In this section we conduct comparative statics analysis of the steady-state partially informative equilibrium. We are particularly interested the effects of a decrease in the cost of divorce δ. By Corollary 1, for a steady-state partially informative equilibrium to exist, δ has to be of intermediate values.

Proposition 5 immediately reveals that a decrease in δ reduces the high type's steady-state equilibrium payoff. In equilibrium, type H is indifferent between keeping and divorcing type L post marriage. By condition (R_H), regardless of how a decrease in δ affects the steady-state fraction p^I of type H, the probability of miscommunication, or the probability of divorce, type H's payoff is pinned down at U_H^* by the indifference condition.

Since type L in equilibrium in indifferent between messages h and l, by the value function iteration changes in δ affect type L's equilibrium payoff $\widehat{U}_L^I$ only through the effects on the steady-state fraction p^I. If we assume away market dynamics by setting $\phi = \frac{1}{2}$, then we have $p^I = \frac{1}{2}$ and by condition (T_L) a decrease in δ has no effect on $\widehat{U}_L^I$. This is in spite of the effects of a decrease in δ will have on the probability of miscommunication by type L and the probability of divorce by type H. In particular, by condition (R_L), with $\widehat{U}_L^I$ unaffected, a decrease in δ reduces k_H^I. This is because a lower cost of divorce makes lying about their type more attractive to type L, and a greater probability of divorce is needed to restore type L's indifference between lying and being truthful at dating. At the same time, by conditions (R_H) and (K_H), a decrease in δ reduces $\widehat{U}_H^I$, and with p^I unaffected, raises m_L^I. Since a lower cost of divorce leads to a decrease in type H's equilibrium payoff, it must be achieved by a greater probability of miscommunication by type L. We summarize the above findings in the corollary below.

Corollary 3. *In any steady-state partially informative equilibrium, a decrease in δ reduces $\widehat{U}_H^I$. In the unique steady-state partially informative equilibrium when $\phi = \frac{1}{2}$, a decrease in δ has no effect on $\widehat{U}_L^I$, but reduces k_H^I and raises m_L^I.*

Recall from our discussion of Corollary 1 that when δ is too high, in the unique steady-state equilibrium dating is uninformative and there is no divorce, and when δ is too low, in the unique steady-state equilibrium dating is fully revealing and again there is no divorce. In either case, a small decrease in δ has no effect on the steady-state equilibrium. When δ is intermediate and satisfies the conditions in Corollary 1, with market dynamics assumed away, the average rate of divorce in the unique steady-state partially informative equilibrium, given by $2p^I(1-p^I)m_L^I(1-k_H^I)$, increases as the cost of divorce δ becomes lower. A higher divorce rate arises both because there is miscommunication at dating and because the high type is more likely to divorce the low type post marriage.

A higher average divorce rate in the steady-state partially informative equilibrium due a lower divorce cost does not translate into a lower fraction of cross-type permanent marriages, given by $2p^I(1-p^I)m_L^I k_H^I$. Although the probability of divorcing type L by type H is higher post marriage, more miscommunication by type L raises the proportion of cross-type marriages. To understand comparative statics with respect to the cost of divorce in greater details, we review the fixed-point argument used in Proposition 5 to establish the existence of a steady-state partially informative equilibrium. We do not assume that $\phi = \frac{1}{2}$ for the remainder of this subsection.

In the construction of Proposition 5, for any given $p^I = p$ between $\min\{\phi, \pi(1)\}$ and $\max\{\phi, \pi(1)\}$, we use conditions (R_L) and (T_L) to determine a unique $m_L^I = m_L(p) \in (0,1)$, use conditions (R_H) and (K_H) to determine a unique $k_H^I = k_H(p) \in (0,1)$, and finally use condition (SD) to construct an equation in p:

$$p = \phi + (1-2\phi)p(1-p)(1-m_L(p)k_H(p)). \tag{FP}$$

Any solution to the above equation corresponds to p^I in a steady-state partially informative equilibrium. It is straightforward to verify that $m_L(p)$ is strictly increasing, because after an increase in p type L has to raise the probability of lying at dating in order for type H to remain indifferent between keeping and divorcing type L post marriage. However, $k_H(p)$ is decreasing, because after an increase in p type H has to reduce the probability of keeping type L post marriage in order for type L to remain indifferent between lying and being truthful at dating. As a result, there may be multiple solutions in p^I to (FP),[12] in which case we restrict to the "extreme" equilibria with the largest and the smallest values of p^I. The comparative statics of the equilibrium fraction of cross-type permanent marriages is the same whether the equilibrium is unique or if we restrict to the extreme equilibria.[13] To reduce the number of parameters we need to consider, we assume the Burdett-Coles (1997)

[12]Under the conditions of Corollary 1, there is a unique steady-state partially informative equilibrium when $\phi \le \frac{1}{2}$. When $\phi > \frac{1}{2}$, the equilibrium is unique if we strengthen the conditions to

$$\phi(\nu_{HH} - \nu_{HL}) < \frac{1-\beta}{\beta}\delta \le \frac{2\pi(1)}{2\pi(1)+1-\beta}\nu_{HH} - \nu_{HL}.$$

The proof of this result is available upon request.

[13]To see this, note that by the properties of $Q(\cdot)$, the right-hand side of (FP) as a function of p is bounded from below by $\min\{\phi, \pi(1)\}$ and from above by $\max\{\phi, \pi(1)\}$. Regardless of whether $\phi < \frac{1}{2}$ or $\phi > \frac{1}{2}$, if the equilibrium is unique, or if the equilibrium has the largest or the smallest value of p^I, the right-hand side of (FP) crosses the 45-degree line from above at the equilibrium p^I.

payoff structure of $\nu_{\Theta H} = \nu_H > \nu_{\Theta L} = \nu_L > 0$ for each $\Theta = H, L$. Assumptions V and D require $\nu_L < \nu_H < \delta$.

Proposition 6. *Suppose that $\nu_{\Theta H} = \nu_H > \nu_{\Theta L} = \nu_L > 0$ for each $\Theta = H, L$, and the conditions in Corollary 1 are satisfied. When δ is sufficiently close to ν_H, a decrease in δ results in a smaller equilibrium fraction of permanent cross-type marriages $2p^I(1-p^I)m_L^I k_H^I$; when ν_L is sufficiently close to ν_H, a decrease in δ results in a greater equilibrium fraction of permanent cross-type marriages.*

As we have argued for Corollary 3, if the equilibrium p^I is unaffected (which is the case if $\phi = \frac{1}{2}$), a decrease in δ requires a smaller probability of not divorcing k_H^I by type H for type L to remain indifferent and simultaneously a greater probability of miscommunication m_L^I by type L for type H to remain indifferent. We show in the proof of Proposition 6 that, under the Burdett-Coles (1997) payoff structure, the first effect dominates when δ is just above ν_H, and the second effect dominates when ν_L is sufficiently close to ν_H, regardless of how a decrease in δ affects the equilibrium p^I. Intuitively, for any fixed degree of vertical differentiation given by $\nu_H - \nu_L$, when δ is sufficiently close to $\nu_H = \nu_{LH}$, type L's indifference between the two messages is sensitive to a marginal decrease in δ, in that a large proportional decrease in $k_H(p)$ is required for type L to remain indifferent at any fixed p. This effect dominates the positive effect of a decrease in δ on $m_L(p)$, and thus the product $m_L(p)k_H(p)$ decreases. If $\phi = \frac{1}{2}$ and thus p^I is fixed at $\frac{1}{2}$, immediately we have that the equilibrium fraction of permanent cross-type marriages $2p^I(1 - p^I)m_L^I k_H^I$ decreases. The decrease in $m_L(p)k_H(p)$ leads to a greater equilibrium value of p^I if $\phi < \frac{1}{2}$, and a lower p^I if $\phi > \frac{1}{2}$; in either case, the effect on the the equilibrium fraction of permanent cross-type marriages remains negative.[14] The opposite occurs when the degree of vertical differentiation is sufficiently small for any given $\delta > \nu_H$. When δ decreases marginally, the dominating effect is a large proportional increase in $a_L(p)$ for type H to remain indifferent. The product $m_L(p)k_H(p)$ increases, and the equilibrium fraction of permanent cross-type marriages increases.

