# Second Leg Home Advantage

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*Abstract.* When a knockout tie of two legs is played between two football (soccer) teams – home and away for each team – teams prefer away first and home second, due to a second leg home advantage. We assume that the home team has an advantage over the visiting team by attacking, and for fixed tactic adopted by the opponents, attacking by a team makes the outcome in each leg more variable than defending. We show that the second leg home advantage is derived from a greater outcome variability when the home team is attacking and the visiting team is defending than the other way around. A smaller home advantage increases rather than decreases the second leg home advantage.

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## 1 Introduction

When a knockout tie of two legs is played between two football teams – home and away for each team – teams prefer away first and home second. Indeed, in UEFA Champion's League, teams that have done better in the group stage get to play their knockout ties away first and home second.[1] This practice confirms a "second leg home advantage" that is separate but connected to the better known "home advantage." In this paper we present a theoretical study of the determinants of the second leg home advantage based on the following three hypotheses.

First, home advantage has a greater impact when the home team is attacking than when it is defending. It is widely observed that teams respond tactically to playing at home versus playing away. Indeed, for nearly 60 years in Champion's League (and the predecessor European Cup), UEFA tried to reduce the second leg home advantage by the so-called "away goal rule." This rule breaks the tie in total goals scored in the two legs between two teams in favor of the team that scored more away goals. The rationale for introducing the rule was that the home team's advantage derives mainly from attacking as opposed to defending. In fact, the most popular explanation for having the home advantage is that the home team do not need to travel, stay and train in an unfamiliar environment. Consistent with this explanation, attackers are more negatively affected than defenders by playing away compared to playing at home. As transportation becomes more convenient in the modern era, the home advantage has declined. This explains why the away goal rule was ended in the Champion's League a few years ago.

Second, attacking makes the outcome in each leg more variable than defending. Outcomes are inherently random, not least because mental and physical conditions of individual players can be unpredictable on a match day. Football being a team sport, the

[1] In the inaugural 2024-25 season of expanded 36-team UEFA Champions League, Arsenal and Barcelona were controversially forced to play the second leg of their semifinals away in spite of achieving high-ranked finishing in the league phase. In the following season, the rules were changed so that top-four finishers in the league phase are rewarded with playing the second legs of their round-of-16 ties and quarterfinals at home, and the second leg home advantage is given to top-two finishers all the way to seminals.

randomness in outcomes can be systematically affected by tactical choices by each team. Against any tactic by the opponents, when a team adopts an attacking tactic, it risks losing control and makes the outcome more variable relative to a defensive tactic. This assumption has immediate implications to tactics used by teams in the second leg. When a team is trailing after the first leg, a greater variability helps it overturn the deficit in the second leg. Everything else being the same, a trailing team therefore favors attacking. Conversely, when a team is leading, a smaller variability helps it maintain its lead and win the tie, implies that it favors defending.

Third, the outcome in each leg is more variable when the home team attacks than when the visiting team attacks. Starting with both teams using a defensive tactic, the outcome becomes more variable when the home team switches to attacking than when the visiting team switches. This asymmetry in outcome variability can arise because the home players, cheered on by the home crowd to seek "glory" on attack, are more willing to take risks than visiting players. As a result, there is more "drama" and less predicability in the outcome when the home team goes on attack than when the visiting team tries to break down home defence.

We develop a two-stage zero-sum game between two ex ante identical teams to study the second leg home advantage. The two teams make simultaneous choices between attacking and defending in the first leg. The score difference is a random variable given by a symmetric noise with zero mean and a positive constant that models the home advantage for the home team in the first leg when it chooses attacking. In the second leg the two teams conditional their simultaneous choices between attacking and defending on the first leg score difference. The final score difference is again random given by an identical and independent noise and a positive constant favoring the home team in the second leg when it chooses attacking. As mentioned above, in each leg, regardless of the tactical choice of the opponent, attacking by each team makes the noise distribution more spread out than defending. When one team attacks and the other defends, the noise distribution is more spread out if it is the home team rather than the visiting team that does the attacking.

We show that in the continuation equilibrium of the second leg, when the first leg score difference remains close relative to the home advantage, both the home team and the visiting team go on attack. The home team attacks even if it leads after the first leg, because the lead is too small to justify sacrificing the home advantage; and the visiting team attacks even if it has a small lead, because the lead is canceled by an anticipated home advantage, which makes increasing the variability in the final score difference through attacking a better tactical choice. When the first leg score difference is not close, the continuation equilibrium features the leading team chooses attacking while the other team chooses defending.

