Dating and Divorce

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Abstract

We introduce private information and divorce in an otherwise standard search and matching model of marriage without transfers. Marriage allows agents to find out about each other and then decide whether or not to incur the cost of divorce and re-enter the market. We show that informative pre-marriage cheap talk communication (dating) can decrease marriage rate and at the same time increase the divorce rate. This is because the parties can partially reveal private information about each other through communication, which makes agents more selective in their marriage decision. At the same time communication raises future prospects after a divorce, and makes them more willing to end less desirable marriages. In the main model with two types, when both a steady state equilibrium with dating and divorce and a babbling equilibrium with no divorce exist, the high type is better off with dating and divorce and the low type is worse off. Both the coexistence of the two steady states and the welfare comparison extend to a model with a continuum of types on both sides of the market.

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1 Introduction

In much of the literature on search and matching, finding the right partners takes time in the market through search, but agents obtain complete information about the match as soon as they meet (Burdett and Coles, 1997; Shimer and Smith, 2000). However, in reality often the only way to find out whether there is a right match is through marriage. In this paper, we introduce private information about one's own match type between two people that have just met on the search market. The private-information assumption raises two issues that have been largely overlooked in the existing search and matching literature.

The first issue is strategic communication. Two people that have just met each have private information about own match characteristics that jointly determines the quality of their match if they marry. They can engage in cheap talk communication a la Crawford and Sobel (1982) prior to their decisions of whether or not to get married. We may think of such strategic information aggregation as "dating."

The second issue is endogenous breakup. As long as cheap talk communication does not perfectly reveal all relevant private information, marriage is a gamble that may or may not work out. Unlike in much of the existing literature on search and matching, divorce is an endogenous choice here. This choice is made based on what the two people in a marriage have learned about each other, as well as on their future prospects from the market after divorce. When there is a cost to divorce, it will also be part of the consideration.

In this paper we construct the simplest model that strategic communication and endogenous breakup interact with each other. The private information that an individual has about his or her own match type is assumed to be perfect, and more importantly, "vertical," in that all other participants in the market have the same ranking of match types. A "high" type is therefore more attractive than a "low" type to all participants on the opposite side of the market. Moreover, we assume that no private information about match types is revealed but only cheap talk messages are exchanged before marriage. So the marriage decision is based on the cheap talk messages only. Further, we assume that the private information about match types is revealed all at once after the first period of the marriage rather than gradually during the marriage. This means that the endogenous breakup decision, i.e., whether or not to divorce, is a one-time decision for the married couple.

We use the model to show that informative pre-marriage strategic communication does not necessarily reduce the divorce rate. There are opposing effects of informative strategic communication compared to uninformative strategic communication, or babbling. On one hand, to the extent that pre-marriage communication is informative about the match types of two people that have met on the market, they will be more selective in their marriage decision. This naturally reduces the incidence of "bad" marriages in which high types are married to low types, and tends to lower the divorce rate. On the other hand, precisely because pre-marriage communication is informative, high types have better future prospects of finding high types on the other side of the market. As a result, so long as pre-marriage communication is imperfect, high types are more willing to divorce if ex post they find out that their partners are actually low types. We show that the second effect of imperfect, informative pre-marriage strategic communication can outweigh the first effect, resulting in a higher divorce rate in a steady state equilibrium.

Indeed, if the divorce cost is sufficiently high, both the high type and the low type will be unwilling to divorce a marriage partner after finding out the latter is of the low type at the end of the first period of the marriage, given that future matching is random as agents do not expect a future meeting to reveal any information about each other's type before the marriage decision. Thus, there is a steady state equilibrium with babbling and no divorce. However, when the divorce cost is not too high, the high type will strictly prefer to divorce the low type after the first period of the marriage, if in any future meeting pre-marriage cheap talk communication perfectly reveals the types of the two agents in the meeting. We show that for intermediate values of the divorce cost, there is another steady state equilibrium in which the low type randomizes between truth-telling and lying about his or her type and the high type randomizes between keeping and divorcing a low-type partner after the first period of the marriage. Compared to the first steady state equilibrium with babbling and no divorce, in this second steady state equilibrium with informative talk and divorce some meetings do not result in a marriage, as the high type rejects any agent who admits to be a low type, but at the same time some cross-type marriages end up in a costly divorce, which ensures that the low type does not always lie.

The increase in the divorce rate across all societies and cultures in modern times has often been attributed to the reduction in the cost of divorce. In this paper we provide a counterpoint. The increase in the divorce rate may have instead resulted from the greater ease and willingness in engaging in meaningful discussions of or examinations about the marriage decision. Stretching our point a little bit, it might be that when social norms move from arranged marriages to dating, the prospects of finding the right partners improve sufficiently so that more bad marriages end up in divorce. For this interpretation to make sense, the private match type information should be something other than wealth or social class that is easily observable. Examples of such private type that is more difficult to ascertain but is at the same time vertical include health status and temperament.

The rest of the paper is organized as follows. In section 2, we set up the basic model with two types. The sufficient conditions for both the steady state equilibrium with babbling and no divorce and the steady state equilibrium with informative talk and divorce are presented in section 3. We show that the two equilibria can coexist, and whenever they do, the high type is better off with dating and divorce than with babbling and no divorce, while the low type is worse off. In section 4, we discuss other stationary and non-stationary equilibria, and show how marriage mechanisms can improve upon the steady state equilibrium payoffs. Section 5 extends the basic model to allow for horizontal differentiation, idiosyncratic shocks, and continuous types, and demonstrates that the insight that informative pre-marriage communication can make the marriage more selective yet at the same time increase the divorcing rate continues to hold. In particular, in the continuous-type extension, we restrict the match payoff to depend only on the partner's type (Burdett and Coles, 1997), and construct a steady state equilibrium with two cheap talk messages and two "classes" in permanent marriages, with the lower second class divided further into three subclasses. First marriages are selective because types in the higher first class and in the top subclass of the second class reject types in the bottom subclass who self-identify by sending a low message. First marriages between types in the first class and types in the top subclass, who both send a high message and accept only the high message, end up in a divorce. When this equilibrium coexists with a babbling equilibrium with no divorce, types in the first class are all better off in the equilibrium with dating and divorce, and all types in the second class are worse off.

2 Model

We study an infinite-horizon dynamic matching market with no transfers. There is a unit measure of men and women. The two sides of the market will be entirely symmetric for simplicity. Time is discrete. All agents are risk-neutral, and have the same per-period discount factor of β.

In each period, participating agents from the two sides, men and women, are randomly paired with each other, with meeting probability equal to one for each male and female agent. This means that the discount factor β measures the magnitude of search frictions in the marriage market. When a man and a woman meet, they do not observe each other's type. Instead, they send cheap talk messages to each other. After exchanging the messages, they independently decide whether or not to marry. If at least one of them decides not to marry, the two agents wait for the opening of the market in the next period. If both decide to marry, they each receive a match value that depends on both of their true types, which we will specify in detail below. After finding out about each other's type, they decide independently whether or not to stay married or divorce. If they both decide to stay, the man and the woman disappear from the market, and their replacements, a man and a woman, take their places in the next period market. If at least one of them decides to leave the marriage, they each incur a divorce cost $\delta > 0$, and go to the next period market.

Now we specify that type and payoff structures in the basic model. There are two types, H and L, of men and women. In section 5.3, we show how to extend the main results of the basic model to a model with a continuum of types on both sides. For any $T, T' = H, L$, the match value between a couple with types T and T' is $\nu_{TT'}$ to type T and $\nu_{T'T}$ to type T'.[1] Throughout the paper, we assume that

$$\nu_{TH} > \nu_{TL} > 0 \tag{1}$$

for each $T = H, L$, so that a type H partner is more attractive than type L regardless of

[1] An implicit assumption here is that the match values to two agents that have met in the market are pinned down by their private match types. In section 5.2, we allow for residual idiosyncratic shocks to the values given their private match types, including both temporary shocks that affect only the values in the first period of the marriage, and permanent shocks that affect the values after the first period.

one's own type, and matching with type L is strictly better than not participating in the market, which for simplicity we have assumed to yield a payoff of zero regardless of type. Assumption (1) means that agents are vertically differentiated, with type H more desirable than type L for every agent regardless of one's own type.[2]

Assumption (1) allows the value to any agent from a match with any fixed type to depend on the agent's own type. The payoff structure in our model is therefore more general than in Burdett and Coles (1997), who assume common match values, with

$$\nu_{HT} = \nu_{LT} \tag{2}$$

for each $T = H, L$. Under (2), the marginal contribution of the type of an agent's marriage partner to the agent's match value is independent of the agent's own type. If instead

$$\nu_{HH} - \nu_{HL} > \nu_{LH} - \nu_{LL}, \tag{3}$$

match values exhibit strict increasing differences (complementarity or supermodularity); and reversing the above inequality yields decreasing differences (submodularity). Unlike in Shimer and Smith (2000), there is no transfer between two agents who have met and must decide whether or not marry. Instead of joint match payoffs, the assumption of increasing differences (or decreasing differences) applies to match values that an individual agent receives. Assumptions such as (2) or (3), or the reverse of (3), are useful in our model to simplify the conditions for our remain result, but are restrictive in that they make it harder to satisfy those conditions.

The final parameter of the model is the fraction of type H in the replacement draw. We denote this exogenous fraction as ϕ, and assume that $\phi \in (0, 1)$. We will restrict to steady states in the main analysis.[3] Let p denote the endogenous steady state fraction of type H agents in the market.

[2]See section 5.1 for a discussion of horizontal differentiation.

[3]In section 4.2 we offer a brief discussion of the robustness of our main result when the fraction of the high type is exogenously given.