[14] In this case, the equilibrium rate of divorce, $2p^I(1 - p^I)m_L^I(1 - k_H^I)$, unambiguously increases as δ decreases. We know from Corollary 3 that this is true when $\phi = \frac{1}{2}$. For $\phi < \frac{1}{2}$, a decrease in δ leads to a greater p^I. This increases $2p^I(1 - p^I)$, and at the same time, increases m_L^I because $m_L(p)$ is a increasing in p for fixed δ, and decreases k_H^I because $k_H(p)$ is decreasing. In the other case of ν_L being sufficiently close to ν_H, the effect of a decrease in δ on the equilibrium divorce rate is ambiguous when $\phi \neq \frac{1}{2}$.

5 Extensions

In this section, we discuss a few extensions of the main model. The first two extensions deal with alternative market dynamics, and the last three focus on the steady-state partially informative equilibrium.

5.1 Exogenous entry size

In our model, the size of the entrants in each period t is "endogenous" in the sense that only those are permanently married are replaced. This ensures that the market size is the same regardless of equilibrium market dynamics. Alternatively, instead of replacement, in each time period t there is a fixed size of agents, equal across the two sides, that enter the market. We normalize the exogenous entry size to 1. As before, a fixed fraction $\phi \in (0,1)$ of new entrants is of type H.

Denote as G_t the market size at the beginning of period t, with $G_1 > 0$ the fixed initial market size. Given any initial fraction p_1 of type H, a collection of infinite sequences $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ generates sequences $\{p_{t+1}\}_{t=1}^{\infty}$ and $\{G_{t+1}\}_{t=1}^{\infty}$ according to

$$p_{t+1}G_{t+1} = \phi + p_t(1-p_t)G_t\left(m_{L,t}(1-a_{L,t}k_{H,t}) + (1-m_{L,t})(1-a_{H,t}a_{L,t}k_{H,t})\right),$$
$$G_{t+1} = 1 + 2p_t(1-p_t)G_t\left(m_{L,t}(1-a_{L,t}k_{H,t}) + (1-m_{L,t})(1-a_{H,t}a_{L,t}k_{H,t})\right).$$

Under the same restriction (S) on $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ as in the main model, the evolution of the "state" $\{p_{t+1}, G_{t+1}\}_{t=1}^{\infty}$ depends only on the sequence of $z_t = 1 - m_{L,t}k_{H,t}$. Then, we can write the new market dynamics as

$$G_{t+1} = 1 + 2p_t(1-p_t)G_t z_t,$$
$$p_{t+1} = \phi + (1-2\phi)\frac{p_t(1-p_t)G_t z_t}{1+2p_t(1-p_t)G_t z_t}.$$

It is immediate from the above two equations that, for any $z \in [0,1]$, at any steady state of a constant fraction of type H and a constant market size, the constant fraction is given by the same function $\pi(z)$ as before. Denote the constant market size in the steady state as

$$\Gamma(z) = \frac{1}{1 - 2\pi(z)(1 - \pi(z))z}.$$

Since incentive conditions of agents depend on the sequence of states $\{p_t, G_t\}_{t=1}^{\infty}$ only through the sequence of fractions $\{p_t\}_{t=1}^{\infty}$, the necessary conditions for a steady-state equilibrium, given from Proposition 1 to Proposition 5 in section 3, as well as our analysis of the steady-state equilibria in section 4, remain the same under exogenous entry. The changes are in the sufficient conditions in construction of the equilibria, from Proposition 1 to Proposition 5, because $\{p_{t+1}\}_{t=1}^{\infty}$ and $\{G_{t+1}\}_{t=1}^{\infty}$ co-evolve. We require the initial market size G_1, as well as the initial fraction p_1, to be close to their corresponding steady state values. As in the main model, the market dynamics jumps from the initial state p_1 and G_1 to the corresponding steady state in one period in a babbling equilibrium with no divorce or randomized divorce and in a partially informative equilibrium, and converges to the corresponding steady state in a babbling equilibrium with divorce and in a fully revealing equilibrium.

5.2 Endogenous replacement

We have assumed that agents exiting the market by getting permanently married are replaced with newborns with an exogenous type distribution. If we interpret replacements as offsprings of exiting agents, then it makes sense that the type distribution of replacements is at least partly correlated with the type of the agents they replace. This alternative can be modeled by assuming that the probability of a newborn agent being type H is equal to a weighted sum of $\phi \in (0, 1)$ in the main model and the type that the agent replaces. Let the weight on ϕ be $\eta \in [0, 1]$; the main model corresponds to $\eta = 1$.

In place of equation (D), given an initial fraction p_1 of type H, and a collection of infinite sequences $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$, the market dynamics becomes

$$\begin{aligned} p_{t+1} = {} & p_t^2(\phi\eta + 1 - \eta) + p_t(1 - p_t)(1 - (m_{L,t} + (1 - m_{L,t})a_{H,t})a_{L,t}k_{H,t}(1 - \phi)\eta) \\ & + (1 - p_t)p_t(m_{L,t} + (1 - m_{L,t})a_{H,t})a_{L,t}k_{H,t}\phi\eta + (1 - p_t)^2(m_{L,t}a_{L,t} + 1 - m_{L,t})^2\phi\eta. \end{aligned}$$

Under the same restriction (S) on $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ as in the main model, the new market dynamics depends on $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}\}_{t=1}^{\infty}$ only through $z_t = 1 - m_{L,t}k_{H,t}$:

$$p_{t+1} = p_t(1-\eta) + (\phi + p(1-p)(1-2\phi)z_t)\eta.$$

Comparing the above with equation (SD), we see that so long as $\eta > 0$, the new market dynamics dampens the evolution of the state p_t without affecting the steady state $\pi(z)$ for any $z \in [0,1]$. All our construction of equilibria, from Proposition 1 to Proposition 5 in section 3, remains unaffected, and so does the steady state analysis of section 4. In the special case of $\eta = 0$, the dynamics disappears completely. This is the "clone" model in the literature, which ensures that dynamics plays no role in equilibrium analysis. Any initial fraction p_1 is a candidate for steady state; the analysis for this special case otherwise proceeds in the same way as in the case of $\phi = \frac{1}{2}$.

5.3 Horizontal differentiation

When agents are horizontally differentiated, there is no common ranking by agents on one side of the market of different types on the other side. With two discrete types labeled as type H and type L, under horizontal differentiation we have Assumption V holding for $\Theta = H$, but the reverse holds for $\Theta = L$. Thus, a cross-type marriage is less desirable than a same-type marriage for both types of agents.

The construction of the partially informative equilibrium in Proposition 5 is asymmetric with respect to type. Indeed, the sufficient conditions given in Corollary 1 are regarding the match values of the high type. Since horizontal differentiation does not change the ranking of the match values for the high type under vertical differentiation — in both cases we have $\nu_{HH} > \nu_{HL}$ — the same two conditions are also sufficient for the low type to mix between the two messages in a way to make the high type indifferent between keeping and divorcing a low-type partner after the first period of the marriage. However, while the proof of Proposition 5 makes it clear that the indifference condition for the low type between the high message and the low message can always be satisfied under vertical differentiation, additional conditions are needed when the ranking of the low type's match values is reversed to $\nu_{LH} < \nu_{LL}$ under horizontal differentiation. In particular, it is straightforward to show that if ν_{LH} is sufficiently smaller than ν_{LL}, at the steady-state partially informative equilibrium given by Corollary 1, the low type would strictly prefer the low message to the high message, regardless of the probability k_H that the high type keeps the low type after the first period of the marriage. Thus, "too much" horizontal differentiation makes it impossible to construct a steady-state partially informative equilibrium.

5.4 Idiosyncratic shocks

We have assumed that the match values to two agents that have met are pinned down by their private match types. A more realistic assumption is that match values are subject to residual shocks given their private match types. In a similar spirit as Harsanyi (1973), these shocks may be used to "purify" the mixed messaging decision by the low type and the mixed keep-divorce decision by the high type in the steady-state partially informative equilibrium.

First, imagine that, upon meeting each other in the market and before making the messaging decision, two agents each independently experiences a meeting-specific shock that affects the match value the agent receives in the first period of the resulting marriage. Such shock is temporary because it has no effect on the match value each of the two agents receives should they remain in a permanent marriage after the first period. Suppose further that the shock is independent of the types of the two agents. To be concrete, suppose that the match value in the first period of marriage is given by $\nu_{\Theta\Theta'} + \lambda_1\sigma_1$ to each agent, whose type is Θ and whose partner's type is Θ', with $\Theta, \Theta' = H, L$, where σ_1 is a real random variable with a continuous density over $[-1, 1]$ and $\lambda_1 > 0$ is a scaling factor. For the steady-state partially informative equilibrium in Corollary 1, since a high-type agent strictly prefers the high message to the low message if λ_1 is sufficiently small, only the low type's messaging decision will be affected. Given that in equilibrium the total probability of marrying a random meeting partner is greater after the high message than after the low message for low type, a positive realization of σ_1 makes the high message a strictly optimal choice, while at the same time a negative shock makes the low message strictly optimal.