Anticipating the continuation equilibrium in the second leg, the home team in the first leg can exploit the home advantage to stochastically shift the first leg score difference in its favor by attacking the visiting team. If the first leg score difference remains close, the continuation equilibrium has both teams attacking each other. This cancels the home advantage in the two legs, but the greater variability in the final score difference benefits the home team in the first leg relative to when it attacks a defensive home team in the second leg without a home advantage. Even if the visiting team in the first leg ends up leading by a relative large score difference, it sacrifices the home advantage by defending the lead when it becomes the home team in the second leg, which again benefits the home team in the first leg. Thus, the home advantage applied sequentially to a two-leg tie favors the home team in the first leg in the absence of the variability asymmetry. This "home first advantage" is smaller when the home advantage is smaller. If there is no home advantage, then the variability asymmetry – the game outcome is more variable when the home team is the only one attacking than when the visiting team is the only one attacking – leads to a second leg home advantage. This happens because when the home team in the first leg has a large lead and defends it against attacking the home team in the second leg, a greater variability in the final score difference benefits the home team in the second leg by giving the latter a greater chance of overcoming any deficit.

By making an additional assumption on the noise distributions that neutralizes the impact of the size of variability asymmetry on tactical choices of the two teams in the first leg, we show that in equilibrium the home team in the first leg employs an attacking tactic regardless of the size of home advantage. As a result, a greater home advantage leads to a greater home first advantage, which shows up as a smaller second leg home advantage. Thus, the model predicts a time trend of gradual increases in second leg home advantage, as home advantage declines due to ease of traveling and accommodation in the modern era and standardizing of stadium facilities. Conversely, the end of the away goal rule leads to a sudden increase in home advantage, and this should show up in the data as a noticeable decrease in second leg home advantage.

## 2 Model

Two ex ante identical teams, A and B, play a knockout tie of two legs. The first leg is played at A, and the second at B. In each leg $l$, $l = 1, 2$, A and B simultaneously choose their tactics, $a_l$ and $b_l$, between two tactics, an offensive tactic denoted as $K$ and a defensive tactic denoted as $D$.

The score difference $s_l$ after each leg depends on a random noise $\epsilon_l$. We assume that $\epsilon_1$ and $\epsilon_2$ identical and independent. The distribution of $\epsilon_l$, $l = 1, 2$, depends on the tactical choices of the two teams. Let $F_{h,v}(\epsilon)$ be the distribution function of $\epsilon$ given the tactical choices $h$ by the home team and $v$ by the visiting team, and let $f_{h,v}(\epsilon)$ be the corresponding density functions.

**Assumption 1.** *Each density function $f_{h,v}(\epsilon)$ is symmetric around and single peaked at $\epsilon = 0$ for any $h, v \in \{K, D\}$, and distribution functions $F_{h,v}(\epsilon)$ are ordered by mean-preserving spread: $F_{K,K}(\epsilon) > F_{K,D}(\epsilon) > F_{D,K}(\epsilon) > F_{D,D}(\epsilon)$ for $\epsilon < 0$, and $F_{K,K}(\epsilon) < F_{K,D}(\epsilon) < F_{D,K}(\epsilon) < F_{D,D}(\epsilon)$ for $\epsilon > 0$.*

By Assumption 1, the noise has zero mean of 0 regardless of the tactical choices of the two teams. All distribution functions $F_{h,v}(\epsilon)$ pass through $\frac{1}{2}$ at $\epsilon = 0$. Fixing the opponent's tactical choice, for each team, whether playing at home or away, there is greater variability in game outcome when the team chooses $K$ than when it chooses $D$. At the same time, the noise distribution $F_{K,D}(\epsilon)$ is more spread out than $F_{D,K}(\epsilon)$: there is greater variability in game outcome when the home team is the only one choosing $K$ than when the visiting team is the only one choosing $K$. To avoid technical issues of modeling home advantage, we assume that the support of the noise is the extended real line regardless of the tactical choices of the two teams. Distributions that satisfy all these assumptions are normal distributions with zero mean, with the highest precision corresponding $(D, D)$, the lowest precision corresponding to $(K, K)$, and the middle precision corresponding to $(D, K)$ greater than the middle precision corresponding to $(K, D)$.

In the first leg, given any tactical choices of $h_1$ by team A and $v_1$ by team B, the score difference $s_1$, defined as A's score minus B's score, is a random variable given by

$$s_1 = \delta \cdot \mathbb{1}_{h_1=K} + \epsilon_1,$$

where $\delta > 0$ represents the home advantage to team A by choosing $K$, and $\epsilon_1$ is distributed according to $F_{K,v_1}$. If A chooses $D$ instead, there is no home advantage for A, with the score difference $s_1$ given by $\epsilon_1$, distributed according to $F_{D,v_1}$. We say that team A is leading after the first leg if $s_1 > 0$, and B is leading is $s_1 < 0$.

After observing $s_1$, the teams A and B simultaneously choose their tactics $v_2$ and $h_2$. The resulting final outcome of the tie is

$$s_2 = s_1 - \delta \cdot \mathbb{1}_{h_2=K} + \epsilon_2,$$

where $\delta$ is B's home advantage from choosing $K$, and $\epsilon_2$ is distributed according to $F_{h_2,v_2}$. Team A wins the tie if $s_2 \geq 0$, and B wins it if $s_2 < 0$.

To simplify the analysis, we make the following assumption.

