Private Career Concerns

Li, Hao & Wei Li
University of British Columbia

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Abstract

In a two-period career concerns model, an agent does not know his own ability but has private information about how much he cares about his second period wage. A first-period principal can offer a menu of wage contracts to screen agent types with high or low marginal utility for the second-period wage from the market that pays the agent his expected ability. We characterize the optimal pooling contract for the first-period principal. The comparison of the optimal pooling with the benchmark where the agent's career-concerns type is public depends on whether ability and effort are complements or substitutes. We show that there is no menu that strictly separates the two types; further, even though full separation is possible, it is never more profitable than the optimal pooling. However, it can be profitable for the first-period principal to use one contract to partially separate out the less career-concerned type.

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1 Introduction

Agents with unknown ability often face implicit incentives provided by the market, especially in careers where ability is crucial to one's success. The more they are perceived to have high ability, the higher are their future wages (or other career rewards). Thus all agents have incentives to work hard to influence the market's perception of their ability. However, while agents work harder, their higher effort is anticipated and discounted in equilibrium. The market forms correct posterior beliefs of the agents when the ability is symmetric information between agents and the market, regardless of whether the information is complete or incomplete, that is, whether agents and the market know the ability of the former or not.

The above paradigm is well-established since Holmstrom (1982).[1] In this paper, we depart from the above paradigm of implicit incentives by assuming that agents have private knowledge about how much they care about their future careers, modeled by their marginal utility of future income. Everything else being equal, one who cares more about his future career is willing to work harder to convince the market she has high ability. This gives rise to a novel problem: how could a principal design wage contracts optimally to take advantage of an agent' desire to impress the market? Unless the principal offers a single contract to all types of career concerns, an agent's choice from the menu can reveal some information about the agent's type of career concerns. That is, the contract chosen by the agent, in addition to the realized output as in the existing literature, can affect the market's expectation of his ability even though the market is not directly interested in the agent's type of career concerns. Such screening effects are moreover endogenously determined as they result from profit maximization by the principal that currently employs the agent.

More specifically, we consider a two-period model in which a principal and an agent have incomplete but symmetric information about whether the agent has high ability or not. The second period is perfectly competitive: the agent is paid the market's posterior belief that she has high ability, conditional on whether she succeeded or failed in the first period and

[1] The literature in this paradigm has assumed competitive labor market and focused on the design of contracts to maximize the agent's utility. For instance, in Gibbons and Murphy (1992) the objective is to optimally insure risk averse agents.

which contract she was under. The agent has private information about his marginal utility for money in the second period, which takes on one of two values, modeling a high career-concerns type and a low career-concerns type. In the first period, the agent either makes a costly effort ("works"), which increases the probability of success for any underlying level of his ability at a fixed cost, or makes no effort at zero cost ("shirks"), with the probability of working representing a greater effort. The principal chooses a menu of contracts to maximize his first period profit, where, under limited liability, each contract specifies a base wage that is paid regardless of the outcome and a bonus that is paid only when the agent succeeds. We stress that in our model the principal is *not* trying to screen the agent for future purpose. Any improved estimate of an agent's ability is completely captured by the agent since the second period is perfectly competitive. Rather, the principal only wants to maximize her current profit by making use of the agent's implicit incentives, especially those who care more about their careers because they respond more strongly to any explicit incentives offered. As for the agent, we assume that the explicit incentives from a first-period wage contract and the implicit incentives from the second-period market wage enter his utility function as perfect substitutes.[2]

Our results on the optimal menu depend on a critical feature of the underlying production function. We call ability and effort "complements" if the ratio of the success probability of the high-ability agent over that of the low-ability one increases in effort; and "substitutes" if the ratio of the failure probability of the high-ability agent over that of the low-ability agent increases in effort. Thus, in the complements case, the probability of success increases faster for the high-ability agent than for the low-ability one as the agent works harder. As a result, success is increasingly a better signal of a high-ability agent, further increasing an agent's implicit incentives to work. In the opposite substitutes case, the probability of success increases slower for the high-ability agent than for the low-ability one as the agent works harder. As effort increases, success becomes a less convincing signal of ability. More of the success is attributed to hard work than to ability, which reduces the agent's implicit incentives.