Next, for the high type's mixed keep-divorce decision, consider match-specific shocks realized after the first period of marriage that affect each agent's permanent match value with the partner, independent of the types of the two agents in the marriage. Suppose that, if two married agents of any two types $\Theta, \Theta' = H, L$ decide to keep each other after the first period of their marriage, the permanent match value per period to the agent of type Θ is given by $\nu_{\Theta\Theta'} + \lambda_2\sigma_2$, where σ_2 is a real random variable with a continuous density over $[-1, 1]$ and $\lambda_2 > 0$ is a scaling factor. For the steady-state partially informative equilibrium in Corollary 1, since all agents strictly prefer to keep a high-type partner in a permanent marriage and a low-type agent strictly prefers to keep a low-type partner, if λ_2 is sufficiently small, only the high type's keep-divorce decision regarding a low-type partner will be affected. Thus, at the original steady-state partially informative equilibrium, a positive shock increases the high-type's payoff in a permanent marriage to the low type and makes keeping the marriage strictly optima, while a negative realized shock makes ending it strictly optimal.

If we have both the temporary meeting-specific shock and the permanent match-specific shock, we can follow a similar analysis to construct a steady-state partially informative equilibrium, where the messaging decision by the low type is conditioned on the realized meeting-specific shock and the mixed keep-divorce decision by the high type is conditioned on the realized match-specific shock. There is no randomization in either decision. Furthermore, for fixed positive but sufficiently small values of λ_1 and λ_2, each decision in equilibrium is represented by a threshold rule, such that the low type chooses the high message if the realized σ_1 is above some threshold s_1, and similarly the high type keeps the low type partner if the realized σ_2 is above the corresponding threshold s_2. The two thresholds s_1 and s_2 are the counterparts of the probability m_L^I of the low type choosing the high message and the probability k_H^I of the high type keeping a low-type partner, and have similar comparative statics interpretations.

5.5 Continuous types

The model in this subsection is otherwise the same as the main model except that there is a continuum of agents' types and that the match value an agent can get from marriage only depends on the spouse's type. The main reason to adopt the match payoff structure of Burdett and Coles (1997) is for simplification of the analysis. As in Shimer and Smith (2000), the presence of search frictions means that marriages will be characterized by a matching correspondence that maps each type to a set of mutually acceptable types. Having pre-marriage cheap talk communication generally will not eliminate search frictions, and can further complicate the analysis.[15] The simple payoff structure makes the analysis more tractable. In particular, permanent marriages have a "class" structure that makes it easier to compare this equilibrium with the one in the main model, where the keep-divorce decision is the same among agents of the same type and jumps discontinuously cross types.

The purpose of this subsection is to establish conditions under which there is a steady-state partially informative equilibrium. Denote the exogenous replacement distribution as Φ over some interval of types $[\underline{x}, \overline{x}]$, with $\overline{x} > \underline{x} \geq 0$. To stay as close as possible to the class of two-type equilibria we have constructed, we further assume that $\delta > \overline{x}$. This is the counterpart of the assumption of $\delta > \max\{\nu_{LH}, \nu_{HL}\}$ in the two-type model, ensuring that there is no benefit of marrying even the highest type for one period if divorce is certain.

For given $\Phi(\cdot)$, consider a steady-state partially informative equilibrium that mimics the one in Proposition 5. Let $P(x)$ be the endogenous steady state fraction of agents whose type is weakly below x in the market. Although informative cheap talk can generally involve many messages, we restrict to just two messages: the equilibrium involves some threshold type $c \in (\underline{x}, \overline{x})$ such that types above c send a high message h and those below send either h or a low message l. In equilibrium, permanent marriages are characterized by a class structure as in Burdett and Coles (1997): for our purpose, we consider a class structure of two classes with type c being the cutoff type: types above c, or "first-class types," keep only types at least as high as c, and types below c, or "second class types," keep all types but are rejected by type c and above.[16] Second class types are further divided into two measurable sets which we call "subclasses": ll types that send the low message and accept senders of both messages, hl types that send the high message and accept senders of both messages and hh types that send the high message and accept only the high message. This class structure

[15]Strategic communication with a continuum of types between two potential marriage partners shares similar characteristics as the strategic information aggregation problem (Li, Rosen and Suen, 2000), as the marriage decision is a binary joint decision, there are no transfers, and payoffs are interdependent. The difference is that the analysis here must incorporate the future prospects of potential marriage partners.

[16] Formally, a class structure in permanent marriages is defined by a right-continuous step function $g(x)$ for all $x \in [\underline{x}, \overline{x}]$ that represents the lowest type kept by type x. With the restriction to two messages in informative cheap talk prior to marriages, in any steady-state equilibrium there are at least two classes. Otherwise, all types would send the high message and the cheap talk is no longer informative.

mimics the equilibrium in Proposition 5 in which l identifies the sender as a second class type while h allows a second-class type to pass themselves as a first class type. But unlike in the main model where type L randomizes between telling the truth and telling a lie, with a continuum of types messages are partially revealing and there is no randomization. Also, subclass hh accepts only message h, while in the main model type L accepts both messages regardless of what message they send.[17]

Given the class cutoff type c, incentive conditions of first class types and the three subclasses of the second class depend on the endogenous type distribution $P(\cdot)$ only through the sizes and conditional averages of three subclasses of the second class and the first class. Let (P_{ll}, A_{ll}), (P_{hl}, A_{hl}) and (P_{hh}, A_{hh}) be the corresponding size and conditional average of the three subclasses, and A_1 be the conditional average of first class types. All first class types have the same equilibrium payoff U_1, satisfying the value iteration

$$U_1 = (1 - P(c))\frac{A_1}{1-\beta} + P_{hl}(A_{hl} - \delta) + P_{hh}(A_{hh} - \delta) + P(c)\beta U_1.$$

where $P(c) = P_{ll} + P_{hl} + P_{hh}$ is the total size of the three subclasses. The above expression follows because all first class types send h, accept only h and keep only first class types. The class cutoff c makes all first class types indifferent between keeping and divorcing c:

$$\frac{\beta c}{1-\beta} = -\delta + \beta U_1.$$

All second class types also have the same equilibrium payoff U_2, even though they send different cheap talk messages and make different accept-reject decisions at marriage. Subclass ll sends l, and marries subclasses ll and hl with no divorce, implying

$$U_2 = \frac{P_{ll}A_{ll} + P_{hl}A_{hl}}{1-\beta} + (1 - P_{ll} - P_{hl})\beta U_2.$$

[17] Subclass hh is an essential part of the equilibrium construction. Unlike in the main model where type L forms a cross-type permanent marriage with a positive probability, in this extension second class types are divorced by first class types. There would be no incentives for second class types to send message h if they accept both messages regardless of their own message.

Subclass hl sends h, accepts both h and l, and is divorced by first class types, implying

$$U_2 = \frac{P(c)A_2}{1-\beta} + (1-P(c))(A_1 - \delta + \beta U_2),$$

where A_2 is the average second class type, satisfying $P(c)A_2 = P_{ll}A_{ll} + P_{hl}A_{hl} + P_{hh}A_{hh}$. Finally, subclass hh sends h, accepts only h and is divorced by first class types, implying

$$U_2 = \frac{P_{hl}A_{hl} + P_{hh}A_{hh}}{1-\beta} + P_{ll}\beta U_2 + (1-P(c))(A_1 - \delta + \beta U_2).$$

The above five incentive conditions place three independent equality restrictions on the equilibrium variable (P_{ll}, A_{ll}), (P_{hl}, A_{hl}), (P_{hh}, A_{hh}) and A_1.