**Assumption 2.** *As a function of $\epsilon$, $F_{K,K}(\epsilon - \delta) - F_{D,K}(\epsilon)$ crosses 0 only once at $\epsilon = s_*$.*

For $\delta = 0$, the above assumption follows directly from Assumption 1, because $F_{K,K}(\epsilon)$ crosses $F_{D,K}(\epsilon)$ at $\epsilon = 0$ from above. For $\delta > 0$, Assumption 1 implies that $s_* < 0$ in Assumption 2.

We assume each team cares only about winning. We have a two-stage zero-sum game with observed actions.

## 3 Analysis

### 3.1 Second-leg continuation

Fix any score difference $s_1$ after the first leg. Consider A's best responses first.

Against $h_2 = K$, team A chooses between $v_2 = K$ and $v_2 = D$ to maximize

$$1 - F_{K,v_2}(-s_1 + \delta) = F_{K,v_2}(s_1 - \delta),$$

where the equality follows because $f_{K,v_2}$ is symmetric around 0 by Assumption 1. Since $F_{K,D}(\epsilon)$ crosses $F_{K,K}(\epsilon)$ from below at $\epsilon = 0$ by Assumption 1, the best response of A to $h_2 = K$ is $v_2 = D$ if $s_1 > \delta$ and $v_2 = K$ if $s_1 < \delta$. That is, team A's best response to the home team B attacking is defending if after the first leg A leads B by more than B's home advantage, and is defending if the lead is less than B's home advantage.

Against $h_2 = D$, which yields no home advantage to B, team A chooses between $v_2 = K$ and $v_2 = D$ to maximize $F_{D,v_2}(s_1)$. Since $F_{D,K}(\epsilon)$ crosses $F_{D,D}(\epsilon)$ from above at $\epsilon = 0$ by Assumption 1, the best response of A to $h_2 = D$ is $v_2 = D$ if $s_1 > 0$ and $v_2 = K$ if $s_1 < 0$. That is, team A's best response to the home team B defending is defending if A leads to B after the first leg, and is attacking if B leads A.

Now, consider B's best responses. Against any $v_2 \in \{K, D\}$, B's best response is $h_2 = K$

if

$$F_{K,v_2}(s_1 - \delta) < F_{D,v_2}(s_1),$$

and $h_2 = D$ otherwise. Since $F_{K,v_2}(\epsilon)$ crosses $F_{D,v_2}(\epsilon)$ from above at $\epsilon = 0$, for all $s_1 \geq 0$,

$$F_{K,v_2}(s_1 - \delta) < F_{K,v_2}(s_1) < F_{D,v_2}(s_1).$$

Thus, B's best response to any $v_2 \in \{K, D\}$ is $K$ for any $s_1 \geq 0$. That is, when B is behind after the first leg, B's dominant strategy is $K$, because it both yields a home advantage and increases the randomness in the final score difference to the benefit of B. Assumption 2 immediately implies that B's best response to $K$ by A is $K$ if $s_1 > s_*$ and is $D$ otherwise.

Putting the above analysis together, we have the following characterization of a unique second-leg continuation equilibrium.

**Lemma 1.** *There is a unique equilibrium in a second leg with any $s_1$, with $v_2 = K$ if $s_1 < \delta$ and $D$ otherwise, and $h_2 = D$ if $s_1 < s_*$ and $K$ otherwise.*

*Proof.* The second-leg continuation game is dominance-solvable for any $s_1$. For $s_1 > \delta$, A's dominant strategy is $D$, and B's dominant strategy is $K$. For $s_1 \in (0, \delta)$, B's dominant strategy is $K$, and A's best response to $K$ is $K$. For $s_1 \in (s_*, 0)$, A's dominant strategy is $K$, and B's best response to $K$ is $K$. For $s_1 < s_*$, A's dominant strategy is $K$, and B's best response to $K$ is $D$. □

By Lemma 1, the continuation equilibrium in the second leg features one team attacking and the other defending when the result from the first leg is not close. When the first leg score difference is greater $\delta > 0$, the home team (team B) is attacking and the visiting team (A) is defending; when the first leg score difference is smaller than $s_* < 0$, team A is attacking and team B is defending.

When the result from the first leg is close, the continuation equilibrium features both teams attacking, but their rationales are different depending on whether A is leading or

B is leading. When A is leading, that is, when the first leg score difference $s_1$ is between 0 and $\delta$, B's dominant strategy is to attack because it both yields a home advantage and makes the final score different more variable, and A's best response is then to attack as well because its lead is not big enough relative to B's home advantage to protect through reducing variability. When B is leading, that is, when the first leg score difference $s_1$ is between $s_*$ and 0, A's dominant strategy is to attack because its only hope is to increase the variability of the final score difference, and B's best response is to attack as well because its own lead is not big enough to protect relative to its home advantage.

### 3.2 Home first or away first

Given the unique continuation equilibrium in the second leg characterized by Lemma 1, team A and team B simultaneously choose their first leg tactics $h_1$ and $v_1$. Consider B's best responses first.