[2]This assumption simplifies interactions between explicit and implicit incentives. It is a standard one in the existing reputation literature. See, e.g., Morris (2001), Koszegi and Li (2008).

We consider first the benchmark case where the agent's career-concerns type is publicly observed. For any given explicit incentives the principal offers, an agent works harder if he values his future career more. As in the existing literature referenced above, this effort is correctly anticipated and discounted by the market. Unsurprisingly, the optimal explicit incentives, which is just a bonus when the agent succeeds, decrease in the agent's type, and the principal's profit increases in the agent's type. However, whether or not the principal wants to induce a higher effort for a higher career-concerns type depends on whether effort and ability are complements or substitutes. In the complements case, the agent's implicit incentives increase in the market's expected effort.[3] Since the probability of success increases in the agent's effort, the higher is the market's expected effort, the more the agent is paid through his implicit incentives. As a result, it is cheaper for the principal to induce the agent to work harder, and the principal's maximum profit is increasing and convex in the effort induced. In the substitutes case, however, the principal faces a tradeoff. While the expected probability of success increases in the agent's effort, a higher expected effort reduces the agent's implicit incentives, and thus requires more explicit incentives from the principal. We show that the principal's profit is concave in the induced effort of the agent.

In the contract design problem under private career concerns, we first characterize the optimal pooling contract. With two possible career-concerns types, the more concerned type—the one with a greater marginal utility for money in the second period — exerts a greater effort in any equilibrium under a pooling contract. Since it is cheaper to induce a given effort level by a more concerned type, there is a cutoff belief about the agent's career-concerned type such that it is optimal for the principal to target the less concerned type for the provision of explicit incentives, if the belief that the agent is less concerned exceeds the cutoff. In this case, since the more concerned type exerts a greater effort in any equilibrium, the principal obtains the same expected profit as in the benchmark when the agent is known to be the less concerned type, regardless of whether ability and effort are complements or

[3]For some contracts there may exist multiple equilibria in which the market's expectation of the agent's effort coincides with the agent's choice, similar to the results in Dewatripont, Jewitt, and Tirole (1999a,b). We show that whenever the principal wants to induce any effort, there exists another equilibrium in which she can induce the agent to choose the maximum effort.

substitutes. In the opposite case when the agent is more likely to be more concerned type relative to the cutoff belief, generally the principal fails to obtain the same expected profit as in the benchmark when the agent is known to be the more concern type, exactly because the less concerned type exerts a smaller effort in any equilibrium. The principal targets the more concern type for the provision of explicit incentives, allowing the less concerned type to free ride. The profit from the optimal pooling is decreasing in the belief that the agent is the less concerned type, as the induced average effort from the two types is lower. When ability and effort are complements, both career-concerns types become better off as a lower average effort leads to weaker implicit incentives and hence stronger explicit incentives. The opposite is true when ability and effort are substitutes.

Our characterization of the optimal pooling naturally leads to the question of whether the first-period principal can do better by separating the two private types of career concerns. We first show that it is impossible to strictly separate the two types, so that each type chooses a different contract from the menu and at least one type strictly prefers his contract to the other one. This may appear counter-intuitive: we have seen that in the benchmark of public career concerns, for the same explicit incentives the more career-concerned type is willing to work harder, which suggests a standard single-crossing property required for successful screening and signaling. However, in any separating equilibrium, the market correctly anticipates each type's effort, and each type's equilibrium expected wage in the second period is just the same common prior belief about his ability. Thus, no type can do strictly worse in terms of the second period wage by deviating to another type's contract and imitating that type's effort choice, than by choosing his own equilibrium contract and making his own equilibrium effort choice. As a result, in a separating equilibrium, implicit incentives cannot be used to screen different types of career concerns. The only possible full separation happens when both types are indifferent between the explicit incentives provided by the two contracts, and is therefore not strict. Furthermore, we show that this kind of full separation is never more profitable than the optimal pooling.