3 Main Analysis

In this section, we first provide sufficient conditions for there to exist a steady state equilibrium in which agents babble before they make their marriage decisions and there is no divorce after the first period of the marriage. Next we provide further sufficient conditions for there to exist another steady state equilibrium in which pre-marriage cheap talk communication is partially informative and the divorce rate is positive after the first period of the marriage. The last subsection compares the welfare of the two types across the two steady states when they co-exist.

3.1 Babbling and no divorce

In this subsection we present the necessary and sufficient condition for a steady state equilibrium in which any two agents who have just met babble to each other, get married, and live happily thereafter. Of course, this is an extreme scenario, as the entire population is replaced in every period. But it is simple to characterize, and more importantly, it serves our main point of contrasting babbling and no divorce with informative cheap talk and divorce after the first marriage.

Given our assumption about the outside option of being single (condition (1)), in any steady state with babbling and no divorce, two agents who have just met accept each other with probability one as marriage partner. To construct such an equilibrium, we only need to ensure that after the two agents find out about each other's type, they find it optimal to keep each other instead of divorce. Clearly, it is optimal for both types to keep type H in a marriage, because this gives the highest possible payoff for an agent regardless of the agent's own type.

For each type $T = H, L$, let U_T be the steady-state payoff of a type T agent. In the proposed steady state, we have

$$U_T = \frac{1}{1-\beta}(p\nu_{TH} + (1-p)\nu_{TL}) \tag{4}$$

for each type $T = H, L$. Since there is no divorce in equilibrium, the expected payoff U_T is independent of δ.

Proposition 1 *The steady state equilibrium with babbling and no divorce exists if and only if, for each $T = H, L$*

$$\frac{1-\beta}{\beta}\delta \geq \phi(\nu_{TH} - \nu_{TL}). \tag{5}$$

Proof. For type $T = H, L$ to keep type L in a marriage, it is necessary and sufficient that

$$\beta U_T - \delta \leq \frac{\beta}{1-\beta}\nu_{TL}.$$

Combining the above with the expression for U_T (equation 4), and noting that $p = \phi$ in the proposed steady state, we have (5). Moreover, given (4), since the match values are all positive by (1), it is optimal for agents to marry anyone they meet. ■

The condition in Proposition 1 requires the divorce cost δ to be high enough for each type $T = H, L$ not to divorce type L, given that type T expects to randomly meet an agent from the other side and engage in babbling subsequently. From the condition, it is easily seen that the steady state equilibrium with babbling and no divorce is more likely to exist if the marginal match value $\nu_{TH} - \nu_{TL}$ to any type T is sufficiently low (the additional gain from marrying a high type is low), if the discount factor β is low (the search friction is large), and the exogenous fraction ϕ of type H agents in the replacement is low (the chance of meeting a high-type agent is low).

In the special case of common match values (when equation (2) holds), condition (5) is the same for $T = H$ and $T = L$. On the other hand, under the increasing-difference assumption (when inequality (3) holds), if condition (5) holds for $T = H$, then it holds for $T = L$. In either case, the necessary and sufficient condition for a steady state equilibrium with babbling and no divorce is condition (5) for $T = H$. Of course, if inequality (3) is reversed, the necessary and sufficient condition is (5) for $T = L$. To the extent that the equilibrium with babbling and no divorce requires (5) to hold for both types, we can could say that either strong supermodularity or strong submodularity makes it harder for such equilibrium to exist.

3.2 Informative talk and divorce

In this subsection we present conditions for a steady state equilibrium in which any two agents who have just met exchange informative cheap talk messages depending on their match type, make marriage decisions based both on their type and the messages, and if they get married, divorce with positive probability.

The strategy profile in a steady state equilibrium with informative talk and divorce is as follows. We allow for two cheap talk messages, a high message ("my type is H") and a low message ("my type is L"). In the steady state equilibrium, type H always sends the high message, while type L tries to mislead by sending the high message with probability $m \in (0, 1)$. In making the marriage decision, the accept-reject decision of an agent of a fixed type depends on the message himself or herself has sent, as well as the message sent by the meeting partner. A type H agent accepts with probability one anyone who has sent the high message and rejects anyone who has sent the low message after himself or herself has sent the high message, and rejects anyone after himself or herself has sent the low message (off the equilibrium path). A type L agent accepts any agent with probability one independent of the message himself or herself has sent.[4] After marriage, the keep-divorce decision of a given type is independent of the messages exchanged prior to the marriage, and depends only on the actual type of the partner. Both types keep another agent of type H with probability one, and while type L keeps a type L partner with probability one, type H keeps a type L partner with probability $k \in (0, 1)$.

A notable feature in the above strategy profile is that type H rejects any agent who has sent the low message and thus identified himself or herself as a type-L agent, even though the former will keep someone like the latter permanently with a positive probability after the first period of marriage. The reason for this, as we show below, is that in our equilibrium the probability of finding another type-H agent in the future is high enough for type H to reject an identified low type, but is not too high to justify the divorce cost if type H is already in a marriage with a type-L agent. In contrast to type H who always sends the high message,

[4] Thus in equilibrium after sending the high message the low type marries with probability one, and after sending the low message he or she is rejected whenever the meeting partner has high type.

type L sends the high message with a positive probability, and after the message, type L accepts any agent who has sent the low message and thus identified himself or herself as a type-L agent. Thus, the two types differ both in their high-low messaging decision and in their accept-reject decision after sending the high message.

Let I_H be the steady-state payoff of a type H agent in informative talk equilibrium. In the proposed steady state, I_H satisfies

$$I_H = \frac{p}{1-\beta}\nu_{HH} + (1-p)\left(m(\nu_{HL} - \delta + \beta I_H) + (1-m)\beta I_H\right). \tag{6}$$

The above assumes that type H divorces a type L partner. Since type H is indifferent between keeping a type L partner and divorcing her or him, we have

$$\beta I_H - \delta = \frac{\beta}{1-\beta}\nu_{HL}. \tag{7}$$

Let I_L be the steady-state payoff of a type L agent. We have

$$I_L = p\left(\frac{k}{1-\beta}\nu_{LH} + (1-k)(\nu_{LH} - \delta + \beta I_L)\right) + \frac{1-p}{1-\beta}\nu_{LL}. \tag{8}$$

The above assumes that type L sends the high message. Since type L is indifferent between the two messages, we also have

$$I_L = p\beta I_L + \frac{1-p}{1-\beta}\nu_{LL}. \tag{9}$$

Note that in the proposed steady state, the expected payoff to type L is the same after meeting another type L agent, regardless of the messages they exchange. This is because they will marry with probability one and divorce with probability zero.

We first show that in any steady state equilibrium, the cost of divorce δ is higher than the payoff to each other from a temporary marriage with the other type. If $\delta \le \nu_{HL}$, type H would never reject an agent who has sent the low message for marriage; if $\delta \le \nu_{LH}$, type L would never send the low message.

Lemma 1 *A steady state equilibrium with informative talk and divorce exists only if $\delta \ge \nu_{HL}$ and $\delta > \nu_{LH}$.*

Proof. Suppose that $\delta < \nu_{HL}$. Then, given the strategies of all other agents in the proposed steady state equilibrium, consider any type H agent who accepts a type L agent who has sent the low message for marriage and then divorces this agent after one period. This deviation makes the type H agent strictly better off, implying that $\delta \geq \nu_{HL}$.

Next, suppose that $\delta \leq \nu_{LH}$. Equating the right-hand side of (8) and (9), we have

$$\frac{1}{1-\beta}\nu_{LH} \leq \beta I_L.$$

But from (9) we have

$$I_L = \frac{1-p}{(1-p\beta)(1-\beta)}\nu_{LL},$$

which contradicts the earlier expression because $\nu_{LH} > \nu_{LL}$ by (1). Thus, $\delta > \nu_{LH}$. ■

In the proposed steady state with informative talk and divorce, for any given exogenous fraction of type H in the replacement, the endogenous fraction p of type H satisfies:

$$p = p^2\phi + (1-p)^2\phi + p(1-p)(1+(2\phi-1)mk). \tag{10}$$

For any $\phi \in (0,1)$, the steady state fraction p of type H depends only on the product mk. This is because, in our proposed steady state, the probability m that type L sends the high message matters to the steady state type distribution only when type H keeps type L after marriage, and likewise, the probability k that type H keeps type L after marriage matters to the steady state type distribution only if type L sends the high message.

We now show that the necessary equilibrium conditions (6), (7), (8), (9), and (10) are also sufficient for a steady state with informative talk and divorce. Type L sends the high message with the requisite probability to make type H willing to mix between keeping and divorcing a type L partner, while type H keeps type L with the requisite probability to make type L willing to mix between the high message and the low message.

Lemma 2 *Suppose $\delta > \max\{\nu_{HL}, \nu_{LH}\}$. A steady state equilibrium with informative talk and divorce exists if there are m, k and p, all strictly between 0 and 1, satisfying (6), (7), (8), (9), and (10).*

Proof. Consider type H's strategy.
(i) For the keep-divorce decision, it is clearly optimal to keep a type H partner, and by construction, type H is indifferent between keeping a type L partner and divorcing her or him. (ii) For the accept-reject decision, if the type H agent himself or herself has sent the

high message, it is optimal for type H to accept anyone who has sent the high message, as it gives a positive probability of being permanently married to another type H agent. Now, suppose that the type H agent has sent the low message and then met someone who has sent the high message, or the type H agent has sent either message and then met someone who has sent the low message. In all these scenarios, accepting the other person means that the type H agent will be married to a type L agent. We claim that it is optimal for the type H agent to reject the other person instead. By assumption, $\delta > \nu_{HL}$. Then, the indifference condition for type H between keeping and divorcing a type L partner (equation (7)) implies that

$$\beta I_H = \delta + \frac{\beta}{1-\beta}\nu_{HL} > \frac{1}{1-\beta}\nu_{HL},$$

so that the continuation payoff from going back to the market after the rejection is higher than being permanently married to a type L agent.