Against $h_1 = K$, which gives a home advantage to A in the first leg, the first leg score difference is $s_1 = \epsilon_1 + \delta$, where $\epsilon_1$ is distributed according to $F_{K,v_1}(\epsilon_1)$ if B chooses $v_1 = K, D$. By Lemma 1, the probability of A winning the tie after the second leg is $F_{D,K}(s_1)$ if $s_1 < s_*$, and $F_{K,K}(s_1 - \delta)$ if $s_1 \in (s_*, 0)$, and $F_{K,D}(s_1 - \delta)$ if $s_1 > 0$. In particular, when $s_1 > s_*$, or equivalently $\epsilon_1 > s_* + \delta$, in the continuation equilibrium we have $h_2 = K$. This gives a home advantage to B and cancels the home advantage to A from the first leg in the final score difference. Thus, B chooses between $v_1 = K$ and $v_1 = D$ in the first leg against $h_1 = K$ to minimize A's probability of winning the tie, given by

$$\int_{-\infty}^{s_*-\delta} F_{D,K}(\epsilon_1 + \delta) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 + \int_{s_*-\delta}^{0} F_{K,K}(\epsilon_1) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 + \int_{0}^{+\infty} F_{K,D}(\epsilon_1) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1.$$

The above is an integral with respect to $f_{K,v_1}(\epsilon_1)$ of a continuous and increasing function, with kinks at $\epsilon_1 = s_* - \delta$ and at $\epsilon_1 = 0$, and is increasing in $\delta$.

Using symmetries in Assumption 1, we have

$$\int_{-\infty}^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,v_1}(\epsilon_1)) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 = 0, \tag{1}$$

and thus

$$\int_{-\infty}^{+\infty} F_{D,K}(\epsilon_1) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 = \int_{-\infty}^{+\infty} F_{K,v_1}(\epsilon_1) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 = \frac{1}{2}.$$

As a result, A's probability of winning the tie when B chooses any $v_1 = K, D$ against $h_1 = K$ can be rewritten as

$$\begin{aligned} &\frac{1}{2} + \int_{-\infty}^{s_*-\delta} (F_{D,K}(\epsilon_1 + \delta) - F_{D,K}(\epsilon_1)) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 + \int_{s_*-\delta}^{0} (F_{K,K}(\epsilon_1) - F_{D,K}(\epsilon_1)) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 \\ &\quad + \int_{0}^{+\infty} (F_{K,D}(\epsilon_1) - F_{D,K}(\epsilon_1)) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1. \end{aligned} \tag{2}$$

The second term in the above expression is positive, because B gives up its home advantage in the second leg to protect its large lead of $s_1 < s_*$, while the third term is also positive, because B exploits its home advantage by attacking the visiting A due to its small lead $s_1 \in (s_*, 0)$. Both the two terms disappear when $\delta = 0$ and thus $s_* = 0$. The fourth term is however negative, because the visiting team is defending its lead from attacking by the home team, which by Assumption 1 makes the game outcome more variable than when the home team is defending its lead from attacking by the visiting team. This term disappears when when $F_{K,D} = F_{D,K}$.

Next, consider B's best response against A's first leg choice of $h_1 = D$. There is no home advantage for A, and the first leg score difference is just $s_1 = \epsilon_1$, distributed according to $F_{D,v_1}(\epsilon_1)$ if B chooses $v_1 = K, D$. By Lemma 1, A's probability of winning the tie is given by

$$\int_{-\infty}^{s_*} F_{D,K}(\epsilon_1) f_{D,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 + \int_{s_*}^{\delta} F_{K,K}(\epsilon_1 - \delta) f_{D,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 + \int_{\delta}^{+\infty} F_{K,D}(\epsilon_1 - \delta) f_{D,v_1}(\epsilon_1) \mathrm{d}\epsilon_1.$$

Using a similar result as (1),

$$\int_{-\infty}^{+\infty} F_{D,K}(\epsilon_1) f_{D,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 = \frac{1}{2},$$

we can write A's probability of winning the tie by choosing $h_1 = D$ against any $v_1 = K, D$ as

$$\begin{aligned} \frac{1}{2} + &\int_{s_*}^{\delta} (F_{K,K}(\epsilon_1 - \delta) - F_{D,K}(\epsilon_1)) f_{D,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 \\ &+ \int_{\delta}^{+\infty} (F_{K,D}(\epsilon_1 - \delta) - F_{D,K}(\epsilon_1)) f_{D,v_1}(\epsilon_1) \mathrm{d}\epsilon_1. \end{aligned} \tag{3}$$

The second term in the above expression is negative, because B exploits its home advantage by attacking the visiting A due to its small lead $s_1 = \epsilon_1 \in (s_*, 0)$. The third term is also negative:

$$F_{K,D}(\epsilon_1 - \delta) < F_{K,D}(\epsilon_1) < F_{D,K}(\epsilon_1)$$

for all $\epsilon_1 > \delta > 0$, because even with $\delta = 0$ the visiting team is defending its lead from attacking by the home team, which by Assumption 1 makes the game outcome more variable than when the home team is defending its lead from attacking by the visiting team. The second term disappears when $\delta = 0$ and thus $s_* = \delta = 0$, and the third term vanishes only when both $\delta = 0$ and $F_{K,D} = F_{D,K}$.