We also consider semi-separation where one of the two career-concerns types randomizes between separation by choosing one contract on the menu and pooling with the other type by choosing the other contract. It may appear that the potentially profitable semi-separation for the principal is where the more concerned type randomizes between a separating contract and a mixing contract, as this would allow the type that the principal prefers under public career concerns to be partially separated out. However, we show that such semi-separation is never more profitable than the optimal pooling, regardless of whether ability and effort are complements or substitutes. The separating contract on its own can be more profitable than the optimal pooling as it is chosen only by the more concern type, while the mixing contract is necessarily less profitable than the optimal pooling because, having chosen the mixing contract the agent is more likely to be the less concerned type than the prior belief. We show that the average profit from the two contracts is always less than the optimal pooling profit. For the case of substitutes, this result comes from concavity of the profit function in the effort level in the benchmark case where the agent is known to be the concerned type. For the case of complements, where the the profit function in the benchmark case is instead convex, we show that the incentive condition for the more concerned type to be indifferent between the separating contract and the mixing contract limits how much more profitable the separating contract can be than the mixing contract. The result is that no semi-separation where the more concerned type is partially separated out is more profitable than the optimal pooling.

We show by construction that the other kind of semi-separation, where the less concerned type is partially separated out, can be more profitable than the optimal pooling, regardless of whether ability and effort are complements or substitutes. The key to this construction is that the prior belief that the agent is the less concerned type exceeds the cutoff given in the benchmark of public career concerns. By our analysis of the optimal pooling, the best the principal could do is to target the less concerned type for provision of the explicit incentives and obtain the same profit as in the benchmark where the agent is known to be the less concerned type. This implies that from the separating contract chosen only by the less concerned type, the principal obtains the same profit as the optimal pooling. By making the less concerned type indifferent between the separating contract and a mixing contract where the less concerned type mixes with the more concerned type, which then allows the principal to control how the less concerned type randomizes, the principal can switch the target to the more concerned type in provision of the explicit incentive. We show through examples that the profit from the mixing contract can exceed the profit in the benchmark where the agent is known to be the less concerned type. Such semi-separation is therefore more profitable than the optimal pooling.

2 Model

2.1 A reputation model with private career concerns

There are two time periods, period one and period two. A principal (she) contracts with an agent (he) in period one. Both the principal and the agent are risk-neutral. For simplicity, we assume that the agent's payoff from the two periods is simply the sum of his first period payoff and the second period payoff, with no discounting.

Once the agent accepts the contract, to be specified in detail below, he chooses either to work at the cost $k > 0$ or to shirk at zero cost. In our analysis we will allow the agent to randomize, and so we denote as $e \in [0, 1]$ the probability of choosing to work. We often refer to $e = 1$ and $e = 0$ as "full effort" and "zero effort" respectively, and $e \in (0, 1)$ as "partial effort." In period one, the agent may succeed in producing some output ($y = s$) or fail ($y = f$). The probability of success depends on the agent's unobservable ability a and his private effort choice. We assume that the agent's ability is either high ($a = h$) or low ($a = l$), with the common prior belief $r \in (0, 1)$ of $a = h$ ("prior reputation") among the principal, the agent, and the period-two market. In our model, the production technology is completely characterized by the probability of success p_{ae} when the agent with ability $a = h, l$ makes the effort choice $e = 0, 1$. We assume that: for each $a = h, l$,

$$p_{a1} \geq p_{a0},$$

with at least one holding strictly. As a result, we have

$$p_1 \equiv rp_{h1} + (1 - r)p_{l1} > rp_{h0} + (1 - r)p_{l0} \equiv p_0.$$

Further, for each $e = 0, 1$,

$$p_{he} \geq p_{le},$$

with at least one holding strictly. Thus, the probability of success is increasing in ability and effort levels. Define

$$\lambda \equiv p_1 - p_0,$$

which is the increase in the probability of success when the agent switches from $e = 0$ to $e = 1$, i.e., the expected productivity of effort. By assumption, $p_{a1} \geq p_{a0}$ for each $a = h, l$ with at least one strictly inequality, and $r \in (0, 1)$, and so $\lambda > 0$.