(iii) For the high-low message decision, it is optimal for type H to send the high message. Sending the low message means the type H agent will be rejected by other type H agents. As we have just argued, type H should reject anyone who sends the low message. It follows that sending the low message yields the maximum payoff of βI_H instead of I_H, implying that it is optimal for the high type to send the high message.

Next, consider type L's strategy.

(i) For the keep-divorce decision, we claim that it is optimal for type L to keep type L. From the payoff after sending the low message (equation (9)), we have

$$I_L = \frac{1-p}{(1-p\beta)(1-\beta)}\nu_{LL} < \frac{1}{1-\beta}\nu_{LL}.$$

Since by assumption $\delta > 0$,

$$\beta I_L - \delta < \frac{\beta}{1-\beta}\nu_{LL}.$$

Thus, it is optimal for type L to keep type L. Given that $\nu_{LH} > \nu_{LL}$, the above implies that it is optimal to keep a type H partner as doing so gives the type L agent a positive probability of keeping type H forever.

(ii) For the accept-reject decision, regardless of the message the type L agent himself or herself has sent, after meeting another agent who has the low message and thus revealed herself or himself to be type L, it is optimal to accept the latter. This is because, as we have just argued, being permanently married to a type L agent is better than getting the continuation payoff from the market:

$$\beta I_L < I_L < \frac{1}{1-\beta}\nu_{LL}.$$

We claim that it is also optimal for type L to accept another agent who has sent the high message, regardless of the message the type L agent himself or herself has sent. Rejecting the latter agent gives the continuation payoff of βI_L. Accepting the agent when she or he is actually type L is equal to $\nu_{LL}/(1-\beta)$, which we have argued is greater than the continuation payoff βI_L. Since type L is indifferent between the two messages, from equations (8) and (9), the expected payoff after meeting an actual type H agent is the same regardless of the message the type L agent himself or herself has sent, given by

$$\frac{k}{1-\beta}\nu_{LH} + (1-k)(\nu_{LH} - \delta + \beta I_L) = \beta I_L.$$

It follows that it is optimal for type L to accept another agent who has sent the high message. (iii) For the high-low message decision, type L is indifferent by construction.

Finally, from equation (7) we have $I_H > 0$, and from (9) we have $I_L > 0$. Given that the outside option is zero, the participation condition is satisfied. ■

To provide sufficient conditions for the equilibrium to exist in terms of the parameters of the model, rewrite equation (10) as

$$p = \phi - p(1-p)(2\phi - 1)(1 - mk).$$

Define

$$\pi(\phi) = \frac{\sqrt{\phi}}{\sqrt{\phi} + \sqrt{1-\phi}}.$$

This is the unique solution strictly between 0 and 1 to equation (10) when $mk = 0$. It represents the steady state fraction of type H when replacement is minimized, as either type H always divorces type L (i.e., $k = 0$), or type H never gets to marry type L in the first place (because $m = 0$). One can easily verify that $\pi(\phi)$ increases with ϕ, with $\pi(\frac{1}{2}) = \frac{1}{2}$, and $\pi(\phi) < \phi$ if and only if $\phi > \frac{1}{2}$. For any $\phi > \frac{1}{2}$, the right-hand side of equation (10) is strictly greater than $\pi(\phi)$ for any $p \geq \pi(\phi)$ so long as $mk > 0$, as

$$\phi - p(1-p)(2\phi-1)(1-mk) > \phi - p(1-p)(2\phi - 1) \geq \phi - \pi(\phi)(1-\pi(\phi))(2\phi-1) = \pi(\phi).$$

At the same time, the right-hand side of (10) is strictly smaller than ϕ for any p. It follows that for any $mk > 0$, equation (10) maps the interval $[\pi(\phi), \phi]$ of values of p into the interval itself. By a symmetric argument, for any $\phi < \frac{1}{2}$, equation (10) maps the interval $[\phi, \pi(\phi)]$ of values of p into the interval itself. In the knife-edge case of $\phi = \frac{1}{2}$, we have $\pi(\frac{1}{2}) = \frac{1}{2}$, and the right-hand side of equation (10) is always $\frac{1}{2}$ regardless of the value of mk.

For any $mk \in (0, 1)$, there is a unique solution p to (10) bounded between ϕ and $\pi(\phi)$. If $\phi > \frac{1}{2}$, this solution increases from $\pi(\phi)$ when $mk = 0$ to ϕ when $mk = 1$; and if $\phi < \frac{1}{2}$, it decreases from $\pi(\phi)$ when $mk = 0$ to ϕ when $mk = 1$. Therefore, the steady state fraction of type H in the search population is always bounded between ϕ and $\pi(\phi)$. The interpretation is that the minimum replacement that happens when $mk = 0$ represents the greatest departure of the steady steady type distribution from the exogenous fraction ϕ of type H in the replacement. Any additional replacement of cross-type couples due to a positive product mk brings the steady state fraction of type H in the search population closer to ϕ.

Proposition 2 *There exists a steady state equilibrium with informative talk and divorce if* $\delta > \max\{\nu_{LH}, \nu_{HL}\}$,

$$\frac{1-\beta}{\beta}\delta > \max\{\phi, \pi(\phi)\}(\nu_{HH} - \nu_{HL}), \tag{11}$$

and

$$\frac{1-\beta}{\beta}\delta < \frac{\min\{\phi, \pi(\phi)\}}{1-(1-\min\{\phi, \pi(\phi)\})\beta}\nu_{HH} - \nu_{HL}. \tag{12}$$

Proof. Combining the two equations (8) and (9) for I_L, we have

$$(1-k)\delta = \left(\frac{k}{1-\beta} + 1 - k\right)\nu_{LH} - \frac{1-p}{1-\beta}\frac{k\beta}{1-p\beta}\nu_{LL}.$$

For any fixed p, both the left-hand side and the right-hand side are linear functions of k. Since $\nu_{HL} > \nu_{LL}$ by (1), the left-hand side is strictly less than the right-hand side at $k = 1$. Since by assumption $\delta > \nu_{LH}$, the reverse is true at $k = 0$. Thus, for any p, there is a unique $k \in (0, 1)$ that satisfies the above equation.

Let Δ be the difference between the right-hand side and the left-hand side of (6), given by

$$\Delta(m, p) = \frac{p}{1-\beta}\nu_{HH} + (1-p)\left(m(\nu_{HL} - \delta) + \beta I_H\right) - I_H,$$

where I_H is given by (7) which is independent of m and p. Since $\delta > \nu_{HL}$, we have that Δ is strictly decreasing in m for any $p < 1$. Thus, for any fixed $p < 1$, there is unique $m \in (0, 1)$ such that $\Delta(m, p) = 0$ if

$$\Delta(1, p) < 0 < \Delta(0, p).$$

It is straightforward to show that

$$\Delta(1, p) = \frac{p}{1-\beta}(\nu_{HH} - \nu_{HL}) - \frac{\delta}{\beta}.$$

Condition (11) implies that $\Delta(1, p) < 0$ for any $p \leq \max\{\phi, \pi(\phi)\}$. Next, we have

$$\Delta(0, p) = \frac{p}{1-\beta}\nu_{HH} - (1 - (1-p)\beta) I_H.$$

With I_H given by (7), condition (12) implies that $\Delta(0, \min\{\phi, \pi(\phi)\}) \geq 0$, and furthermore,

$$\frac{\nu_{HH}}{1-\beta} > \beta I_H.$$

Thus, $\Delta(0, p) > 0$ for all $p \geq \min\{\phi, \pi(\phi)\}$. It follows that there is unique $m \in (0, 1)$ that together with some I_H satisfies both (6) and (7) for any $p \in [\min\{\phi, \pi(\phi)\}, \max\{\phi, \pi(\phi)\}]$.

We have shown that for any fixed $p \in [\min\{\phi, \pi(\phi)\}, \max\{\phi, \pi(\phi)\}]$, there is a unique pair of values for m and k strictly between 0 and 1 such that type H is indifferent between keeping and divorcing a low-type marriage partner and type L is indifferent between the high and the low messages. Together with the fixed value of p, these values of m and k map into a new value of the steady state fraction $p' \in [\min\{\phi, \pi(\phi)\}, \max\{\phi, \pi(\phi)\}]$ of high-type agents through the right-hand side of equation (10). The mapping from p in the interval $[\min\{\phi, \pi(\phi)\}, \max\{\phi, \pi(\phi)\}]$ to $p' \in [\min\{\phi, \pi(\phi)\}, \max\{\phi, \pi(\phi)\}]$ is continuous. By Brower's Fixed Point Theorem, there is a value of $p \in [\min\{\phi, \pi(\phi)\}, \max\{\phi, \pi(\phi)\}]$ that satisfies equation (10). If $\phi = \frac{1}{2}$, then the fixed point is $\frac{1}{2}$; otherwise, the fixed point is an interior point of the interval $[\min\{\phi, \pi(\phi)\}, \max\{\phi, \pi(\phi)\}]$ because we have established the value of mk is strictly between 0 and 1 for any p on the interval. By Lemma 2, this value of p corresponds to a steady state equilibrium with informative talk and divorce. ■

Proposition 2 does not make any claim about the uniqueness of the steady state equilibrium with informative talk and divorce. That is, in a given model that satisfies the two conditions (11) and (12), there may be multiple sets of values for m, k and p for which the equilibrium conditions in Lemma 2 hold. This is the case despite the fact that, for fixed values of m and k strictly between 0 and 1, there is a unique value of p that satisfies equation (10). The reason is that the values of m and k that make the high type indifferent between keeping and divorcing a low-type marriage partner and the low type indifferent between the high and the low messages depend on p. In other words, the mapping from a given fraction p of the high type to a new fraction p' through the two indifference conditions and equation (10) constructed in the proof of Proposition 2 may have multiple fixed points.[5]

Given $\delta > \max\{\nu_{LH}, \nu_{HL}\}$, the two conditions (11) and (12) in Proposition 2, are sufficient but not necessary for the existence of a steady state equilibrium with informative talk and no divorce. From the proof of the proposition, the necessary conditions are obtained by replacing $\max\{\phi, \pi(\phi)\}$ with $\min\{\phi, \pi(\phi)\}$ in (11) and replacing $\min\{\phi, \pi(\phi)\}$ with $\max\{\phi, \pi(\phi)\}$ in (12). For $\phi = \frac{1}{2}$, we have $\max\{\phi, \pi(\phi)\} = \min\{\phi, \pi(\phi)\} = \frac{1}{2}$, and thus (11) and (12) together with $\delta > \max\{\nu_{LH}, \nu_{HL}\}$ are both necessary and sufficient for there to exist a a steady state equilibrium with informative talk and no divorce.