The above analysis immediately leads to the following result. We say that there is a second leg home advantage if in any subgame perfect equilibrium A wins the tie with probability less than $\frac{1}{2}$, and there is a home first advantage if A wins the tie with probability greater than $\frac{1}{2}$.

**Proposition 1.** *There is a second leg home advantage for $\delta$ sufficiently close to 0; there is a home first advantage for $F_{K,D}$ sufficiently close to $F_{D,K}$.*

*Proof.* Team A's probability of winning the tie is less than $\frac{1}{2}$ when team A chooses $D$ in the first leg, regardless of the value of $\delta$ or whether $F_{K,D}$ is close to $F_{D,K}$, and regardless of whether team B chooses $K$ or $D$ in the first leg. When team A chooses $K$ in the first leg, its probability of winning the tie is less than $\frac{1}{2}$ when $\delta = 0$, regardless of whether team B chooses $K$ or $D$ in the first leg. By continuity, when $\delta$ is sufficiently close to 0, team A's probability of winning is less than $\frac{1}{2}$, establishing the existence of a second leg home advantage.

By choosing $K$ in the first leg, team A ensures a probability of winning the tie greater than $\frac{1}{2}$, regardless of whether B chooses $K$ or $D$, when $F_{K,D} = F_{D,K}$. By continuity, when $F_{K,D}$ is sufficiently close to $F_{D,K}$, there is a home first advantage. $\square$

The special case of $\delta = 0$, with $F_{K,D}(\epsilon) < F_{D,K}(\epsilon)$ for all $\epsilon > 0$, can be thought of as the "pure" model of second leg home advantage, while the special case of $F_{K,D}(\epsilon) = F_{D,K}(\epsilon)$ for all $\epsilon$, with $\delta > 0$, can be thought of as the "pure" model of home first advantage. Of course, when $\delta = 0$ and $F_{K,D}(\epsilon) = F_{D,K}(\epsilon)$ for all $\epsilon$, there is no advantage to either team, with each team winning with probability. Furthermore, the two teams are indifferent between the two tactics in the first leg regardless of the tactics of its opponent. The second-leg tactics, however, continue to be determined by Lemma 1.

### 3.3 First-leg tactics

Consider first the special case of $\delta = 0$. Using integration by parts, we have

$$\int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1))(f_{K,D}(\epsilon) - f_{D,K}(\epsilon))\mathrm{d}\epsilon_1 = 0. \tag{4}$$

With $s_* = \delta = 0$, condition (3) implies that $D$ is B's best response to $D$ in the first leg, anticipating the second leg continuation given by Lemma 1, if and only if

$$\int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1))f_{D,D}(\epsilon_1)\mathrm{d}\epsilon_1 > \int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1))f_{D,K}(\epsilon_1)\mathrm{d}\epsilon_1. \tag{5}$$

By (4), the above is equivalent to

$$\int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1)) f_{D,D}(\epsilon_1) \mathrm{d}\epsilon_1 > \int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1)) f_{K,D}(\epsilon_1) \mathrm{d}\epsilon_1.$$

It then follows from (2) and (3) that A's best response to $D$ is $K$ if and only if B's best response to $D$ is $D$.

By a symmetric argument, in any subgame perfect equilibrium, $D$ is B's unique best response to $K$ in the first leg if and only if

$$\int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1)) f_{K,D}(\epsilon_1) \mathrm{d}\epsilon_1 > \int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1)) f_{K,K}(\epsilon_1) \mathrm{d}\epsilon_1. \tag{6}$$

By (4), the above is equivalent to

$$\int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1)) f_{D,K}(\epsilon_1) \mathrm{d}\epsilon_1 > \int_0^{+\infty} (F_{D,K}(\epsilon_1) - F_{K,D}(\epsilon_1)) f_{K,K}(\epsilon_1) \mathrm{d}\epsilon_1.$$

Thus, $K$ is A's best response to $K$ if and only if $D$ is B's best response to $K$.

We have the following characterization of equilibrium tactics in the first leg for the special case of $\delta = 0$.

**Lemma 2.** *Suppose that $\delta = 0$. In the unique subgame perfect equilibrium, $h_1 = K$ and $v_1 = D$ if conditions (5) and (6) both hold, $h_1 = D$ and $v_1 = K$ if (5) and (6) both fail, and the two teams randomize between $K$ and $D$ otherwise.*

*Proof.* Condition (5) implies that $D$ is B's best response to $D$ and $K$ is A's best response to $D$, while condition (6) implies that $D$ is B's best response to $K$ and $K$ is A's best response to $K$. It follows that when (5) and (6) both hold, $D$ is B's dominant strategy and $K$ is A's dominant strategy. Thus, in the unique subgame perfect equilibrium, $h_1 = K$ and $v_1 = D$. Symmetrically, when (5) and (6) both fail, $K$ is B's dominant strategy and $D$ is A's dominant strategy, and in the unique subgame perfect equilibrium, $h_1 = D$ and $v_1 = K$. Finally, if condition (5) holds but condition (6) fails, B's best response is $D$ against