In period two, the agent faces a competitive market. The market observes the set of contracts offered by the principal, the contract chosen by the agent, and the realized output in period one, without of course observing his effort choice, and computes the posterior belief r' of $a = h$ ("posterior reputation"). The agent then receives a payoff of tr', where $t \geq 0$ is the agent's marginal utility for period-two income, or the career concerns type. We assume that the value of t is private information to the agent, representing his private career concerns. Moreover, t takes two possible values, m ("motivated") and n ("not motivated"), with $m > n \geq 0$. Let $\eta \in [0, 1]$ be the prior belief that $t = n$.[4]

Now we describe in detail contracts that the principal may offer in the first period. We assume that the agent has limited liability in that any payment to the agent must be non-negative. Given that the output is binary, a contract is simply a base wage $w \geq 0$ that is made to the agent regardless of the output, and a bonus payment $b \geq 0$ that is made only upon success. The value of success to the principal is normalized to 1, so we also have $b \leq 1$. Since there are two possible private career-concerns types, we restrict the principal to two contracts when a menu is offered. Finally, if the agent rejects the offers made by the principal, both the agent and the principal receive 0.[5]

The timeline of our model is illustrated in Figure 1.

One important assumption we have made is that in period one the principal has all the

[4] With a continuum of types of career concerns, the agent's equilibrium behavior generally depends on the distribution of t (and the conditional distribution if some types choose the same contract). This complicates the analysis without yielding more insight about how the monopoly principal can best profit from the agent's career-oriented implicit incentives.

[5] Together with the assumption of limited liability, this ensures that the agent will always participate by choosing a contract, and thus eliminates the need to consider how the market might compute the posterior reputation if the agent had rejected all contract offers.

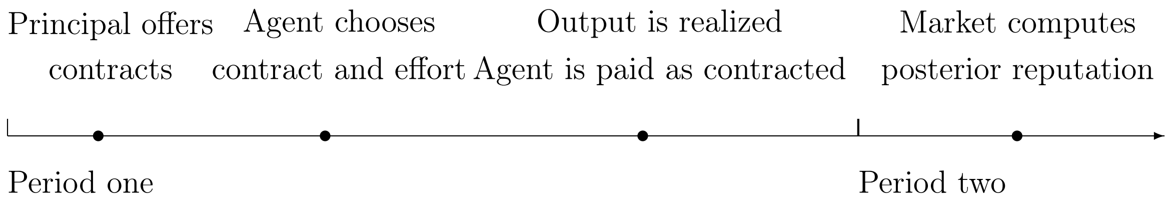

Figure 1: Timeline

bargaining power over the agent. Contrasting with this assumption, we have also assumed that in period two the agent faces a competitive market so we abstract away from any further moral hazard problems. The combination of these two assumptions allows us to consider the issue of contract design in a standard career-concerns model. The principal wishes to exploit the implicit incentives faced by the agent with career concerns in designing explicit wage-output incentives to maximize her profits in the first period. However, because career concerns are private, the principal must cope with the agent's incentive to use his contract choice for signaling to the second period market.

Even though career concerns are modeled in terms of marginal utility of money, they are intended as a stand-in for the agent's intrinsic valuation of the rewards or recognition his career may bring. Alternatively, the marginal utility of money may represent the length of the agent's planned career. For instance, a value of zero means that the agent plans to quit the labor market at the end of period one. It is well-documented that changes in people's personal life circumstances may affect how much they value their career achievements; also, such commitment to careers may differ at different stages of professional development and across organizations. For instance, elite physicists' commitment to their careers increase through time while the others decrease.[6] Observe from the agent's payoff function that, due to the heterogeneity in marginal utility, the agents respond to the same wage contract *differently*: those who value that wage more will work harder for it. This implies that the contract chosen in the first period can be a signal of career commitment, and will be taken into account by the market in evaluating an agent's ability.

[6]Hermanowicz (2003) found in a longitudinal study of physicists that in their transitions from mid to late career, elite physicists "remained consistent in their identification with science and in their scientific ability." Their research output continued to accelerate, while others became less committed to their careers and their output stagnated.