For an example where all sufficient conditions in Proposition 2 are satisfied, suppose that $\phi = \frac{1}{2}$ and (2) holds with ν_T denoting the common match value for each $T = H, L$. Fix any $\nu_H > \nu_L > 0$. It is straightforward to verify that there is an open interval of values of β contained in $[0, 1]$ such that

$$1 - \frac{2(1-\beta)}{\beta} < \frac{\nu_L}{\nu_H} < \frac{1}{2-\beta} - \frac{1-\beta}{\beta}.$$

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[5] The indifference condition of type H obtained from (6) and (7) determines an increasing relationship from p to m, while the indifference condition of type L from (8) and (9) determines a decreasing relationship from p to k. To see the former claim, note that (7) implies that I_H is independent of p and m, and that the right-hand side of (6) is increasing in p (this is implied by condition (12) as we show in the proof of Proposition 2) and decreasing in m (because $\nu_{HL} < \delta$). To see the latter claim, note that (9) implies that I_L is independent of k and decreasing in p, and equating (8) and (9) gives another expression of I_L that is independent of p and increasing in k (because $\nu_{LH} < \delta$). Thus, even though the solution in p to (10) is monotone in mk, there can be multiple fixed points in the mapping from p to p' through the indifference conditions of types H and L.

Then for any β on this interval, both (11) and (12) hold strictly with $\delta = \nu_H$. Then, for intermediate values of δ satisfying

$$\nu_H < \delta < \frac{\beta}{1-\beta}\left(\frac{1}{2-\beta}\nu_H - \nu_L\right),$$

all conditions in Proposition 2 are satisfied.

More generally, condition (11) parallels (5) for $T = H$. Indeed, for $\phi \geq \frac{1}{2}$, we have $\max\{\phi, \pi(\phi)\} = \phi$, so the two conditions are identical. As in Proposition 1, condition (11) ensures that when $m = 1$ so that cheap talk is uninformative, type H is unwilling to divorce type L after marriage, as babbling reduces type H's future prospect of meeting another type H from the market. On the other hand, condition (12) ensures that when $m = 0$ so that cheap talk is perfectly informative, type H strictly prefers to divorce type L if the latter deviates and sends the high message, as type H's future prospect of meeting another type H outweighs the cost of divorce. The steady state equilibrium with informative talk and babbling does not exist when $k = 0$, as this would imply that type L strictly prefers the low message given that $\delta > \nu_{LH}$.

Finally, the two equilibrium conditions (11) and (12) involve only type H's incentives to mix between keeping and divorcing a type L-partner after the first period of marriage. For fixed values of δ and ν_{LH} such that $\delta > \nu_{LH}$, strong supermodularity in match values satisfying (3) or strong submodularity satisfying the reverse of (3) will make it harder to satisfy both (11) and (12). The reason for the absence of conditions on type L's match values is that type L's equilibrium condition to mix between the high message and the low message can always be satisfied for some probability k of type H keeping the low type, as Lemma 2 shows. Broadly speaking, type L and type H play different roles in the construction of the equilibrium.

3.3 Welfare comparison

We have constructed two possible steady state equilibria, one with babbling and no divorce and one with informative talk and divorce. Condition (11) in Proposition 2 implies condition (5) for $T = H$ in Proposition 1. Thus, under either the assumption (2) of common match values or the assumption (3) of increasing differences, the same sufficient conditions in Proposition 2 imply that there exist both the steady state with babbling and no divorce and the steady state with informative talk and divorce.

In the steady state equilibrium of Proposition 1, the marriage decision is not selective, but there is no divorce. In contrast, in the steady state equilibrium of Proposition 2, type H rejects the fraction of type L agents who choose the low message so the former's marriage decision is selective. The cheap talk communication is imperfect, as the remaining fraction of type L agents send the high message and marry type H agents. But type H ends up divorcing a fraction of type L in these cross-type marriages. Thus, costly divorce impacts both type H and type L in the steady state equilibrium of Proposition 2. The following result offers a welfare comparison between the two steady states.

Proposition 3 *Suppose that there exist both a steady state equilibrium with babbling and no divorce and a steady state equilibrium with informative talk and divorce. Then,* $U_L > I_L$ *and* $U_H \leq I_H$*, with the latter holding with strict inequality when (5) is strict for* $T = H$*.*

Proof. From the payoff after sending the low message (equation (9)), we have

$$I_L = \frac{1-p}{(1-p\beta)(1-\beta)}\nu_{LL} < \frac{1}{1-\beta}\nu_{LL}.$$

By Assumption (1), we have $I_L < U_L$, where U_L is given by (4) for $T = L$.

In the steady state equilibrium with babbling and no divorce, type H weakly prefers to keep a type L partner:

$$\beta U_H - \delta \leq \frac{\beta}{1-\beta}\nu_{LH}.$$

In contrast, we have (7) in a steady state equilibrium with informative talk and divorce, as type H is indifferent between keeping and divorcing type L, Thus, $U_H \leq I_H$. The inequality is strict whenever (5) is for $T = H$. ■

In any such steady state equilibrium the divorce cost is higher than the match value to both type H and type L from a temporary cross-type marriage (Lemma 1). However, the fact that it is type H who weakly prefers to divorce type L implies that type H is better off in the steady state equilibrium with informative talk and divorce than in the steady state with babbling and no divorce. At the same time, type L is worse off even though type L does not completely lose the opportunity to marry the high type permanently due to the positive probability that the high type keeps in the low type. However, for the cheap talk to be informative, the low type has to be indifferent between masquerading as the high type and self-identifying as the low type by sending the low message, which means the low type not only will never marry the high type but also lose the one-period payoff after being rejected by the high type.

4 Further Analysis

4.1 Other steady states

We have focused on establishing the sufficient conditions on parameter values of the model for the simultaneous existence of one steady state with babbling and no divorce and another with informative talk and divorce. It is straightforward to characterize other "less interesting" steady states equilibria. Instead of providing a complete characterization of steady state equilibria for all parameter values, we highlight the class of steady state equilibria in which type L keeps a type L partner with probability one on and off the equilibrium path, which includes both the steady state with babbling and no divorce of Proposition 1 and the steady state with informative talk and divorce of Proposition 2. This class of steady states can be represented the probability m that type L sends the high message and probability k that type H keeps a type L partner. The steady state of Proposition 1 corresponds to $m = 1$ and $k = 1$; the steady state of Proposition 2 corresponds to $m \in (0, 1)$ and $k \in (0, 1)$.

When cheap talk communication is uninformative, i.e., $m = 1$, the accept-reject decision for marriage is trivial. Instead of no divorce as in Proposition 1, we have a steady state equilibrium in which type H divorces type L with probability one, i.e., $k = 0$. This requires the cost of divorce to be low enough for type H to divorce type L, but high enough for type L to keep type L. The conditions for such equilibrium are thus

$$\pi(\phi)\beta(\nu_{LH} - \nu_{LL}) \leq \delta \leq \pi(\phi)\frac{\beta}{1-\beta}(\nu_{HH} - \nu_{HL}).$$

In this steady state, all cross-type marriages result in divorce. We can also have a steady state equilibrium with babbling and type H divorcing type L with a probability strictly between 0 and 1. That is, $m = 1$ and $k \in (0, 1)$. This requires that type H to be indifferent between keeping and divorcing type L. The necessary and sufficient conditions for such an equilibrium are

$$\min\{\phi, \pi(\phi)\}\frac{\beta}{1-\beta}(\nu_{HH} - \nu_{HL}) < \delta < \max\{\phi, \pi(\phi)\}\frac{\beta}{1-\beta}(\nu_{HH} - \nu_{HL}).$$

The above conditions can be satisfied so long as $\phi \neq \frac{1}{2}$.

When cheap talk communication is perfectly informative, i.e., $m = 0$, the accept-reject decision for marriage on the equilibrium path is again trivial. Type H accepts only agents who send the high message, and type L is accepted only by those who send the low message. As in the steady state equilibrium with informative talk and divorce (Lemma 1), a necessary equilibrium condition is that $\delta > \max\{\nu_{HL}, \nu_{LH}\}$. On the equilibrium path, there is no divorce because there is no cross-type marriage. Depending on whether type H divorces type L off the path after type L sends the high message, the equilibrium conditions are different. For the steady state with divorce off the path, we require

$$\frac{1-\beta}{\beta}\delta \leq \frac{\pi(\phi)}{1-(1-\pi(\phi))\beta}\nu_{HH} - \nu_{HL}.$$

Unsurprisingly, the above is identical to (12) for $\phi \geq \frac{1}{2}$. For the steady state with no divorce off the path, we require the opposite of the above inequality.

4.2 Non-stationary equilibria

In the above steady state analysis, our goal has been to establish that, if the fraction of the high type happens to be a steady state fraction, there are probabilities respectively of the low type sending the high message and the high type keeping the low-type partner in a permanent marriage, with the associated payoffs to the high and low types, that form an equilibrium with the fraction unchanged from one period to the next. Now we ask what happens in equilibrium if the starting fraction of the high type is instead different from a steady state fraction or at least is close to it.