$D$ and $K$ against $K$, while A's best response is $K$ against $D$ and $D$ against $K$. There is no profile of first-leg tactics that forms part of a subgame perfect equilibrium, and in the unique subgame perfect equilibrium, both teams randomize. The argument is the same if condition (5) fails but condition (6) holds. $\square$

In the special case of $\delta = 0$, B's second leg home advantage arises because as in the home team in the second leg B benefits from an asymmetry in outcome variability in the second-leg continuation equilibrium. Since B attacks and A defends when A leads after the first leg and the opposite happens when B leads, the assumption that $F_{K,D}$ is more spread out than $F_{D,K}$ benefits B, both when B plays catchup and B defends a lead. Although there is also an asymmetry in outcome variability in the first leg, A does not receive the corresponding benefit because there is no lead to attack or defend.

To see that a greater outcome asymmetry leads to a greater second leg home advantage for B, define a new noise distribution function

$$\tilde{F}_{K,D}(\epsilon) = (1 - \alpha_{K,D})F_{K,D}(\epsilon) + \alpha_{K,D}F_{K,K}(\epsilon),$$

where $\alpha_{K,D} \in [0, 1]$. For any $\alpha_{K,D}$, $\tilde{F}_{K,D}$ is a legitimate distribution function and satisfies Assumption 1 and 2 together with the unchanged distributions $F_{D,K}$, $F_{D,D}$ and $F_{K,K}$. For given $F_{D,K}$, an increase in $\alpha_{K,D}$ moves $\tilde{F}_{K,D}$ closer to $F_{K,K}$, and represents a greater outcome variability. Against $h_1 = D$ in the first leg, for each choice of $v_1 = K, D$, the derivative with respect to $\alpha_{K,D}$ of B's probability of winning, given by the left-hand side of (5) for $v_1 = D$ and by the right-hand side for $v_1 = K$, is

$$\int_0^{+\infty} (F_{K,D}(\epsilon_1) - F_{K,K}(\epsilon_1))f_{D,v_1}(\epsilon_1)\mathrm{d}\epsilon_1 > 0.$$

Against $h_1 = K$, using (4), the derivative with respect to $\alpha_{K,D}$ of B's probability of winning, given by (7), is

$$\int_0^{+\infty} (F_{K,D}(\epsilon_1) - F_{K,K}(\epsilon_1)) f_{v_1,K}(\epsilon_1) \mathrm{d}\epsilon_1 > 0.$$

Thus, regardless equilibrium tactics used by A and B in the first leg, moving $F_{K,D}$ closer to $F_{K,K}$ increases B's second leg home advantage.

Similarly, we can define another noise distribution

$$\tilde{F}_{D,K}(\epsilon) = (1 - \alpha_{D,K}) F_{D,K}(\epsilon) + \alpha_{D,K} F_{D,D}(\epsilon)$$

for $\alpha_{D,K} \in [0,1]$. An increase in $\alpha_{D,K}$ moves $\tilde{F}_{D,K}$ closer to $F_{D,D}$, and also represents a greater outcome variability. Against $h_1 = D$ in the first leg, for each $v_1 = K, D$, the derivative with respect to $\alpha_{D,K}$ of B's probability of winning is

$$\int_0^{+\infty} (F_{D,D}(\epsilon_1) - F_{D,K}(\epsilon_1)) f_{v_1,D}(\epsilon_1) \mathrm{d}\epsilon_1 > 0.$$

Against $h_1 = K$, the derivative with respect to $\alpha_{D,K}$ of B's probability of winning, is

$$\int_0^{+\infty} (F_{D,D}(\epsilon_1) - F_{D,K}(\epsilon_1)) f_{K,v_1}(\epsilon_1) \mathrm{d}\epsilon_1 > 0.$$

Thus, moving $F_{D,K}$ closer to $F_{D,D}$ also increases B's second leg home advantage.

We summarize the above comparative statics results formally below.

**Corollary 1.** *Suppose that $\delta = 0$. Moving $F_{K,D}$ closer to $F_{K,K}$ or $F_{D,K}$ closer to $F_{D,D}$ increases the second leg home advantage.*

To characterize equilibrium tactics in the first leg beyond the special case of $\delta = 0$, we assume that both $F_{K,D}$ and $F_{D,K}$ are convex combinations of the most concentrated noise distribution $F_{D,D}$ and the most spread out distribution $F_{K,K}$. That is,

$$F_{K,D}(\epsilon) = (1-\beta_{K,D})F_{K,K}(\epsilon) + \beta_{K,D}F_{D,D}(\epsilon),$$
$$F_{D,K}(\epsilon) = (1-\beta_{D,K})F_{K,K}(\epsilon) + \beta_{D,K}F_{D,D}(\epsilon)$$

for all $\epsilon$. By Assumption 1, we need $0 < \beta_{K,D} < \beta_{D,K} < 1$. The corresponding density functions $f_{K,D}$ and $f_{D,K}$ are also convex combinations of $f_{D,D}$ and $f_{K,K}$ with the same weights. As a result, all density functions intersect at the same points. The assumption of convex combinations "neutralizes" strategic interactions between the two teams in the first leg. More precisely, both conditions (5) and (6) become equalities, so that both A and B are indifferent between $K$ and $D$ regardless the tactical choice of their opponent: for (5), we have