2.2 Rational expectations equilibrium

The objective of this paper is to study optimal contracting by the principal when the agent has private career concerns. For a given menu of contracts offered by the principal, the agent sends a signal of his private career concerns by making a contract choice. As in standard sender-receiver signaling models with reduced-form modeling of the receiver, in our model the second-period market is a not a player. Nonetheless, it is important to model how the market forms the posterior reputation about the agent's ability. We define a "posterior reputation system" for the given menu, which gives a posterior reputation of the market about the agent for each contract on the menu and for each realization of the output under the contract.

We say that the agent's type-dependent choice of a contract from the menu and the agent's subsequent type-dependent choice of effort under the chosen contract form a "rational expectations equilibrium," if there is a posterior reputation system such that: (i) the posterior reputation is derived from the agent's contract choice and effort choice using Bayes' rule whenever it applies; and (ii) the agent's contract choice and the effort choice maximize the agent's utility given the posterior reputation system. This definition is of course an adaptation of the standard perfect Bayesian equilibrium. We call it rational expectations equilibrium in order to highlight the role of posterior reputation. In addition, this equilibrium concept applies in a natural way to the case when the career-concerns type space is degenerate, as in the analysis of the complete-information benchmark below.

The above definition allows for the existence of multiple rational expectations equilibria for a given menu. We say that a menu is "optimal" for the principal, if there exists a rational expectations equilibrium under the menu such that no other rational expectations equilibrium under any other menu gives the principal a greater expected profit.

2.3 Complements versus substitutes

Before our main analysis, we impose natural restrictions on the production parameters p_{ae}, $a = h, l$ and $e = 0, 1$. The restrictions allow us to simplify the analysis and focus on the underlying economic insights.

For any effort choice $e \in [0,1]$ of the agent taken as fixed by the market, a posterior reputation system can then be represented by a pair of beliefs: the probability $R_s(e)$ of $a = h$ conditional on $y = s$ and the probability $R_f(e)$ of $a = h$ conditional on $y = f$. Whenever Bayes' rule applies, we have

$$R_s(e) = \frac{r(ep_{h1} + (1-e)p_{h0})}{ep_1 + (1-e)p_0}, \tag{1}$$

and

$$R_f(e) = \frac{r(e(1-p_{h1}) + (1-e)(1-p_{h0}))}{e(1-p_1) + (1-e)(1-p_0)}. \tag{2}$$

By the law of iterated expectations,

$$(ep_1 + (1-e)p_0)R_s(e) + (e(1-p_1) + (1-e)(1-p_0))R_f(e) = r. \tag{3}$$

In words, given that the market expects e and forms the posterior reputation system accordingly, the expected reputation of the agent who chooses e is his prior reputation r.

Since the probability of success is increasing in the worker's ability a for any effort level, we have the following likelihood ratio comparison:

$$\frac{ep_{h1} + (1-e)p_{h0}}{ep_{l1} + (1-e)p_{l0}} > \frac{e(1-p_{h1}) + (1-e)(1-p_{h0})}{e(1-p_{l1}) + (1-e)(1-p_{l0})}.$$

Regardless of the effort level expected by the market, success is a better signal for high ability rather than low quality than failure is. Therefore

$$\Delta(e) \equiv R_s(e) - R_f(e) > 0.$$

We refer to $\Delta(e)$ as the "reputation gap" between success and failure as the result of the agent's effort.

The reputation gap plays a critical role in our analysis of the optimal menu for the first-period principal. It is a function of e, the effort level that the market uses in forming the posterior reputation. Our analysis depends on whether the reputation gap function $\Delta(e)$ increases or decreases in e, that is, whether the reputation gap increases or decreases in the effort level expected by the market. Below we provide sufficient conditions for Δ to be monotone in e regardless of the prior reputation r.