Given the class cutoff type c, there are also three independent steady state restrictions on the equilibrium variables. Since first class types are those above c, we have

$$(1-\Phi(c))\overline{Q} = (1-P(c))^2,$$

where $\overline{Q} = P^2(c) - 2P_{ll}P_{hh} + (1-P(c))^2$ is the average probability of forming a permanent marriage. For the same reason,

$$A_1 = \int_c^{\overline{x}} \frac{xd\Phi(x)}{1-\Phi(c)}.$$

Finally, second class types are those below c, we have

$$\overline{Q}\int_{\underline{x}}^{c} xd\Phi(x) = P^2(c)A_2 - P_{ll}P_{hh}(A_{ll} + A_{hh}).$$

For given parameters $\beta \in (0,1)$ and $\delta > \overline{x}$, and a replacement type distribution $\Phi(\cdot)$ over $[\underline{x}, \overline{x}]$, for any $c \in (\underline{x}, \overline{x})$, we have an extra degree of freedom to find equilibrium variables (P_{ll}, A_{ll}), (P_{hl}, A_{hl}), (P_{hh}, A_{hh}) and A_1 to satisfy all six equality restrictions.[18] With continuous types, to complete the equilibrium construction we have to specify the endogenous distribution $P(x)$ for each $x \in [\underline{x}, \overline{x}]$, and how to split $P(x)$ into the three subclasses

[18] The details of the equilibrium construction are available upon request. In addition to the six equality restrictions, we need all second class types to be willing to keep type $\underline{x}$, that is, $\beta\underline{x}/(1-\beta) \geq -\delta + \beta U_2$. We can show that these equality and inequality restrictions are sufficient for the equilibrium. In particular, the incentive conditions of first class types at marriage are implied by the equality restrictions.

$s = ll, hl, hh$. For each $x \in [c, \overline{x}]$, this is given by

$$\frac{dP(x)}{dx} = \frac{\overline{Q}}{1 - P(c)} \frac{d\Phi(x)}{dx}.$$

For each $x \in [\underline{x}, c]$, denoting as $dP_s(x)/dx$ the density of subclass $s = ll, hl, hh$, we require

$$(P_{ll} + P_{hl}) \frac{dP_{ll}(x)}{dx} + (P_{ll} + P_{hl} + P_{hh}) \frac{dP_{hl}(x)}{dx} + (P_{hl} + P_{hh}) \frac{dP_{hh}(x)}{dx} = \overline{Q} \frac{d\Phi(x)}{dx}.$$

The subclass structure is indeterminate so long as the above conditions are satisfied.

The equilibrium class structure of permanent marriages in our model is different from the class structure of Burdett and Coles (1997), which does not have either informative cheap talk or costly divorce.[19] Despite being in the same class and having the same expected payoff, type ll agents and type hh agents send different messages and make exclusive accept-reject decisions at marriage and thus never form permanent marriages with each other. If cheap talk prior to marriage is uninformative, we would have the same kind of class structure in permanent marriages as in Burdett and Coles (1997), so costly divorce alone is not the reason for the difference. Without the divorce cost, however, cheap talk would not be informative, so the difference in the equilibrium class structure from Burdett and Coles (1997) is a result of the interaction between information cheap talk and costly divorce. We leave the full characterization of how dating affects marriage and class to future research.

References

[1] Keera Allendorf (2013), "Schemas of Marital Change: From Arranged Marriages to Eloping for Love," *Journal of Marriage and Family* 75(2), pp. 453-469.

[2] Keera Allendorf and Roshan K. Pandian (2016), "The Decline of Arranged Marriage? Marital Change and Continuity in India," *Population and Development Review* 42(3), pp. 435-464.

[19] Our equilibrium with dating and divorce can coexist with a segregation equilibrium with the same class structure of Burdett and Coles (1997). Indeed, if we set $P_{hh} = P_{hl} = 0$ in the above construction, we can have an equilibrium with two classes and no divorce on the equilibrium path. This is analogous to the fully-revealing equilibrium of Proposition 4.

[3] Yair Antler, Daniel Bird and Daniel Fershtman (2022), "Learning in the Marriage Market: The Economics of Dating," *Working paper*.

[4] Francis Bloch and Harl Ryder (2000), "Two-Sided Search, Marriages, and Matchmakers," *International Economic Review* 41(1), pp. 93-116.

[5] Martin Browning, Pierre-Andre Chiappori, and Yoram Weiss (2014), "Economics of the Family," Cambridge University Press.

[6] Ken Burdett and Melvyn G. Coles (1997), "Marriage and Class," *The Quarterly Journal of Economics* 112(1), pp. 141-168.

[7] Ken Burdett and Randall Wright (1998), "Two-Sided Search with Nontransferable Utility," *Review of Economic Dynamics* 1(1), pp. 220-245.

[8] Hector Chade (2006), "Matching with Noise and the Acceptance Curse," *Journal of Economic Theory* 129(1), pp. 81-113.

[9] Pierre-Andre Chiappori and Yoram Weiss (2006), "Divorce, Remarriage, and Welfare: A General Equilibrium Approach," *Journal of the European Economic Association* 4(2-3), pp. 415-426.

[10] Vincent P. Crawford and Joel Sobel (1982), "Strategic Information Transmission," *Econometrica* 50(6), pp. 1431-1451.

[11] Ettore Damiano, Hao Li and Wing Suen (2006), "Unraveling of Dynamic Sorting, " *Review of Economic Studies* 72(4), pp. 1057-1076.

[12] Premchand Dommaraju (2016), "Divorce and Separation in India," *Population and Development Review* 42(2), pp. 195-223.

[13] Jan Eeckhout (1999), "Bilateral Search and Vertical Heterogeneity," *International Economic Review* 40(4), pp. 869-887.

[14] John Harsanyi (1973), "Games with Randomly Disturbed Payoffs: A New Rationale for Mixed-strategy Equilibrium Points," *International Journal of Game Theory* 2, pp. 1-23.

[15] Nicolas L. Jacquet and Serene Tan (2007), "On the Segmentation of Markets," *Journal of Political Economy* 115(4), pp. 639-664.

[16] Stephan Lauermann and Georg Noldeke (2014), "Stable Marriages and Search Frictions," *Journal of Economic Theory* 151(3), pp. 163-195.

[17] Hao Li, Sherwin Rosen and Wing Suen (2001), "Conflicts and Common Interests in Committees, " *American Economic Review* 91(5), pp. 1478-1497.

[18] Giuseppe Moscarini and Fabien Postel-Vinay (2013), "Stochastic Search Equilibrium," *The Review of Economic Studies* 80(4), pp. 1545-1581.

[19] Christopher Sandmann and Nicolas Bonneton (2024), "Existence of a Non-Stationary Equilibrium in Search-and-Matching Models: TU and NTU," *Working paper.*

[20] Robert Shimer and Lones Smith (2000), "Assortative Matching and Search," *Econometrica* 68(2), pp. 343-369.

[21] Lones Smith (2006), "The Marriage Model with Search Frictions," *Journal of Political Economy* 114(6), pp. 1124-1144.

Appendix

Proof of Proposition 1

(i) (Necessity) Post marriage in any time period $t \geq 1$, there is no profitable one-shot deviation for type $\Theta = H$ with respect to type $\Theta' = L$ only if condition (K) is satisfied. This is equivalent to $\widehat{U}_H^N \leq U_H^*$.

(ii) (Sufficiency) Post marriage in any time period $t \geq 1$, by Assumption V, there is no profitable one-shot deviation for type $\Theta = H, L$ if condition (K) is satisfied for Θ and $\Theta' = L$. This is equivalent to

$$\delta \geq \beta\phi\frac{\nu_{\Theta H} - \nu_{\Theta L}}{1-\beta}.$$

By assumption, the above holds for $\Theta = H$. By Assumption C, it also holds for $\Theta = L$.

At marriage in period $t \geq 2$, for each type $\Theta = H, L$, condition (A) is satisfied because $U^N_{\Theta,t} = \widehat{U}^N_\Theta > 0$. In period 1, condition (A) becomes $U^N_{\Theta,1} \geq \beta \widehat{U}^N_\Theta$. This is satisfied when p_1 is sufficiently close to ϕ.

Proof of Proposition 2

We first prove a result that will be used in this proof and the proof of Proposition 4.