$$\begin{aligned}&\int_0^{+\infty}(F_{D,K}(\epsilon_1)-F_{K,D}(\epsilon_1))(f_{D,D}(\epsilon_1)-f_{D,K}(\epsilon_1))\mathrm{d}\epsilon_1\\ =&(\beta_{D,K}-\beta_{K,D})(F_{D,D}(\epsilon_1)-F_{K,K}(\epsilon_1))(1-\beta_{D,K})(f_{D,D}(\epsilon_1)-f_{K,K}(\epsilon_1))\mathrm{d}\epsilon_1,\end{aligned}$$

which is equal to 0 using a similar symmetric result as (4), and similarly for (6).

Under the assumption of convex combinations, we immediately have the following characterization of the equilibrium tactic of the home team.

**Proposition 2.** *Suppose that $F_{K,D}$ and $F_{D,K}$ are convex combinations of $F_{D,D}$ and $F_{K,K}$. In any subgame perfect equilibrium, $h_1 = K$.*

*Proof.* Against any $v_1 = \{K, D\}$, the payoff difference for A between $h_1 = K$ and $h_1 = D$ in the first leg is given by the difference between (2) and (3). Under the assumption of convex combinations, the payoff difference is 0 when $\delta = 0$. The derivative of the payoff difference with respect to $\delta$ is

$$\int_{-\infty}^{s_*-\delta} f_{D,K}(\epsilon_1+\delta)f_{K,v_1}(\epsilon_1)\mathrm{d}\epsilon_1 + \int_{s_*}^{\delta} f_{K,K}(\epsilon_1-\delta)f_{D,v_1}(\epsilon_1)\mathrm{d}\epsilon_1 + \int_{\delta}^{+\infty} f_{K,D}(\epsilon_1-\delta)f_{D,v_1}(\epsilon_1)\mathrm{d}\epsilon_1,$$

which is strictly positive. Thus, for any $\delta > 0$, A's dominant strategy in the first leg is $K$. The proposition follows. $\square$

The assumption of convex combinations is of course restrictive. However, even without the assumption, A's payoff difference between attacking and defending against any fixed tactic of B is strictly increasing in $\delta$, so that the conclusion of Proposition 2 is likely to hold with the restrictive assumption for sufficiently large home advantage. That is, a greater home advantage gives A a greater incentive to choose attacking instead of defending. Indeed, for any $v_1$, an increase in $\delta$ increases A's payoff from attacking and simultaneously decreases A's payoff from defending. This is because, as shown in Proposition 1, by choosing attacking instead of defending, A not only exploits the home advantage in the first leg but also increases the variability of first leg score difference. A greater home advantage reinforces the advantage of choosing attack in the first leg for A.

Proposition 2 allows us to conclude that a greater home advantage $\delta$ unambiguously increases A's payoff. We have yet to characterize B's equilibrium tactic in the first leg, but attacking is A's dominant strategy in the first leg, and A's payoff increases with $\delta$ regardless of B's choice of $v_1$. This implies that a greater home advantage reduces the second leg home advantage. This may seem paradoxical but recall that home advantage is the source of home first advantage and therefore a source of second leg home "disadvantage." We formally state this result below.

**Corollary 2.** *Suppose that $F_{K,D}$ and $F_{D,K}$ are convex combinations of $F_{D,D}$ and $F_{K,K}$. A greater home advantage reduces the second leg home advantage.*

Finally, consider B's equilibrium tactic in the first leg. By Proposition 2, we will focus on the case where $h_1 = K$. Under the assumption of convex combinations, we have

$$\int_0^{+\infty} (F_{K,D}(\epsilon_1) - F_{D,K}(\epsilon_1))(f_{K,K}(\epsilon_1) - f_{K,D}(\epsilon_1))\mathrm{d}\epsilon_1 = 0.$$

and

$$\int_{-\infty}^{0}(F_{K,K}(\epsilon_1)-F_{D,K}(\epsilon_1))(f_{K,K}(\epsilon_1)-f_{K,D}(\epsilon_1))\mathrm{d}\epsilon_1=0.$$

It then follows from (2) that B's response to $h_1 = K$ is $K$ in the first leg if and only if

$$\int_{-\infty}^{s_*-\delta}(F_{K,K}(\epsilon_1)-F_{D,K}(\epsilon_1+\delta))(f_{K,K}(\epsilon_1)-f_{K,D}(\epsilon_1))\mathrm{d}\epsilon_1>0. \tag{7}$$

We provide sufficient conditions on the home advantage $\delta$ for condition (7) to hold in the result below.