We say that the agent's ability and effort are "complements" if success is a stronger signal for high ability when the agent exerts full effort than when he shirks. That is, the likelihood ratio p_{he}/p_{le} is greater when $e = 1$ than when $e = 0$, or

$$p_{h1}p_{l0} \geq p_{l1}p_{h0}. \tag{4}$$

The above implies[7]

$$p_{h1} - p_{h0} > p_{l1} - p_{l0},$$

which means that ability and effort are "weak complements" in the sense that the increase in the probability of success when the agent switches from shirk to work is greater when the agent has high ability than when he has low ability. Equivalently,

$$(1 - p_{h0}) - (1 - p_{h1}) > (1 - p_{l0}) - (1 - p_{l1}),$$

so that the decrease in the probability of failure when the agent switches from $e = 0$ to $e = 1$ is greater when he has high ability than when he has low ability.

We say that ability and effort are "substitutes" if failure is a stronger signal for low ability when the agent shirks than he exerts full effort, or

$$(1 - p_{h1})(1 - p_{l0}) \geq (1 - p_{l1})(1 - p_{h0}). \tag{5}$$

The above implies

$$(1 - p_{h0}) - (1 - p_{h1}) < (1 - p_{l0}) - (1 - p_{l1}),$$

which means that ability and effort are "weak substitutes" in the sense that the decrease in the probability of failure when the agent switches from shirk to work is smaller when the

[7]There are two cases. In the first case, we have $p_{l0} = 0$. Then (4) implies that either $p_{l1} = 0$ or $p_{h0} = 0$, or both. If $p_{l1} = 0$, then since by assumption the agent's effort strictly increases the probability of success at least for one ability level, we have $p_{h1} > p_{h0}$ and therefore $p_{h1} - p_{h0} > p_{l1} - p_{l0}$; if $p_{h0} = 0$, then since by assumption the agent's ability strictly increases the probability of success at least for one effort level, we have $p_{h1} > p_{l1}$ and again $p_{h1} - p_{h0} > p_{l1} - p_{l0}$. In the second case, $p_{l0} > 0$ and therefore $p_{h0} \geq p_{l0} > 0$. If $p_{h0} = p_{l0}$, then $p_{h1} > p_{l1}$; if $p_{h0} > p_{l0}$, then $p_{h1}/p_{h0} > 1$ and thus $p_{h0}(p_{h1}/p_{h0} - 1) > p_{l0}(p_{l1}/p_{l0} - 1)$ (otherwise (4) implies $p_{l1}/p_{l0} = 1$ and contradicts the assumption that effort strictly increases the probability of success for at least one ability level). Either way, $p_{h1} - p_{h0} > p_{l1} - p_{l0}$.

agent has high ability than when he has low ability. Equivalently,

$$p_{h1} - p_{h0} < p_{l1} - p_{l0},$$

so that the increase in the probability of success when the agent switches from shirk to work is smaller when the agent has high ability than when he has low ability.

Since (4) implies that ability and efforts are weak complements while (5) implies that ability and efforts are weak substitutes, there is no intersection between the set of production parameters p_{ae} for which ability and efforts are complements and the set for which ability and efforts are substitutes.[8] By taking derivatives, one can easily establish the following lemma.

Lemma 1 *When ability and effort are complements, $R_s(e)$ is weakly increasing, $R_f(e)$ is strictly decreasing, and $\Delta(e)$ is strictly increasing. When ability and effort are substitutes, $R_s(e)$ is strictly decreasing, $R_f(e)$ is weakly increasing, and $\Delta(e)$ is strictly decreasing.*

When ability and effort are complements, $R_s(e)$ is weakly increasing because success is a stronger signal for high ability when the agent exerts a greater effort. Correspondingly, since (4) implies that the opposite of (5) holds, failure becomes a stronger signal for low ability when the agent exerts a greater effort, and thus $R_f(e)$ is strictly decreasing. Symmetrically, when ability and effort are substitutes, $R_f(e)$ is weakly increasing because (5) implies that failure is a weaker signal for low ability when the agent exerts a greater effort, and the opposite of (4) implies that $R_s(e)$ is strictly decreasing because success is a weaker signal for high ability when the agent exerts a greater effort. In what follows we will show that the reputation gap provides the implicit incentive for the agent to exert effort, and so Lemma 1 implies that it is easier to provide such incentive for a greater effort when ability and effort are complements, but harder to do the same when they are substitutes.