Denote time period by $t = 1, 2, \ldots$. In Proposition 1, we have shown that under the conditions (5), there is a steady state equilibrium with babbling and no divorce where the fraction of the high type is ϕ. Now, suppose that we start off with $p(1) \neq \phi$ in the first period. We claim that under the same conditions (5), it is an equilibrium for the low type to babble in period 1 with $m(t) = 1$, and for the high type to keep the low-type partner with probability $k(t) = 1$, followed by the stationary equilibrium with babbling and no divorce, with $p(t) = \phi$, $m(t) = 1$ and $k(t) = 1$ for all $t \geq 2$. To establish this claim, note that the transition equation for the fraction of the high type is given by adding the appropriate time indices in (10):

$$p(t+1) = p^2(t)\phi + (1 - p(t))^2\phi + p(t)(1 - p(t))(1 + (2\phi - 1)m(t)k(t)).$$

The above implies that $p(t + 1) = \phi$ for any $p(t)$ if $m(t) = k(t) = 1$ for any $t = 1, 2 \ldots$. Given that the continuation equilibrium is the steady state equilibrium with babbling and no divorce, conditions (5) then imply that $m(1) = 1$ is optimal for the low type and $k(1) = 1$ is optimal for the high type in the first period. Thus, we have a non-stationary equilibrium with babbling and no divorce, in which the equilibrium fraction of the high type jumps to ϕ regardless of the starting point $p(1)$.

Next, we consider the non-stationary counterpart of the equilibrium with informative talk and divorce we have constructed in Proposition 2. For a collection of infinite sequences $\{p(t), m(t), k(t), I_H(t), I_L(t)\}_{t=1}^{\infty}$ to be an equilibrium for a given $p(1)$, it is necessary that they satisfy the same transition equation for $p(t)$ as above, as well as the counterparts of the equilibrium conditions in Lemma 2. Specifically, the equilibrium payoff to the high type satisfies

$$I_H(t) = \frac{p(t)}{1-\beta}\nu_{HH} + (1 - p(t))\left(m(t)(\nu_{HL} - \delta + \beta I_H(t + 1)) + (1 - m(t))\beta I_H(t + 1)\right);$$

the indifference of the high type between keeping and divorcing a low-type partner requires

$$\beta I_H(t + 1) - \delta = \frac{\beta}{1-\beta}\nu_{HL};$$

the low type's payoff from sending the high message satisfies

$$I_L(t) = p(t)\left(\frac{k(t)}{1-\beta}\nu_{LH} + (1 - k(t))(\nu_{LH} - \delta + \beta I_L(t + 1))\right) + \frac{1 - p(t)}{1-\beta}\nu_{LL};$$

and the low type's payoff from sending the low message satisfies

$$I_L(t) = p(t)\beta I_L(t + 1) + \frac{1 - p(t)}{1-\beta}\nu_{LL}.$$

Let $\{p, m, k, I_H, I_L\}$ be a steady state equilibrium with informative talk and divorce as constructed in Proposition 2. Suppose that for the exogenous starting fraction $p(1)$ of the high type, there is an $m(1) \in [0,1]$ satisfying the following version of the transition of the fraction of the high type:

$$p = p^2(1)\phi + (1 - p(1))^2\phi + p(1)(1 - p(1))(1 + (2\phi - 1)m(1)k).$$

By construction, the above holds if $p(1) = p$ and $m(1) = m$, so by continuity there is an $m(1)$ in the neighborhood of m such that the above transition equation holds for $p(1)$ in a sufficiently small neighborhood of p.[6] We claim that if for any starting fraction $p(1)$ in the first period is such a neighborhood of p, there is an equilibrium in which the fraction of the high type jumps to p in the second period, and the continuation equilibrium starting in the second period is the same as $\{p, m, k, I_H, I_L\}$.

To establish the claim, note that by construction, the high type's indifference condition is satisfied with $I_H(t+1) = I_H$ for all t, because it is independent of $p(t)$. Further, the low type is indifferent between the high and the low messages if and only if

$$\frac{k(t)}{1-\beta}\nu_{LH} + (1 - k(t))(\nu_{LH} - \delta + \beta I_L(t+1)) = \beta I_L(t+1),$$

which is independent of $p(t)$, and is satisfied if we set $k(t) = k$ and $I_L(t+1) = I_L$ for all t. Since $m(1)$ enters only in the expression for the high type's equilibrium payoff at the beginning of the first period, we can use $m(1)$ as a free variable to satisfy the transition of the fraction of the high type from $p(1)$ in the first period to p in the second period. The rest of the argument follows immediately from Lemma 2.

The above construction of an equilibrium with the fraction of the high type jumps from somewhere in a neighborhood of the steady state fraction in the first period to the steady state fraction from the second period onwards suggests that we can construct a non-stationary equilibrium away from the steady state fraction of the high type that takes longer to move to the steady state. One key observation is that the equilibrium payoffs $I_H(1)$ and $I_L(1)$ to the high type and the low type respectively, as well as the probability $m(1)$ of the low type sending

[6] Indeed, in such a neighborhood, a higher $p(1)$ requires a higher $m(1)$ and a lower $p(1)$ requires a lower m, regardless of whether $\phi > \frac{1}{2}$ or $\phi < \frac{1}{2}$.

the high message in the first period, are "free" equilibrium variables in a non-stationary equilibrium. In contrast, the equilibrium payoff $I_H(t)$ to the high type for any $t \geq 2$ is pinned down by the high type's indifference condition between keeping and divorcing a low-type marriage partner. Another key issue is that, besides the above non-stationary counterparts of the equilibrium conditions in the lemma, we need to ensure that the counterpart of Lemma 2 holds for the equilibrium collection of sequences $\{p(t), m(t), k(t), I_H(t), I_L(t)\}_{t=1}^{\infty}$.

4.3 Marriage mechanisms

The outcomes of cheap talk communication between two agents who have just met on the market has no binding power on their accept-reject marriage decisions. This is the reason that in the steady state equilibrium with informative talk and divorce constructed in Proposition 2, type L will accept anyone who has sent the low message and thus identified as type L — in the steady state equilibrium type L is better off marrying another type-L agent in spite of having just made the claim to be type H. This lack of commitment power naturally raises the question of how a mechanism might use commitment to improve the steady state welfare. We assume that the mechanism designer has no control over a married couple's keep-divorce decision after the first period of the marriage. Also, the designer can not alter the random matching technology, or remember the history of marriages and divorces of any agent. All that the designer can do is to solicit reports from two agents who have just met in the market about their types, and make the joint marriage decision for them according to a pre-committed plan.

Our first step is to construct a marriage mechanism that induces an incentive compatible steady state outcome that replicates the one given by Proposition 2. By revelation principle, we need just two reports in the message space of the mechanism: I am a high type (message H), and I am a low type (message L). The joint marriage decision depends on the two messages from the two who have just met in the market. Let $\alpha^{TT'}$ be the probability of marriage if the two messages are $T, T' = \{H, L\}$. Then, if in a steady state equilibrium with informative cheap talk and divorce m represents the cheap talk strategy of the low type, k represents the divorce strategy of the high type, and p is the steady state fraction of the high type in the search population, the designer induces the same steady state by setting

$\alpha^{HH} = \alpha^{LL} = 1$, and $\alpha^{HL} = m$, and recommending type H to send message H, and type L to send L. Each type follows the respective recommendation and after the first marriage the keep-divorce decision of each type remains the same as in the steady state equilibrium with informative cheap talk and divorce.

We now show that, by adjusting α^{HL} appropriately without changing the other two joint marriage probabilities α^{HH} and α^{LL}, the designer can induce an incentive compatible steady state in which the payoff to type H is unchanged but the payoff to type L is increased compared to the steady state equilibrium $\{p_*, m_*, k_*, I_H, I_L\}$ of Proposition 2. We will first construct the steady state and then show that it is incentive compatible. In this steady state, type H remains indifferent between keeping and divorcing a type L partner, and the counterpart of (7) holds,

$$\beta W_H - \delta = \frac{\beta}{1-\beta}\nu_{HL}.$$

which pins down the expected payoff W_H for the high type in the steady state at the same level I_H as in the equilibrium with cheap talk and divorce, regardless of the values of α^{HL} and p. Given the indifference between keeping and divorcing a type L partner, in the steady state of the marriage mechanism, W_H for type H is obtained by sending message H, followed by divorcing a low-type partner. Thus,

$$W_H = \frac{p}{1-\beta}\nu_{HH} + (1-p)\left(\alpha^{HL}(\nu_{HL} - \delta) + \beta W_H\right).$$

With α^{HL} in place of m, the above is identical to (6). It follows that the mapping from p to the value of α^{HL} that keeps type H indifferent between keeping and divorcing a type L partner and $W_H = I_H$ is the same as the mapping $M(p)$ from p to m in the equilibrium in Proposition 2.

For type L, given a steady state proportion p of type H and given any α^{HL}, the expected payoff W_L from sending message L in an incentive compatible steady state satisfies

$$W_L = p\left(\alpha^{HL}\left(\frac{\kappa^{HL}}{1-\beta}\nu_{LH} + (1-\kappa^{HL})(\nu_{LH} - \delta + \beta W_L)\right) + (1-\alpha^{HL})\beta W_L\right) + (1-p)\frac{\nu_{LL}}{1-\beta},$$

where κ^{HL} is the probability that type H keeps a type-L partner after the first marriage in the steady state. Let $K(p)$ be the mapping from p to k in the steady equilibrium with informative talk and divorce, determined by type L's indifference between the two cheap talk messages. We have $K(p_*) = k_*$, and this value of k satisfies (8) and (9), and thus

$$\frac{k_*}{1-\beta}\nu_{LH} + (1-k_*)(\nu_{LH} - \delta + \beta I_L) = \beta I_L,$$

with $\beta I_L < \nu_{LL}/(1-\beta) < \nu_{LH}/(1-\beta)$. We construct a steady state with $\kappa^{HL} = k_* + \eta$ for a small positive η. Then

$$(W_L - I_L)(1 - p(1-\alpha^{HL}k_*)\beta) = p\alpha^{HL}\left(\frac{\nu_{LH}}{1-\beta} - (\nu_{LH} - \delta + \beta W_L)\right)\eta.$$

Since $\nu_{LH} \leq \delta$, for sufficiently small η, the above implies that $W_L > I_L$, regardless of the values of α^{HL} and p.