**Proposition 3.** *Suppose that $F_{K,D}$ and $F_{D,K}$ are convex combinations of $F_{D,D}$ and $F_{K,K}$. In the unique subgame perfect equilibrium, $v_1 = K$ if $\delta$ is sufficiently small or sufficiently large.*

*Proof.* For $\delta = 0$, we have $s_* = 0$, and by the assumption convex combinations, the left-hand side of condition (7) is 0. The derivative with respect to $\delta$ at $\delta = 0$ is given by

$$\int_{-\infty}^{0}f_{D,K}(\epsilon_1)(f_{K,D}(\epsilon_1)-f_{K,K}(\epsilon_1))\mathrm{d}\epsilon_1=\int_{-\infty}^{0}(F_{K,K}(\epsilon_1)-F_{K,D}(\epsilon_1))f'_{D,K}(\epsilon)\mathrm{d}\epsilon_1>0,$$

where the equality follows from integration by parts, and the inequality follows because by Assumption 1, $F_{K,K}(\epsilon_1) > F_{K,D}(\epsilon_1)$ and $f'_{D,K}(\epsilon) > 0$ for all $\epsilon_1 < 0$. Thus, for sufficiently small $\epsilon$, condition (7) is satisfied.

By Assumption 2, for all $\epsilon_1 < s_* - \delta$ we have

$$F_{K,K}(\epsilon_1) > F_{D,K}(\epsilon_1+\delta).$$

Thus, a sufficient condition for condition (7) is that

$$s_* - \delta < -\hat{\epsilon}$$

where by Assumption 1, we can find $\hat{\epsilon} > 0$ such that $f_{K,K}(\epsilon_1) \geq f_{D,K}(\epsilon_1)$ for all $\epsilon_1 \leq -\hat{\epsilon}$ with equality if and only if $\epsilon_1 = -\hat{\epsilon}$. This sufficient condition is satisfied for all $\delta > \hat{\epsilon}$. $\square$

The proof of Proposition 3 makes it clear that condition (7) can hold for all $\delta$. All we need is that, against $h_1 = K$ in the first leg, the derivative with respect to $\delta$ of the difference in B's winning probabilities between attacking and defending, represented by the left-hand side of (7), can change sign only once. That is, the sign of

$$\int_{-\infty}^{s_*-\delta} f_{D,K}(\epsilon_1+\delta)(f_{K,D}(\epsilon_1)-f_{K,K}(\epsilon_1))\mathrm{d}\epsilon_1$$

can only change from positive to negative as $\delta$ increases. Although we cannot reduce this condition further by imposing additional assumptions on the noise distributions, it is relatively easy to verify for specific examples of distributions.

## 4 Empirics

Empirical implications of the model are based on two hypotheses. They can be used for comparative statics analysis. The model can be tested in this way.

First, there has been gradual decline in home advantage across football leagues. While the home team is still favored to win a given game, the difference in winning probability between the home team and the visiting team has decreased steadily.[2] This declining home advantage carries over knockout stages in cup competitions such as the UEFA Champions League. In the model, this can be taken as steady decreases in the parameter $\delta$ in two-leg ties. By Corollary 2, we should expect to see that over time the home first advantage decreases, or equivalently, the second leg home advantage increases. That is, the model predicts that in knockout stages in the UEFA Champions League, teams playing at home in the second leg win with probabilities that have increased in recent years. Of course, since the decline is gradual, one has to identify a time trend in this second leg home advantage. It is also necessary to control for qualities of teams in competition and other factors that may the time trend.

Second, the end of the away goal rule led to a sudden increase in home advantage. In

[2]See a data analysis for the English leagues in https://blog.engora.com/2025/07/vanishing-home-field-advantage-in.html.

the UEFA Champions League, the away goal rule was abolished in the 2021-2022 season. Unlike the graduate decline in home advantage, this change was sudden, and should have noticeable negative impact on the second leg home advantage.

## 5 Discussion

Our model has completely ignored the fact that football is a low scoring sport. By adopting a model where the score difference is a continuous variable, we have assumed away the possibility of draws, and penalty shootouts after extra time in the second leg. One prominent explanation of second leg home advantage is that the home team in the second leg gets to play the extra time and shootouts in front of their own supporters.[3] This may well be correct, but is a constant factor that does not affect either the time trend of declining home advantage or the end of the away goal rule. The predictive power of our model is based on comparative statics that would be invalidated by the importance extra time and penalty shootouts in football.

We have also assumed away effort choices by teams and focused on tactics. There is a long economics literature on tournaments starting from Lazear and Rosen (1981) where effort choices are paramount.[4] However, it is not empirically clear that effort choices are of first-order importance in high-stake games like UEFA Champions League ties.

Finally, in our model tactical choices by teams are made once-for-all for this leg. Of course teams do make in-game adjustments of tactics. However, such adjustments are often made as Plan B, in response to game changing events such as red cards and injuries to players central to chosen tactics. As a result, in-game adjustments can be costly, and may affect neither our explanation of the source of second leg home advantage nor matching predications of the model to empirical evidence.

[3]See Jost (2024) for an analysis of second leg home advantage based on the tie-breaking rule.

[4]See Chan, Courty and Li (2009) for a dynamic extension of the model that explains the use of rank order as opposed to piece rate in rewarding winners of sports competition.

## References

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