Lemma A. *Fix any $W > 0$, $W' \geq 0$ and $\beta \in [0,1]$. Suppose that for sequence $\{q_t\}_{t=1}^\infty$ satisfying $0 < q_t < 1$ for each $t \geq 1$, $\{V_t\}_{t=1}^\infty$ satisfies*

$$V_t = q_t W - W' + (1 - q_t)\beta V_{t+1}.$$

If $\{q_t\}_{t=1}^\infty$ is increasing (decreasing) and converges to $\hat{q}$ as t goes to infinity, then $\{V_t\}_{t=1}^\infty$ is increasing (decreasing) and converges to

$$\widehat{V} = \frac{\hat{q}W - W'}{1 - (1 - \hat{q})\beta}.$$

Proof. We first establish the following claim: if $\{\widetilde{V}_t\}_{t=1}^\infty$ satisfies

$$\widetilde{V}_t = \tilde{q}_t W - W' + (1 - \tilde{q}_t)\beta \widetilde{V}_{t+1},$$

for sequence $\{\tilde{q}_t\}_{t=1}^\infty$ satisfying $q_t < \tilde{q}_t < 1$, then $V_1 \leq \widetilde{V}_1$, if strict inequality if $\widetilde{V}_1$ is finite. By repeated substitutions, we have

$$V_1 = \sum_{t=1}^{\infty} \prod_{\tau=1}^{t-1} (1 - q_\tau)\beta^{t-1}(q_t W - W').$$

For any $T \geq 2$, let V_1^T be the first T terms in V_1, given by

$$\begin{aligned} V_1^T &= \sum_{t=1}^{T} \prod_{\tau=1}^{t-1} (1-q_\tau)\beta^{t-1}(q_t W - W') \\ &= \sum_{t=1}^{T-1} \prod_{\tau=1}^{t-1} (1-q_\tau)\beta^{t-1}(q_t W - W') + \prod_{\tau=1}^{T-1} (1-q_\tau)\beta^{T-1}(q_T W - W'), \end{aligned}$$

where we have separated out the last term in the sum. Since $q_T < \tilde{q}_T$ and $W > 0$,

$$\begin{aligned} V_1^T &< \sum_{t=1}^{T-1} \prod_{\tau=1}^{t-1} (1-q_\tau)\beta^{t-1}(q_t W - W') + \prod_{\tau=1}^{T-1} (1-q_\tau)\beta^{T-1}(\tilde{q}_T W - W') \\ &= \sum_{t=1}^{T-2} \prod_{\tau=1}^{t-1} (1-q_\tau)\beta^{t-1}(q_t W - W') + \prod_{\tau=1}^{T-2} (1-q_\tau)\beta^{T-2}(q_{T-1} W - W') \\ &\quad + \prod_{\tau=1}^{T-2} (1-q_\tau)\beta^{T-2}(1-q_{T-1})\beta(\tilde{q}_T W - W'), \end{aligned}$$

where we have separated out also the second to the last term. Since $q_{T-1} < \tilde{q}_{T-1}$ and $\beta(\tilde{q}_T W - W') < W$, we have

$$\begin{aligned} V_1^T &< \sum_{t=1}^{T-2} \prod_{\tau=1}^{t-1} (1-q_\tau)\beta^{t-1}(q_t W - W') + \prod_{\tau=1}^{T-2} (1-q_\tau)\beta^{T-2}(\tilde{q}_{T-1} W - W') \\ &\quad + \prod_{\tau=1}^{T-2} (1-q_\tau)\beta^{T-2}(1-\tilde{q}_{T-1})\beta(\tilde{q}_T W - W') \\ &= \sum_{t=1}^{T-2} \prod_{\tau=1}^{t-1} (1-q_\tau)\beta^{t-1}(q_t W - W') \\ &\quad + \sum_{t=T-1}^{T} \prod_{\tau=1}^{T-2} (1-q_\tau)\beta^{T-2} \prod_{\tau'=T-1}^{t-1} (1-\tilde{q}(\tau'))\beta^{\tau'-(T-2)}(\tilde{q}_t W - W'). \end{aligned}$$

If $T = 2$, we have already shown that $V_1^2 < \widetilde{V}_1^2$. Since the above argument holds for any T, we have $V_1 < \widetilde{V}_1$.

Now, suppose that $\{q_t\}_{t=1}^{\infty}$ is an increasing sequence converging to $\hat{q}$; the argument is symmetric if $\{q_t\}_{t=1}^{\infty}$ is decreasing. Fix any $t \geq 1$. Compare sequences $\{V_\tau\}_{\tau=t}^{\infty}$ and $\{\widetilde{V}_\tau\}_{\tau=t}^{\infty}$, where $\widetilde{V}_\tau = \widehat{V}$ for all $\tau \geq t$. By construction, $\{\widetilde{V}_\tau\}_{\tau=t}^\infty$ satisfies

$$\widetilde{V}_\tau = \tilde{q}_\tau W - W' + (1 - \tilde{q}_\tau)\beta \widetilde{V}_{\tau+1},$$

for sequence $\{\tilde{q}_\tau\}_{\tau=t}^\infty$ with $\tilde{q}_\tau = \hat{q}$ for all $\tau \geq t$. Since $q_\tau < \tilde{q}_\tau$ for all $\tau \geq t$, applying the result we have established above, we have $V_t < \widehat{V}$, that is, $\{V_t\}_{t=1}^\infty$ is bounded from above.

Next, compare sequences $\{V_\tau\}_{\tau=t}^\infty$ and $\{\widetilde{V}_\tau\}_{\tau=t}^\infty$, where $\widetilde{V}_\tau = V_{\tau+1}$ for all $\tau \geq t$. By construction, $\{\widetilde{V}_\tau\}_{\tau=t}^\infty$ satisfies

$$\widetilde{V}_\tau = \tilde{q}_\tau W - W' + (1 - \tilde{q}_\tau)\beta \widetilde{V}_{\tau+1},$$

for sequence $\{\tilde{q}_\tau\}_{\tau=t}^\infty$ with $\tilde{q}_\tau = q_{\tau+1}$ for all $\tau \geq t$. Since $\{q_t\}_{t=1}^\infty$ is an increasing sequence, $q_\tau < q_{\tau+1} = \tilde{q}_\tau$ for all $\tau \geq t$. Applying the result we have established above, we have $V_t < V_{t+1}$, that is, $\{V_t\}_{t=1}^\infty$ is an increasing sequence. By the Monotone Convergence Theorem, $\{V_t\}_{t=1}^\infty$ converges. Since $\{q_t\}_{t=1}^\infty$ converges to $\hat{q}$, $\{V_t\}_{t=1}^\infty$ converges to $\widehat{V}$. □

Now we prove Proposition 2.

(i) (Necessity) Post marriage in period $t \geq 1$, consider condition (K) for type $\Theta = H$ with respect to type $\Theta' = L$. By Lemma A, $\{U_{H,t+1}^D\}_{t=1}^\infty$ converges to $\widehat{U}_H^D$ as t goes to infinity. Since $\widehat{U}_H^D < U_H^*$, for sufficiently large t, condition (K) holds, contradicting the equilibrium incentive condition for type H that they optimally divorce type L.

At marriage in any period $t \geq 1$, consider condition (A) for type L. By Lemma A, $\{U_{L,t}^D\}_{t=1}^\infty$ converges to $\widehat{U}_L^D$ as t goes to infinity. If $\widehat{U}_L^D < 0$, for sufficiently large t, condition (A) is violated for type L.

(ii) (Sufficiency) Post marriage in period $t \geq 1$, consider condition (K) for type L. By Lemma A, $U_{L,t+1}^D$ for any period $t \geq 1$ is bounded between $\widehat{U}_L^D$ and

$$\frac{1}{1 - p_t\beta}\left((1 - p_t)\frac{\nu_{LL}}{1 - \beta} - p_t(\delta - \nu_{LH})\right).$$

By Assumption D, $U_{L,t+1}^D < \nu_{LL}/(1 - \beta)$. Thus, condition (K) holds for type L with respect to type L in any period $t \geq 1$. By a similar argument, we can show that $U_{H,t+1}^D < \nu_{HH}/(1-\beta)$

for all $t \geq 1$ and thus condition (K) holds for type H with respect to type H. For type $\Theta = H$ with respect to type $\Theta' = L$, there are two cases. If $p_2 = Q(p_1|1) > \pi(1)$, then by Lemma A, $\{U^D_{H,t+1}\}_{t=1}^{\infty}$ decreases and converges to $\widehat{U}^D_H$ as t goes to infinity. Since by assumption the opposite of condition (K) holds in the steady state, the opposite of (K) holds for all $t \geq 1$. If $p_2 = Q(p_1|1) < \pi(1)$, then by Lemma A, $\{U^D_{H,t+1}\}_{t=1}^{\infty}$ increases and converge to $\widehat{U}^D_H$ as t goes to infinity. The opposite of (K) holds for all $t \geq 1$ if and only if $U^D_{H,2} \geq U^*_H$. In the proof of Lemma A, we show that

$$U^D_{H,2} > \frac{1}{1-(1-p_2)\beta}\left(p_2\frac{\nu_{HH}}{1-\beta} - (1-p_2)(\delta - \nu_{HL})\right).$$

Since by assumption $\widehat{U}^D_H > U^*_H$, we have $U^D_{H,2} \geq U^*_H$ when p_1 is sufficiently close to $\pi(1)$.