The final equilibrium condition is the counterpart of (10):

$$p = p^2 + (1-p)^2 + p(1-p)\left(1 + (2\phi - 1)\alpha^{HL}\kappa^{HL}\right).$$

As in Proposition 2, the above determines a steady value of p as a fixed point, where $\alpha^{HL} = M(p)$, and $\kappa^{HL} = k_* + \eta$. We have argued that for small η, at the steady steady $W_H = I_H$ and $W_L > I_L$. The steady state involves a value of p that differs from p_*, which then requires the designer to adjust α^{HL} from m_* to $M(p)$, and type H to keep a type L partner with probability $\kappa^{HL} = k_* + \eta$. Since by construction type H is indifferent between keeping and rejecting a type L partner, such κ^{HL} is incentive compatible.

It remains to argue that each type is willing to submit a truthful report. For type H, the expected payoff from sending message L is

$$p\left(\alpha^{HL}\frac{\nu_{HH}}{1-\beta} + (1-\alpha^{HL})\beta W_H\right) + (1-p)\left(\nu_{HL} - \delta + \beta W_H\right).$$

Since $\beta W_H < \nu_{HH}/(1-\beta)$ and $\nu_{HL} \leq \delta$, this is strictly less than W_H regardless of α^{HL} and p. For type L, incentive compatibility in a steady state requires that W_L to be greater than the expected payoff from sending message H, given by

$$p\left(\frac{\kappa^{HL}}{1-\beta}\nu_{LH} + (1-\kappa^{HL})(\nu_{LH} - \delta + \beta W_L)\right) + (1-p)\left(\alpha^{HL}\frac{\nu_{LL}}{1-\beta} + (1-\alpha^{HL})\beta W_L\right).$$

At $\eta = 0$, the above is strictly less than I_L because $\beta I_L < \nu_{LL}/(1-\beta)$. For sufficiently small η, the incentive compatibility constraint is thus satisfied for type L.

5 Extensions

5.1 Horizontal differentiation

When agents are horizontally differentiated, there is no common ranking by agents on one side of the market of different types on the other side. Retaining the assumption of two discrete types and the notation of type H and type L, under horizontal differentiation we have assumption (1) holding for $T = H$, but the reverse holds for $T = L$. Thus, a cross-type marriage is less desirable than a same-type marriage for both types of agents.

For the steady state equilibrium with babbling and no divorce constructed in Proposition 1, the change to horizontal differentiation from vertical differentiation simply means that, for the low type as well as for the high type, the divorce cost is required to be high enough to deter divorcing a high-type marriage partner instead of another low type marriage partner. To the extent that this equilibrium requirement is symmetric with respect to type and that the requirement for the high type is unchanged, horizontal differentiation does not make it more or less likely to have a steady state equilibrium with babbling and no divorce compared to vertical differentiation.

In contrast, the construction of the steady state equilibrium with informative talk and divorce given in Proposition 2 is asymmetric with respect to type. Indeed, the conditions (11) and (12) given in the proposition are both regarding the match values of the high type. Since horizontal differentiation does not change the ranking of the match values for the high type under vertical differentiation — in both cases we have $\nu_{HH} > \nu_{HL}$ — the same two conditions are also sufficient for the low type to mix between the two messages in a way to make the high type indifferent between keeping and divorcing a low-type partner after the first period of the marriage. However, while the proof of Proposition 2 makes it clear that the indifference condition for the low type between the high message and the low message can always be satisfied under vertical differentiation, additional conditions are needed when the ranking of the low type's match values is reversed to $\nu_{LH} < \nu_{LL}$ under horizontal differentiation. In particular, it is straightforward to show that if ν_{LH} is sufficiently smaller than ν_{LL}, at the steady state equilibrium with informative talk and divorce given by Proposition 2, the low type would strictly prefer the low message to the high message, regardless of the probability

k that the high type keeps the low type after the first period of the marriage.[7] Thus, "too much" horizontal differentiation makes it impossible to construct a steady state equilibrium with informative talk and divorce.

5.2 Idiosyncratic shocks

We have assumed that the match values to two agents that have met in the market are pinned down by their private match types. A more realistic assumption is that match values are subject to residual shocks given their private match types. In a similar spirit as Harsanyi (1973), these shocks may be used to "purify" the mixed messaging decision by the low type and the mixed keep-divorce decision by the high type in the steady state with informative talk and divorce.

First, imagine that, upon meeting each other in the market and before making the messaging decision, two agents each independently experiences a meeting-specific shock that affects the match value the agent receives in the first period of the resulting marriage. Such shock is temporary because it has no effect on the match value each of the two agents receives should they remain in a permanent marriage after the first period. Suppose further that the shock is independent of the types of the two agents. To be concrete, suppose that the match value in the first period of marriage is given by $\nu_{TT'} + \lambda_1\sigma_1$ to each agent, whose type is T and whose partner's type is T', with $T, T' = H, L$, where σ_1 is a real random variable with a continuous density over $[-1, 1]$ and $\lambda_1 > 0$ is a scaling factor. Since the shock is temporary, so long as λ_1 is small enough to leave the participation in the market unaffected, the steady state equilibrium with babbling and no divorce constructed in Proposition 1 stays the same. For the steady state equilibrium with informative talk and divorce in Proposition 2, since a high-type agent strictly prefers the high message to the low message if λ_1 is sufficiently small, only the low type's messaging decision will be affected. Given that in equilibrium the total probability of marrying a random meeting partner is greater after the high message than after the low message for low type, a positive realization of σ_1 makes the high message a strictly optimal choice, while at the same time a negative shock makes the low message

[7] More precisely, type L strictly prefers the low message if $\delta > \nu_{LH}$ and $\nu_{LL}(1-p)\beta > \nu_{LH}(1-p\beta)$ for $p = \min\{\phi, \pi(\phi)\}$.

strictly optimal.

Next, for the high type's mixed keep-divorce decision, consider match-specific shocks realized after the first period of marriage that affect each agent's permanent match value with the partner, independent of the types of the two agents in the marriage. Suppose that, if two married agents of any two types $T, T' = H, L$ decide to keep each other after the first period of their marriage, the permanent match value per period to the agent of type T is given by $\nu_{TT'} + \lambda_2\sigma_2$, where σ_2 is a real random variable with a continuous density over $[-1, 1]$ and $\lambda_2 > 0$ is a scaling factor. If condition (5) holds strictly for both types, then for sufficiently small λ_2, the steady state equilibrium with babbling and no divorce constructed in Proposition 1 is unaffected. For the steady state equilibrium with informative talk and divorce in Proposition 2, since all agents strictly prefer to keep a high-type partner in a permanent marriage and a low-type agent strictly prefers to keep a low-type partner, if λ_2 is sufficiently small, only the high type's keep-divorce decision regarding a low-type partner will be affected. Therefore, at the original steady state equilibrium with informative talk and divorce, a positive shock increases the high-type's payoff in a permanent marriage to the low type and makes it strictly optimal to keep the marriage, while a negative realized shock makes it strictly optimal to end the marriage.

If we have both the temporary meeting-specific shock and the permanent match-specific shock, we can follow a similar analysis to Proposition 2 to construct a steady state equilibrium with informative talk and divorce, where the messaging decision by the low type is conditioned on the realized meeting-specific shock and the mixed keep-divorce decision by the high type is conditioned on the realized match-specific shock. There is no randomization in either decision. Furthermore, for fixed positive but sufficiently small values of λ_1 and λ_2, each decision in equilibrium is represented by a threshold rule, such that the low type chooses the high message if the realized σ_1 is above some threshold s_1, and similarly the high type keeps the low-type partner if the realized σ_2 is above the corresponding threshold s_2. The two thresholds s_1 and s_2 are the counterparts of the probability m of the low type choosing the high message and the probability k of the high type keeping a low-type partner, and have similar comparative statics interpretations.

5.3 Continuous types

The model is otherwise the same as the basic model except that there is a continuum of agents' types and that the match value an agent can get from marriage only depends on the spouse's type. The main reason to adopt the match payoff function of Burdett and Coles (1997) is for simplification of the analysis. As in Shimer and Smith (2000), the presence of search frictions means that permanent marriages will be characterized by a matching correspondence that maps each type to a set of mutually acceptable types. Having pre-marriage cheap talk communication generally will not eliminate search frictions, and can further complicate the analysis.[8] The simple payoff structure makes the analysis more tractable. In particular, permanent marriages have a "class" structure that makes it easier to compare this equilibrium with the one in the basic model, where the keep-divorce decision is the same among agents of the same type and jumps discontinuously cross types.

The purpose of this subsection is to show that our main results in the two-type model are robust to continuous types. In particular, we provide sufficient conditions on the primitives of the model for the coexistence of a steady state equilibrium with informative talk and divorce and a babbling equilibrium with no divorce. Denote the exogenous replacement distribution as Φ over some interval of types $[\zeta, 1]$, with $\zeta \in [0, 1)$.[9] For a fixed steady state equilibrium, let $P(x)$ be the endogenous steady state fraction of agents whose type is weakly below x in the market.