[8] We do not consider production parameters p_{ae} for which ability and effort are neither complements nor substitutes. In particular, we exclude from our analysis the case in which the reputation gap $\Delta(e)$ is non-monotone in e.

3 Public Career Concerns

Our main analysis starts with the benchmark case of public career concerns, where the agent's marginal utility for the second-period income t is common knowledge among the agent, the principal and the market. It is without loss to assume that the principal offers a single contract, and so the agent's contract choice is trivial. We provide a characterization of the optimal contract in the benchmark case. That is, we ask: what is the wage-bonus contract that induces a rational expectations equilibrium with an expected profit for the first-period principal that is no lower than the expected profit in any rational expectations equilibrium induced by any wage-bonus contract?

Fix $t = m, n$. For any wage-bonus contract (w, b), when the second-period market expects effort level $\hat{e}$, the agent of type t weakly prefers working to shirking if

$$w + bp_1 - k + t(p_1 R_s(\hat{e}) + (1 - p_1)R_f(\hat{e})) \geq w + bp_0 + t(p_0 R_s(\hat{e}) + (1 - p_0)R_f(\hat{e})).$$

It follows that a rational expectations equilibrium under public career concerns is given by $\hat{e}$ such that:

$$\lambda(b + t\Delta(\hat{e})) \geq k \text{ if } \hat{e} > 0 \text{ and } \lambda(b + t\Delta(\hat{e})) \leq k \text{ if } \hat{e} < 1. \tag{6}$$

In particular, $\lambda(b + t\Delta(\hat{e})) = k$ if $\hat{e} \in (0, 1)$.

Condition (6) says that the agent weakly prefers to work if the sum of the explicit and implicit incentives is greater than the cost of effort. The first term on the left-hand side of (6), λb, represents the explicit incentive for work provided by the first period contract (w, b). It depends only on the bonus b, not on the base wage w. It also depends on λ, i.e., how productive the agent's effort is. Finally, the explicit incentive depends on the prior belief r about the agent's ability, but not on the effort level $\hat{e}$ perceived by the market.

The second term on the left-hand side of (6), $\lambda t\Delta(\hat{e})$, represents the implicit incentive for work provided by the second period market. It depends on the reputation gap $\Delta(\hat{e})$ at the equilibrium effort level $\hat{e}$. This is multiplied by t, which represents how much the agent in the first period cares about his career in the second period. Finally, like the explicit incentive, the implicit incentive also depends on how productive the agent's effort is.

Given a continuation rational expectations equilibrium $\hat{e}$ for any wage contract (w, b), we can characterize the optimal contract under public career concerns. The principal's equilibrium expected profit is

$$(1-b)(\hat{e}p_1+(1-\hat{e})p_0)-w.$$

For given (w, b), there may be multiple continuation equilibria. Indeed, since $\Delta(\cdot)$ is increasing when ability and effort are complements, the equilibrium condition (6) implies that if there is a mixed-strategy equilibrium $\hat{e} \in (0,1)$ for some contract (w, b), then $\hat{e}=1$ (as well as $\hat{e}=0$) is also a pure strategy equilibrium. Intuitively, a higher expected effort makes success a stronger signal of high ability, and failure a stronger signal for low ability, enhancing his expected reputation and increasing his future pay. Thus the agent will indeed work more, fulfilling the expectations. In contrast, when ability and effort are substitutes, there is always a unique continuation rational expectation equilibrium.

Instead of considering all continuation rational expectation equilibria for all possible contracts, we search for the optimal contract for the first-period principal by considering all possible equilibrium effort. To avoid uninteresting parameter cases, we assume

$$m\overline{\Delta} < k/\lambda < 1+n\underline{\Delta}, \tag{7}$$

where $\overline{\Delta}=\Delta(1)$ and $\underline{\Delta}=\Delta(0)$ in the case of complements, and $\overline{\Delta}=\Delta(0)$ and $\underline{\Delta}=\Delta(1)$ in the case of substitutes. These assumptions imply that explicit incentives are necessary for the principal to induce even the more career-concerned type to work even when implicit incentives reach the maximum, while at the same time explicit incentives are potentially profitable for her to even for the less career-concerned type and even when implicit incentives are at the minimum. Then, to induce any equilibrium effort level $\hat{e} \in (0,1]$, the principal chooses bonus b such that the inequalities in (6) hold as equality. Since the base wage w does not affect the continuation equilibrium $\hat{e}$, it is optimal for the principal to set $w=0$. This implies that the principal's expected profit is given by