At marriage in any period $t \geq 1$, consider condition (A). We distinguish two cases. In the first case, $\phi < \frac{1}{2}$ and $p_1 \in \left(\pi(1), \frac{1}{2}\right)$, or $\phi > \frac{1}{2}$ and $p_1 \in (\pi(1), 1)$. From the properties of Q, we have $p_2 = Q(p_1|1) > \pi(1)$ and $\{p_t\}_{t=1}^{\infty}$ decreases and converges to $\pi(1)$. By Lemma A, as t goes to infinity, $\{U^D_{H,t}\}_{t=1}^{\infty}$ decreases and converges to $\widehat{U}^D_H$, while $\{U^D_{L,t}\}_{t=1}^{\infty}$ increases and converges to $\widehat{U}^D_L$. Since by assumption $\widehat{U}^D_H \geq U^*_H > 0$, condition (A) is satisfied for type H. For type L, we have

$$U^D_{L,t} - \beta U^D_{L,t+1} = \frac{1-p_t}{p_t}\left(\frac{\nu_{LL}}{1-\beta} - U^D_{L,t}\right) - (\delta - \nu_{LH}) > \frac{1-p_t}{p_t}\left(\frac{\nu_{LL}}{1-\beta} - \widehat{U}^D_L\right) - (\delta - \nu_{LH}),$$

where the inequality follows because $U^D_{L,t} < \widehat{U}^D_L$. By assumption $\widehat{U}^D_L > 0$, and so the above is strictly positive for $p_t = \pi(1)$. Since $\{p_t\}_{t=1}^{\infty}$ is a decreasing sequence, we have $U^D_{L,t} - \beta U^D_{L,t+1} > 0$ for all $t \geq 1$ if $p_1 > \pi(1)$ is sufficiently close to $\pi(1)$. The second case is symmetric. Assume that $\phi < \frac{1}{2}$ and $p_1 \in (0, \pi(1))$, or $\phi > \frac{1}{2}$ and $p_1 \in \left(\frac{1}{2}, \pi(1)\right)$. From the properties of Q, we have $p_2 = Q(p_1|1) < \pi(1)$ and $\{p_t\}_{t=1}^{\infty}$ increases and converges to $\pi(1)$. By Lemma A, as t goes to infinity, $\{U^D_{H,t}\}_{t=1}^{\infty}$ increases and converges to $\widehat{U}^D_H$, while $\{U^D_{L,t}\}_{t=1}^{\infty}$ decreases and converges to $\widehat{U}^D_L$. Given that $\widehat{U}^D_L > 0$, condition (A) is always satisfied for type L. For type H, condition (A) is satisfied if $p_1 < \pi(1)$ is sufficiently close to $\pi(1)$.

Proof of Proposition 3

(i) (Necessity) Post marriage in period $t \geq 1$, consider condition (K) for type $\Theta = H$ with respect to type $\Theta' = L$. We have argued that a necessary condition for there to be a babbling equilibrium with randomized divorce is that there exists p^R strictly between ϕ and $\pi(1)$ that satisfies condition (R). Suppose that $\phi < \frac{1}{2}$ and thus $\pi(1) > \phi$. By Assumption V, it is necessary that

$$\phi \frac{\nu_{HH}}{1-\beta} + (1-\phi)\frac{\nu_{HL}}{1-\beta} < U_H^* < \pi(1)\frac{\nu_{HH}}{1-\beta} + (1-\pi(1))\frac{\nu_{HL}}{1-\beta}.$$

Using the definitions of $\widehat{U}_H^N$, U_H^* and $\widehat{U}_H^D$, we find that the above is equivalent to $\widehat{U}_H^N < U_H^* < \widehat{U}_H^D$. For the other case of $\phi > \frac{1}{2}$, condition (R) requires $\widehat{U}_H^D < U_H^* < \widehat{U}_H^N$. In either case, we have a steady state p^R with $z^R = \pi^{-1}(p^R) \in (0,1)$. The market jumps to the steady state in period 2, with $k_{H,t} = 1 - z^R$ for all $t \geq 2$.

At marriage in any period $t \geq 2$, condition (A) for type L is satisfied only if $\widehat{U}_L^D \geq 0$.

(ii) (Sufficiency) Post marriage in any time period $t \geq 1$, there is no profitable one-shot deviation for each type $\Theta = H, L$ if condition (K) holds for type Θ with respect to type $\Theta' = L$. For $\Theta = H$, condition (K) holds with equality because $U_{H,t+1}^R = U_H^*$ for all $t \geq 1$. For $\Theta = L$, condition (K) holds with strict inequality because $U_{L,t+1}^R = \widehat{U}_L^R$ for all $t \geq 1$, and

$$\widehat{U}_L^R - \frac{\nu_{LL}}{1-\beta} < p^R \frac{(\nu_{LH} - \nu_{LL})}{1-\beta} \leq p^R \frac{(\nu_{HH} - \nu_{HL})}{1-\beta} = \frac{\delta}{\beta},$$

where the first inequality follows from $\widehat{U}_L^R$ being strictly decreasing in z^R, the second inequality from Assumption C, and the equality from $\widehat{U}_H^R = U_H^*$.

At marriage in the steady state, with $t \geq 2$, condition (A) holds strictly for each type $\Theta = H, L$, with $U_{\Theta,t}^R = U_{\Theta,t+1}^R = \widehat{U}_\Theta^R$, because $\widehat{U}_H^R = U_H^* > 0$ and by assumption $\widehat{U}_L^R > 0$. For period $t = 1$, by the properties of Q, for any $p_1 \in \left(\phi, \frac{1}{2}\right)$ when $\phi < \frac{1}{2}$ or $p_1 \in \left(\frac{1}{2}, \phi\right)$ when $\phi > \frac{1}{2}$, there exists a unique $z_1 \in (0,1)$, with $k_{H,1} = 1 - z_1$ such that $p^R = Q(p_1|z_1)$. Given that $k_{H,1} \in (0,1)$, we have

$$U_{H,1}^R = p_1 \frac{\nu_{HH}}{1-\beta} + (1-p_1)\frac{\nu_{HL}}{1-\beta}.$$

Thus, condition (A) is satisfied for type H if $U^R_{H,1} \geq \beta U^*_H$. By condition (R) and Assumption V, it suffices if $p_1 \geq \beta p^R$, which is true if p_1 is close to p^R. For type L, we have

$$U^R_{L,1} = (1-p_1)\frac{\nu_{LL}}{1-\beta} + p_1\left(k_{H,1}\frac{\nu_{LH}}{1-\beta} + (1-k_{H,1})(\nu_{LH} - \delta + \beta\widehat{U}^R_L)\right).$$

At $p_1 = p^R$, we have $k_{H,1} = 1 - z^R = 1 - \pi^{-1}(p^R)$, and $U^R_{L,1} = \widehat{U}^R_L$. Since $\widehat{U}^R_L > 0$ by assumption, condition (A) is satisfied for type L in period $t = 1$ with a strict inequality at $p_1 = p^R$, and thus remains satisfied for all p_1 sufficiently close to p^R.

Proof of Proposition 4

(i) (Necessity) Post marriage in period $t \geq 1$, consider condition (K) for type $\Theta = H$ with respect to type $\Theta' = L$. By Lemma A, $\{U^F_{H,t+1}\}_{t=1}^{\infty}$ converges to $\widehat{U}^F_H$ as t goes to infinity. If $\widehat{U}^F_H < U^*_H$, then for any t sufficiently large, condition (K) holds, and requires $k_{H,t} = 1$. This contradicts the restriction we have imposed on the equilibrium.