Consider a steady state equilibrium with babbling in which everyone accepts the first marriage and chooses to keep everyone as in Proposition 1. Since all agents exit after one period, the steady state condition then implies $P(x) = \Phi(x)$ for all $x \in [\zeta, 1]$. The equilibrium payoff is the same for all types because the match value does not depend on the agent's own

[8]Strategic communication with a continuum of types between two potential marriage partners shares similar characteristics as the strategic information aggregation problem (Li, Rosen and Suen, 2000), as the marriage decision is a binary joint decision, there are no transfers, and payoffs are interdependent. The difference is that the analysis here must incorporate the future prospects of potential marriage partners from the market.

[9]We keep ζ as a free parameter instead of setting it to 0. This helps us find parameter ranges for the coexistence of an equilibrium with informative talk and divorce and a babbling equilibrium with no divorce.

type and all the agents have the same acceptance decision as well as the divorce decision. Denoting the payoff as U, we have

$$U = \int_{\zeta}^{1} \frac{x d\Phi(x)}{1-\beta}. \tag{13}$$

Since the match value increases in the spouse's type, a babbling equilibrium with no divorce exists if and only if agents are willing to keep a type-ζ partner, that is, if and only if

$$\beta U - \delta \leq \frac{\beta}{1-\beta}\zeta.$$

We write the necessary and sufficient condition for a babbling equilibrium with no divorce as

$$\frac{1-\beta}{\beta}\delta \geq \mu - \zeta, \tag{14}$$

where μ is the unconditional mean of Φ. Condition (14) is the continuous-type version of condition (5) in the basic model. In particular, the incentive to keep a type-ζ agent is stronger when divorce is more costly, when the search friction is larger or when the expected upside of going back to the marriage market is smaller because μ is small.

Now consider a steady state equilibrium with informative cheap talk and divorce that mimics the one in Proposition 2. Although informative cheap talk can generally involve many messages, we restrict to just two messages: the equilibrium involves a threhold type $s \in (z, 1)$ such that types above s send the high message and those below send the low message. Furthermore, to mimic the equilibrium in Proposition 2 in which the low message identifies the sender as the low type while the high message allows a low-type sender to pass himself or herself as the high type, we construct an equilibrium where there is a threshold type $r \in (s, 1)$ such that types above r accept only senders of the high message, while types below r accept senders of both messages. Types above r correspond to the high type in the main model, and those below correspond to the low type. Unlike in the main model where the low type randomizes between telling the truth and telling a lie, with a continuum of types messages are partially revealing and there is no randomization.

Given the matching correspondence in first marriages determined by the equilibrium cutoffs s and r, the permanent marriages are generally characterized by a class structure as in Burdett and Coles (1997). For our purpose, we consider a class structure with two classes, where there is a cutoff type $c \in (r, 1)$ such that types above c keep only types at least as high as c, and types below c keep all types but are rejected by type c and above.[10] Instead of mixing between keeping and divorcing their low-type partners in the first marriage in the main model, with continuous types those above r that have sent the high message and have been selective in their first marriages are subdivided into two groups, with types in the first class (above c) divorcing their partners in the second class (below c).

All types above c (the first class) have the same equilibrium payoff I_1, given by

$$I_1 = \int_c^1 \frac{x dP(x)}{1-\beta} + \int_s^c (x-\delta) dP(x) + \beta P(c) I_1. \tag{15}$$

The above expression follows because all types above c send the high message, accept only types above r that in equilibrium send the high message, and keep only those whose realized types are above c. The definition of c as the lowest type in the first class leads to the indifference condition of types above c between keeping type c and divorcing type c:

$$\frac{\beta c}{1-\beta} = -\delta + \beta I_1. \tag{16}$$

All types below c (the second class) also have the same equilibrium payoff I_2, even though they send different cheap talk messages and make difference accept-reject decisions in the first marriage. Those below s send the low message, and marry any type below r with no divorce, implying

$$I_2 = \int_\zeta^r \frac{x dP(x)}{1-\beta} + \beta(1 - P(r)) I_2. \tag{17}$$

Types between s and r send the high message, marry all types they meet, and are divorced by types above c, implying

$$I_2 = \int_\zeta^c \frac{x dP(x)}{1-\beta} + \int_c^1 (x-\delta) dP(x) + \beta(1 - P(c)) I_2. \tag{18}$$

[10] Formally, a class structure in permanent marriages is defined by a right-continuous step function $g(x)$ for all $x \in [\zeta, 1]$ that represents the lowest type kept by type x. With the restriction to two messages in informative cheap talk prior to first marriages, in any steady state equilibrium there are at least two classes, and type s and type r belong to the same class, that is, $g(1) > r$ and $g(r) < s$. The proof of this result is available upon request.

Finally, types between r and c send the high message, marry only types above s, and are divorced by types above c, implying

$$I_2 = \int_s^c \frac{xdP(x)}{1-\beta} + \int_c^1 (x-\delta)dP(x) + \beta(P(s) + (1-P(c))I_2. \tag{19}$$

Equations (17) and (18) together imply indifference between the two messages by all types in the second class, while equations (18) and (19) together imply indifference between accepting and rejecting agents that have sent the low message conditional on having sent the high message. Since we assume there are only two classes, we need all types below c are willing to keep type ζ:

$$\frac{\beta\zeta}{1-\beta} \geq -\delta + \beta I_2. \tag{20}$$

Given the exogenous replacement distribution $\Phi(x)$ over $[\zeta, 1]$, the endogenous distribution $P(x)$ satisfies:

$$\frac{d\Phi(x)}{dx} \int_\zeta^1 Q(y)dP(y) = \frac{dP(x)}{dx} Q(x), \tag{21}$$

where $Q(x)$ is the probability that type x forms a permanent marriage in equilibrium, given by

$$Q(x) = \begin{cases} P(r) & \text{if} \quad x \in [\zeta, s) \\ P(c) & \text{if} \quad x \in [s, r) \\ P(c) - P(s) & \text{if} \quad x \in [r, c) \\ 1 - P(c) & \text{if} \quad x \in [c, 1]. \end{cases}$$

The following proposition establishes that the results in the main model extend to continuous types: a two-message, two-class equilibrium can coexist with the babbling equilibrium with no divorce. The proof of the proposition is relegated to the appendix. We first formally present the complete strategies for each interval of types $[\zeta, s)$, $[s, r)$, $[r, c)$ and $[c, 1]$. We then show that conditions (15)-(21) are sufficient for a two-message, two-class equilibrium if they are satisfied by s, r, c, I_1, I_2 and $P(\cdot)$. The rest of the proof consists of identifying parameter values to satisfy these conditions.

Proposition 4 *There exist β, δ and $\Phi(\cdot)$ such that a steady state equilibrium with two messages and two classes coexist with the babbling equilibrium with no divorce, and furthermore, all types in the first class are better off in the steady state with two messages and two classes, and all types in the second class are worse off.*

We have included in Proposition 4 the welfare comparison by type between the steady state equilibrium with two messages and two classes and the babbling equilibrium with no divorce. Similar to Proposition 3, all types in the first class are better off in the cheap-talk equilibrium than in the babbling equilibrium, because they are indifferent between keeping and divorcing type c in the former equilibrium while they prefer to keep the low type ζ in the latter equilibrium. In contrast, types in the second class are all worse off. Their payoff can be determined by sending the low message and getting accepted only by types below r without no divorce, as types below s do. This is worse than the payoff they receive in the babbling equilibrium with no divorce, as they miss out not only types above c who are in the first class but also types on $[r, c]$ who are in the second class.

The equilibrium class structure of permanent marriages in our model is different from the class structure of Burdett and Coles (1997), which does not have either informative cheap talk or costly divorce. Despite being in the same class and having the same expected payoff, types below s and types between r and c send different messages and make exclusive accept-reject decisions in the first marriage and thus never form permanent marriages with each other. If cheap talk prior to the first marriage is uninformative, we would have the same kind of class structure in permanent marriages as in Burdett and Coles (1997), so costly divorce alone is not the reason for the difference. Without the divorce cost, however, cheap talk would not be informative, so the difference in the equilibrium class structure from Burdett and Coles (1997) is a result of the interaction between information cheap talk and costly divorce. We leave the full characterization of how dating affects marriage and class to future research.

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6 Appendix: Proof of Proposition 4

The steady state equilibrium is as follows: types on $[\zeta, s)$ send the low message, accept both messages regardless of their own message, and keep all realized types; types on $[s, r)$ send the high message, accept both messages regardless of their own message, and keep all realized types; types on $[r, c)$ send the high message, accept only the high message after sending the high message and accept both messages after sending the low message, and keep all realized types; types on $[c, 1]$ send the high message, accept only the high message after sending the high message and reject both messages after sending the low message, and keep only realized types that are least as high as c.

Proposition 4 follows immediately from Lemmas 3, 4, 5 and 6 below.

Lemma 3 *Given β, δ and $\Phi(\cdot)$, a steady state equilibrium with two messages and two classes exists if there are s, r, c, I_1, I_2 and $P(\cdot)$ with $\zeta < s < r < c < 1$ such that (15)-(21) are satisfied.*

Proof. Consider first an agent with type between $[c, 1]$. We work backwards and consider all three decisions.

(i) For the keep-divorce decision, equations (15) and (16) together imply that the agent will keep types at least as high as c and divorce lower types.

(ii) For the accept-reject decision, if the agent has sent the high message, relative to rejecting the high message, accepting it implies an additional payoff which is proportional to

$$\int_s^c (x - \delta)dP(x) + \int_c^1 \left(\frac{1}{1-\beta}x - \beta I_1\right) dP(x) = (1-\beta)I_1 > 0,$$

where the equality follows from (15) and the inequality from (16). Thus the agent strictly prefers to accept anyone who has sent the high message.