(ii) (Sufficiency) Post marriage in period $t \geq 1$, consider condition (K) for type $\Theta = L$ with respect to type $\Theta' = L$. By Lemma A, $U^F_{L,t}$ for any period $t \geq 2$ is bounded between $\widehat{U}^F_L$ and

$$\frac{1-p_t}{1-p_t\beta} \cdot \frac{\nu_{LL}}{1-\beta}.$$

Thus, $U^F_{L,t} < \nu_{LL}/(1-\beta)$, and condition (K) holds for type L with respect to type L in any period $t \geq 2$. By a similar argument, we can show that $U^F_{H,t} < \nu_{HH}/(1-\beta)$ for all $t \geq 2$ and thus condition (K) holds for type H with respect to type H. For type H with respect to type L, which is off the equilibrium path, there are two cases. If $p_2 = Q(p_1|1) > \pi(1)$, then by Lemma A, $\{U^F_{H,t+1}\}_{t=1}^{\infty}$ decreases and converges to $\widehat{U}^F_H$ as t goes to infinity. Since by assumption the opposite of condition (K) holds in the steady state, the opposite of (K) holds for all $t \geq 1$. If $p_2 = Q(p_1|1) < \pi(1)$, then by Lemma A, $\{U^F_{H,t+1}\}_{t=1}^{\infty}$ increases and converge to $\widehat{U}^F_H$ as t goes to infinity. The opposite of (K) holds for all $t \geq 1$ if and only if $U^F_{H,2} \geq U^*_H$. In the proof of Lemma A, we show that

$$U^F_{H,2} > \frac{p_2}{1-(1-p_2)\beta} \cdot \frac{\nu_{HH}}{1-\beta}.$$

Since by assumption $\widehat{U}_H^F > U_H^*$, we have $U_{H,2}^F \geq U_H^*$ when p_1 is sufficiently close to $\pi(1)$ so that $p_2 = Q(p_1|1) < \pi(1)$ is close to $p(1)$.

At marriage in any period $t \geq 1$, consider one-shot deviations by each type $\Theta = H, L$ after receiving a message. Regardless of their own message, rejecting the message means skipping to period $t+1$ and receiving a payoff of $\beta U_{\Theta,t+1}^F$. For type L with respect to message l, given that $a_{L,t} = 0$ in equilibrium, a one-shot deviation makes a difference to type L's payoff only after sending message l on the equilibrium path. Since in equilibrium only type L sends message l, accepting it by a type-L agent who has sent message l implies a payoff of $\nu_{LL}/(1-\beta)$. There is no profitable one-shot deviation for type L from accepting message l because we have already shown above that $U_{L,t+1}^F < \nu_{LL}/(1-\beta)$ for all $t \geq 1$. Now, consider type L's one-shot deviation after receiving message h. Given that $a_{H,t} = 0$ in equilibrium, a one-shot deviation makes a difference to type L's payoff only after sending message h off the path. Since in equilibrium only type H sends message h, and since $k_{H,t} = 0$, by Assumption D, it is not profitable to deviate from $a_{L,t} = 0$. Symmetrically, for type H after receiving message h, since $a_{H,t} = 0$, a one-shot deviation from accepting message h makes a difference to type H only after sending message h on the equilibrium path. Such deviation is not profitable, as we have shown above that $U_{H,t+1}^F < \nu_{HH}/(1-\beta)$ for all $t \geq 1$. For type H with respect to message l, since $a_{L,t} = 0$, a one-shot deviation from accepting message h makes a difference only after deviating and sending message l. There is no profitable one-shot deviation from $a_{H,t} = 0$ by Assumption D, because in equilibrium only type L sends message l and $k_{H,t} = 0$.

At dating in any period $t \geq 1$, the only one-shot deviation for type H is to send message l. By our equilibrium restrictions, this leads to a payoff of $\beta U_{H,t+1}^F$ as $a_{L,t} = 0$, and $a_{H,t} = 0$ regardless of their own message sent by type H. As we have already shown above, $U_{H,t+1}^F < \nu_{HH}/(1-\beta)$ for all $t \geq 1$, which implies that

$$U_{H,t}^F = p_t \frac{\nu_{HH}}{1-\beta} + (1-p_t)\beta U_{H,t+1}^F > \beta U_{H,t+1}^F.$$

Thus condition (T) is satisfied. For type L, the only one-shot deviation from $m_{L,t} = 0$ is to send message h. Since $a_{H,t} = 0$, and $a_{L,t} = 0$ regardless of their own message, this deviation leads to a payoff $\beta U^F_{L,t+1}$ instead of $U^F_{L,t}$. As we have already shown above, $U^F_{L,t+1} < \nu_{LL}/(1-\beta)$ for all $t \geq 1$, which implies that

$$U^F_{L,t} = (1-p_t)\frac{\nu_{LL}}{1-\beta} + p_t \beta U^F_{L,t+1} > \beta U^F_{L,t+1}.$$

Thus condition (T) is satisfied.

Proof of Proposition 5

(i) (Necessity) Consider first the case of $\phi < \frac{1}{2}$. This implies that $\phi < \pi(1)$. Since $z_t < 1$ for all $t \geq 1$ in a partially informative equilibrium, by the properties of $Q(\cdot|z)$, for any $t \geq 2$ we have

$$p_{t+1} = Q(p_t|z_t) < Q(p_t|1) < Q(\overline{\phi}|1).$$

Since $Q^t(\overline{\phi}|1)$ converges to $\pi(1)$, we have $\limsup_{t\to\infty} p_t = \pi(1)$. At the same time, since $z_t > 0$ for all $t \geq 1$, by the properties of $Q(\cdot|z)$, for any $t \geq 1$ we also have

$$p_{t+1} = Q(p_t|z_t) > Q(p_t|0) = \phi.$$

Consider permanent marriage in period $t \geq 1$ for type H with respect to type L. For type H to be indifferent between keeping and divorcing type L, we need

$$U^*_H = p_t \frac{\nu_{HH}}{1-\beta} + (1-p_t)\left(m_{L,t}\frac{\nu_{HL}}{1-\beta} + (1-m_{L,t})\beta U^*_H\right).$$

By Assumption D, the right-hand side of the above condition is decreasing in $m_{L,t}$. Since $m_{L,t} \in (0,1)$, it is necessary that

$$p_t \frac{\nu_{HH}}{1-\beta} + (1-p_t)\frac{\nu_{HL}}{1-\beta} < U^*_H < p_t \frac{\nu_{HH}}{1-\beta} + (1-p_t)\beta U^*_H.$$

Since $p_t > \phi$ for $t \geq 2$, by Assumption V, the first inequality requires that $U^*_H > \widehat{U}^N_H$. Since $\limsup_{t\to\infty} p_t = \pi(1)$, the second inequality requires $U^*_H \leq \widehat{U}^F_H$.

The case of $\phi > \frac{1}{2}$ is symmetric. We have $\liminf_{t\to\infty} p_t = \pi(1)$ and $p_t < \phi$. The necessary equilibrium conditions are $U_H^* > \widehat{U}_H^D$ and $U_H^* \leq \widehat{U}_H^f$.

(ii) (Sufficiency) Suppose that $\phi < \frac{1}{2}$, and fix any $p \in [\phi, \pi(1)]$. Since $\max\{\widehat{U}_H^N, \widehat{U}_H^D\} < U_H^* < \min\{U_H^F, U_H^f\}$, by the same argument in part (i), there is a unique value $m \in (0,1)$ for m_L^I such that conditions (R_H) and (K_H) are satisfied for $p^I = p$. From condition (T_L) for $p^I = p$, we can solve for a value U for $\widehat{U}_L^I$. We have $U < \nu_{LL}/(1-\beta) < \nu_{LH}/(1-\beta)$, where the second inequality follows from Assumption V. Then, from condition (R_L) for $\widehat{U}_L^I = U$, we find a unique value $k \in (0,1)$ for k_H^I. To summarize, for any value $p \in [\phi, \pi(1)]$, there are unique values m for m_L^I and k for k_H^I, both strictly between 0 and 1, such that conditions (R_H), (K_H), (R_L) and (T_L) are all satisfied for $p^I = p$. Since $z = 1 - mk \in (0,1)$, by the properties of Q, we have

$$\phi = Q(p|0) < Q(p|z) < Q(p|1) \leq Q(\pi(1)|1) = \pi(1).$$

We have thus constructed a continuous mapping from the interval $[\phi, \pi(1)]$ to itself. By Brower's Fixed Point Theorem, the mapping has a fixed point. This fixed point, which we denote as p^I, uniquely determines $m_L^I \in (0,1)$, $k_H^I \in (0,1)$, together with $\widehat{U}_H^I$ and $\widehat{U}_L^I$ that satisfy all conditions (R_H), (K_H), (R_L) and (T_L). Further, since $m_L^I \in (0,1)$ and $k_H^I \in (0,1)$, we have $p^I \in (\phi, \pi(1))$.

Now, we construct a partial informative equilibrium that jumps to the steady state in period 2 from p_1. Consider $\{m_{L,t}, a_{H,t}, a_{L,t}, k_{H,t}, p_{t+1}\}_{t=1}^{\infty}$, where $a_{H,t} = 0$, $a_{L,t} = 1$, $m_{L,t+1} = m_L^I$, $k_{H,t} = k_H^I$ and $p_{t+1} = p^I$ for all $t \geq 1$. Let $z^I = 1 - m_L^I k_H^I$. By construction, we have $p^I = \pi(z^I)$, and therefore the condition of market dynamics in the definition of perfect Bayesian equilibrium is satisfied in the steady state, that is, in any period $t \geq 2$. For period $t = 1$, by the properties of Q, for any p_1 such that $Q(p_1|1 - k_H^I) < p^I < Q(p_1|1)$, there exists a unique $m_{L,1} \in (0,1)$ satisfying

$$Q(p_1|1 - m_{L,1}k_{H,}^I) = p^I.$$

We verify below that all incentive conditions are satisfied. For each $t \geq 1$, let $U_{H,t+1}^I = \widehat{U}_H^I = U_H^*$ and $U_{L,t+1}^I = \widehat{U}_L^I$.

Post marriage in period $t \geq 1$, condition (K) is satisfied for type L. This is because by construction, $U^I_{L,t+1} = \widehat{U}^I_L < \nu_{LL}/(1-\beta) < \nu_{LH}/(1-\beta)$, where the second inequality follows from Assumption V. It is optimal for type L to keep both type L and type H. For type H, by construction $U^I_{H,t+1} = U^*_H < \nu_{HH}/(1-\beta)$ for all $t \geq 1$. Condition (K) holds with equality with respect to type L, and holds with a strictly inequality with respect to type H by Assumption V. There is no profitable one-shot deviation for type H.

At marriage in any period $t \geq 1$, consider the one-shot deviation for each type $\Theta = H, L$ after receiving message l. In equilibrium, only type L sends message l. For type L, the one-shot deviation of rejecting message l is not profitable, because $U^I_{L,t+1} = \widehat{U}^I_L < \nu_{LL}/(1-\beta)$. For type H, the one-shot deviation of accepting message l is not profitable, because $\nu_{HL}/(1-\beta) < \beta U^I_{H,t+1} = \beta U^*_H$ by condition (R$_H$) and Assumption D. Now, consider the one-shot deviation for each type $\Theta = H, L$ after receiving message h. By Bayes' rule, the updated belief that the meeting partner is of type H is given by

$$p'_t = \frac{p_t}{p_t + (1-p_t)m_{L,t}}.$$

For type L, given conditions (R$_L$) and (T$_L$), the one-shot deviation of rejecting message h after sending message h is not profitable (there is no profitable one-shot deviation after sending message l because $a_{H,t} = 0$) if

$$\begin{aligned}
\beta U^I_{L,t+1} &\leq (1-p'_t)\frac{\nu_{LL}}{1-\beta} + p'_t\left(k_{H,t}\frac{\nu_{LH}}{1-\beta} + (1-k_{H,t})\left(\nu_{LH} - \delta + \beta U^I_{L,t+1}\right)\right) \\
&= (1-p'_t)\frac{\nu_{LL}}{1-\beta} + p'_t\beta U^I_{L,t+1},
\end{aligned}$$

where the equality follows from type L's indifference condition between message h and message l. The above holds for all $t \geq 1$ because $U^I_{L,t+1} = \widehat{U}^I_L < \nu_{LL}/(1-\beta)$. For type H, given condition (R$_H$), the one-shot deviation of rejecting message h after sending message h is not profitable (there is no profitable one-shot deviation after sending message l because $a_{H,t} = 0$) if

$$\beta U^*_H \leq p'_t\frac{\nu_{HH}}{1-\beta} + (1-p'_t)\frac{\nu_{HL}}{1-\beta}.$$

The above is equivalent to

$$\beta U_H^* \le U_{H,t}^I.$$

This holds with a strict inequality for $t \ge 2$ as $U_{H,t}^I = U_H^* > 0$. By continuity, it holds for $t = 1$ when p_1 is sufficiently close to p^I and $m_{L,1}$ to m_L^I.

At dating in any period $t \ge 1$, the only one-shot deviation that needs to be considered is for type H to send message l. Since $a_{H,t} = 0$ regardless of one's own message, by condition (R_H), the deviation payoff is βU_H^* regardless the type of the meeting partner or the message sent by a type-L partner. Such deviation is not profitable if $\beta U_H^* \le U_{H,t}^I$, which as we have just argued holds strictly for all $t \ge 2$, and for $t = 1$ when p_1 is sufficiently close to p^I and $m_{L,1}$ to m_L^I.

The argument for sufficiency in the case of $\phi > \frac{1}{2}$ is symmetric. Under the conditions of $\max\{\widehat{U}_H^N, \widehat{U}_H^D\} < U_H^* < \min\{U_H^F, U_H^f\}$, there is a $p^I \in (\pi(1), \phi)$ such that conditions (R_H), (K_H), (R_L) and (T_L) are satisfied by some $m_L^I \in (0,1)$ and $k_H^I \in (0,1)$. The verification of the incentive conditions for both the steady state $t \ge 2$ and period $t = 1$ is the same.

Proof of Proposition 6

Fix any p^I that is either unique, or an extreme. For now, assume that $\phi \neq \frac{1}{2}$. We establish three claims.

First, rewrite equation (FP) as

$$p = \phi + (1 - 2\phi)(p(1-p) - \Omega(p)),$$

where

$$\Omega(p) = p(1-p) m_L(p) k_H(p).$$

At the equilibrium, $\Omega(p^I)$ gives the fraction of permanent cross-type marriages. For $\phi < \frac{1}{2}$, the derivative of $\Omega(p^I)$ with respect to δ has the opposite sign of the derivative p^I with respect to δ; for $\phi > \frac{1}{2}$, the derivative of $\Omega(p^I)$ with respect to δ has the same sign of the derivative p^I with respect to δ.

Second, by properties of $Q(\cdot)$, for $\phi < \frac{1}{2}$, the derivative of p^I with respect to δ has the opposite sign as the derivative of $m_L(p)k_H(p)$ with respective to δ at $p = p^I$; for $\phi > \frac{1}{2}$, the derivative of p^I with respect to δ has the same sign as the derivative of $m_L(p)k_H(p)$ with respective to δ at $p = p^I$.

Third, for fixed p, the derivative of $m_L(p)k_H(p)$ with respect to δ has the same sign as

$$-\frac{C_m(p)}{(\delta-\nu_L)(C_m(p)-(\delta-\nu_L))}+\frac{C_k(p)}{(\delta-\nu_H)(C_k(p)+\delta-\nu_H)},$$

where

$$C_m(p) = \frac{\nu_H-\nu_L}{1-\beta} - \frac{\nu_H}{1-\beta(1-p)},$$
$$C_k(p) = \frac{\nu_H-\nu_L}{1-\beta} + \frac{\nu_L}{1-\beta p}.$$

Since $C_k(p) > C_m(p)$, the derivative of $m_L(p)k_H(p)$ with respect to δ is strictly positive if

$$(\nu_H-\nu_L)(C_m(p)-(\nu_H-\nu_L)) > 2(\delta-\nu_H)(\delta-\nu_L).$$

Given any $\nu_L < \nu_H$, the above holds if δ is sufficiently close ν_H. Similarly, since $C_m(p) < C_k(p)$, the derivative of $m_L(p)k_H(p)$ with respect to δ is strictly negative if

$$(\nu_H-\nu_L)(C_k(p)-(\nu_H-\nu_L)) < 2(\delta-\nu_H)(\delta-\nu_L).$$

Given any $\delta > \nu_H$, the above holds if ν_L is sufficiently close ν_H.

Now we combine the above three claims. Suppose that δ is sufficiently close ν_H for given $\nu_L < \nu_H$. By the third claim, a decrease in δ leads to a decrease in $m_L(p)k_H(p)$ for all p. If $\phi = \frac{1}{2}$, then $p^I = \frac{1}{2}$, and $\Omega(p^I)$ decreases. If $\phi < \frac{1}{2}$, then by the second claim p^I increases, and by the first claim $\Omega(p^I)$ decreases. If $\phi > \frac{1}{2}$, then by the second claim p^I decreases, and by the first claim $\Omega(p^I)$ decreases. The argument is the same when ν_L is sufficiently close ν_H for given $\delta > \nu_H$.