If the agent has sent the low message and later met someone who sent the high message, accepting will lead to marriage if the meeting partner's type is between s and r. Relative to rejecting, the additional payoff from accepting is proportional to $\int_s^r (x-\delta)dP(x)$. This is strictly negative, because combining (17) and (18) we have

$$\int_c^1 (x-\delta)dP(x) = \int_r^c \left(\beta I_2 - \frac{x}{1-\beta}\right) dP(x) < 0,$$

where the inequality follows because combining (18) and (19) yields

$$\int_\zeta^s \left(\beta I_2 - \frac{x}{1-\beta}\right) dP(x) = 0. \tag{22}$$

Thus the agent will reject the high message.

If the agent has sent either message and met someone who sent the low message, accepting means marrying to someone with type below s for one period and then divorce. Relative to rejecting, the additional payoff from accepting is proportional to $\int_\zeta^s (x-\delta)dP(x)$. This is strictly negative because we have just shown that $\int_s^r (x-\delta)dP(x) < 0$. Thus the agent will reject the low message.

(iii) For the high-low message decision, as shown in (ii), after sending the low message, the agent will reject everyone and hence get the payoff of βI_1. The payoff from sending the high message is I_1, which is strictly greater because $I_1 > 0$ by (16). Thus the agent will send the high message.

Now consider an agent with type between $[\zeta, c)$. Again we work backwards and consider all three decisions.

(i) For the keep-divorce decision, (20) implies that the agent is willing to keep all types.

(ii) For the accept-reject decision, if the agent has sent the high message and met someone who sent the high message, relative to rejecting, accepting yields additional payoff proportional to

$$\int_s^c \left(\frac{x}{1-\beta} - \beta I_2\right) dP(x) + \int_c^1 (x-\delta)dP(x) = (1-\beta)I_2 > 0,$$

where the equality follows from (19) and the inequality follows from (17). Thus the agent will accept the high message after sending the high message.

If the agent has sent the low message and met someone who sent the high message, relative to rejecting, accepting leads to an additional payoff proportional to

$$\int_s^r \left(\frac{x}{1-\beta} - \beta I_2\right) dP(x),$$

which is strictly positive because of (22). Thus, the agent will accept the high message after sending the low message.

(iii) For the high-low message decision, the agent is indifferent by (17) and (18). ■

Lemma 4 *There exist* $(\beta, \delta, \Phi(\cdot))$ *and* $(s, r, c, I_1, I_2, P(\cdot))$ *with* $\zeta < s < r < c < 1$ *that satisfy (15)-(21).*

Proof. For convenience, denote as $A_{x_1}^{x_2}$ the conditional average type on the interval $[x_1, x_2] \subseteq [\zeta, 1]$ under $P(\cdot)$. Eliminating I_1 through (16) and I_2 through (22), we can rewrite remaining independent sufficient conditions (15), (17), (18) and (20) as

$$(1-\beta P(c))\left(\frac{c}{1-\beta} + \frac{\delta}{\beta}\right) + (P(r) - P(s))(\delta - A_s^r) = (1-P(c))\frac{A_c^1}{1-\beta} \tag{23}$$

$$(1-P(c))(\delta - A_c^1) = (P(c) - P(r))\frac{A_r^c - A_\zeta^s}{1-\beta} \tag{24}$$

$$\left(\frac{1}{\beta} - (1-P(r))\right) A_\zeta^s = P(s)A_\zeta^s + (P(r) - P(s))A_r^s \tag{25}$$

$$\frac{A_\zeta^s}{1-\beta} \leq \frac{\beta\zeta}{1-\beta} + \delta, \tag{26}$$

together with (21). Fix any δ, c, and $(A_\zeta^s, A_s^r, A_r^c, A_c^1)$ such that

$$0 < A_\zeta^s < A_s^r < A_r^c < c < A_c^1 < \delta < 1.$$

By solving for $P(c)$, then $P(s)$, and finally $P(r)$, we can further rewrite conditions (23), (24) and (25) as

$$(1-P(c))\left(\frac{A_c^1 - c}{(1-\beta)^2} - \frac{\delta - c}{1-\beta} - \frac{(\delta - A_c^1)(\delta - A_r^c)}{A_r^c - A_\zeta^s}\right) = \frac{\delta - A_s^r}{\beta}\frac{A_\zeta^s}{A_s^r - A_\zeta^s} + \frac{c}{1-\beta} + \frac{\delta}{\beta}$$

$$P(s) = P(c) - (1-P(c))(1-\beta)\frac{\delta - A_c^1}{A_r^c - A_\zeta^s} - \frac{1-\beta}{\beta}\frac{A_\zeta^s}{A_s^r - A_\zeta^s},$$

together with (25). It is straightforward to verify from the above two equations that when β is sufficiently close to 1, we have $P(c) < 1$ and $P(s) > 0$. Then, condition (24) implies that

$P(r) < P(c)$ because $\delta > A_c^1$, and condition (25) implies that $P(s) < P(r)$. Thus, (23), (24) and (25) are satisfied by the solutions $(P(s), P(r), P(c))$ with $0 < P(s) < P(r) < P(c) < 1$. Since $A_\zeta^s < \delta$, condition (26) is satisfied if we choose ζ sufficiently close but strictly smaller than A_ζ^s. Finally, given ζ and c, for any $s \in (A_\zeta^s, A_s^r)$ and $r \in (A_s^r, A_r^c)$ we can choose the density function $dP(x)/dx$ conditional on each interval $[\zeta, s)$, $[s, r)$, $[r, c)$ and $[c, 1]$ to be consistent with A_ζ^s, A_s^r, A_r^c, and A_c^1 respectively. The desired exogenous replacement distribution $\Phi(\cdot)$ can be then found by (21). ■

Lemma 5 *There exist $(\beta, \delta, \Phi(\cdot))$ and $(s, r, c, I_1, I_2, P(\cdot))$ with $\zeta < s < r < c < 1$ that satisfy (15)-(21) and (14).*

Proof. From the proof of Lemma 4, given the values of $(A_\zeta^s, A_s^r, A_r^c, A_c^1)$ used in the proof, we only need to show that when β is sufficiently close to 1, we can choose ζ to satisfy both (16) and (14).

By (21), the unconditional mean type μ under $\Phi(\cdot)$ satisfies

$$\mu\overline{Q} = P(r)P(s)A_\zeta^s + P(c)(P(r) - P(s))A_s^r + (P(c) - P(s))(P(c) - P(r))A_r^c + (1 - P(c))^2 A_c^1, \tag{27}$$

where

$$\overline{Q} = P(r)P(s) + P(c)(P(r) - P(s)) + (P(c) - P(s))(P(c) - P(r)) + (1 - P(c))^2.$$

It is straightforward to verify from conditions (23), (24) and (25) that $P(s)$, $P(r)$ and $P(c)$ all approach 1 as β goes to 1, and thus μ approaches A_ζ^s. We have

$$\begin{aligned}
\lim_{\beta\to 1} \frac{\partial \mu}{\partial \beta} &= \lim_{\beta\to 1} \left(\frac{\partial P(r)}{\partial \beta} + \frac{\partial P(s)}{\partial \beta} - \frac{\partial \overline{Q}}{\partial \beta} \right) A_\zeta^s + \lim_{\beta\to 1} \left(\frac{\partial P(r)}{\partial \beta} - \frac{\partial P(s)}{\partial \beta} \right) A_s^r \\
&= \lim_{\beta\to 1} \left(\frac{\partial P(r)}{\partial \beta} - \frac{\partial P(s)}{\partial \beta} \right) (A_s^r - A_\zeta^s) \\
&= -\frac{A_\zeta^s}{A_s^r - A_\zeta^s} (A_s^r - A_\zeta^s) \\
&= -A_\zeta^s,
\end{aligned}$$

where the third equality comes from taking derivative with respect to β of both sides of condition (25). It follows from L'Hopital's rule that

$$\lim_{\beta\to 1} \frac{\beta\mu - A_\zeta^s}{1 - \beta} = 0.$$

Thus, when β is sufficiently close to 1, when we choose ζ to satisfy (16), condition (14) is also satisfied. ■

Lemma 6 *Suppose that for some* $(\beta, \delta, \Phi(\cdot))$ *there exist both a babbling equilibrium with payoff* U *for all types and a two-message, two-class steady state equilibrium*$(s, r, c, I_1, I_2, P(\cdot))$. *Then,* $I_1 > U > I_2$.

Proof. For types above c, conditions (16) and (14) together imply that $I_1 > U$.

For types below c, from equation (17) we have

$$I_2 = \frac{1}{1-\beta(1-P(r))} \int_{\zeta}^{r} \frac{xdP(x)}{1-\beta} < \frac{A_{\zeta}^{r}}{1-\beta}.$$

Since

$$(P(r) - P(s))(A_s^r - A_{\zeta}^r) = P(s)(A_{\zeta}^r - A_{\zeta}^s),$$

we have

$$\begin{aligned} P(c)(P(r) - P(s))A_s^r =& P(c)(P(r) - P(s))(A_s^r - A_{\zeta}^r) + P(c)(P(r) - P(s))A_{\zeta}^r \\ >& P(r)(P(r) - P(s))(A_s^r - A_{\zeta}^r) + P(c)(P(r) - P(s))A_{\zeta}^r \\ =& P(r)P(s)(A_{\zeta}^r - A_{\zeta}^s) + P(c)(P(r) - P(s))A_{\zeta}^r. \end{aligned}$$

By (27), the above implies

$$\mu\overline{Q} > (P(r)P(s) + P(c)(P(r) - P(s)))A_{\zeta}^r + (P(c) - P(s))(P(c) - P(r))A_r^c + (1 - P(c))^2 A_c^1.$$

Thus $\mu > A_{\zeta}^r$. Equation (13) implies that $I_2 < U$. ■