$$\Pi_t(\hat{e}) = (1-(k/\lambda - t\Delta(\hat{e}))(\hat{e}p_1+(1-\hat{e})p_0) \tag{8}$$

for $\hat{e} \in (0,1]$. For $\hat{e}=0$, for convenience we denote as $\Pi_t(0)$ the limit of (8) as $\hat{e}$ goes to 0. This ensures $\Pi_t(\hat{e})$ is a continuous function for $\hat{e} \in [0,1]$. By assumption (7), to induce $\hat{e}=0$ it is optimal for the principal to set $b=0$, and thus $\Pi_t(0) < p_0$. In our model, small explicit incentives do not pay.

The optimal explicit contract under public career concerns depends whether ability and effort are complements or substitutes. In the case of complements, the reputation gap $\Delta(\cdot)$ is strictly increasing. The profit function $\Pi_t(\hat{e})$ given by (8) is increasing and convex. A higher expected effort not only increases the probability of success but also increases the reputation gap and hence the implicit incentives that the agent faces, making the agent willing to put in even more effort. The optimal equilibrium effort the principal wants to induce is either full effort or no effort, depending on whether the following condition holds:

$$(1 - (k/\lambda - t\Delta(1))\, p_1 > p_0, \tag{9}$$

Denote e_t^* as the equilibrium effort under the optimal contract. We have established the following result.

Lemma 2 *When ability and effort are complements,* $\Pi_t(e)$ *is strictly increasing and convex for each* $t = m, n$*. The optimal contract under public career concerns for* t *is:* $w_t^* = 0$*, and* $b_t^* = k/\lambda - t\Delta(1)$ *with* $e_t^* = 1$ *if (9) holds, and otherwise* $b_t^* = 0$ *with* $e_t^* = 0$*.*

As a function of the agent's career concerns t, both b_t^* and e_t^* are discontinuous: condition (9) determines a threshold value of the agent's career concerns t such that b_t^* jumps from 0 for t just below the threshold to a positive level for t just above the threshold, while e_t^* jumps from 0 to 1. Of course, the optimal profit of the principal, given by

$$\pi_t^* = (1 - b_t^*)(e_t^* p_1 + (1 - e_t^*)p_0),$$

is a continuous function of t. Both b_t^* and π_t^* increase as t increases when the former is continuous. When the principal induces a continuation rational expectations equilibrium $\hat{e} = 1$, an agent who is more career-concerned allows the principal to reduce the implicit incentive and increase the profit.

When ability and effort are substitutes, the reputation gap $\Delta(\cdot)$ is strictly decreasing. By taking derivatives, we can easily show that $\Pi_t(\hat{e})$ is strictly concave. The concavity of $\Pi_t(\cdot)$ is a critical feature when ability and effort are substitutes. It reflects the trade off between a greater probability of success when the principal induces the agent to work harder and a higher explicit pay because the agent faces a weaker implicit incentive as the reputation gap Δ decreases. The concavity of $\Pi_t(\cdot)$ implies that we can uniquely define

$$\hat{e}_t = \arg\max_{\hat{e}} \Pi_t(\hat{e}).$$

Denote e_t^* as the equilibrium effort under the optimal contract. The following result is then immediate.

Lemma 3 *When ability and effort are substitutes, $\Pi_t(e)$ is strictly concave for each type $t = m, n$. The optimal contract under public career concerns for each t is: $w_t^* = 0$, and $b_t^* = 0$ with $e_t^* = 0$ if $\Pi_t(\hat{e}_t) \leq p_0$, and otherwise $b_t^* = k/\lambda - t\Delta(\hat{e}_t)$ with $e_t^* = \hat{e}_t$.*

Unlike in the case of complements, the optimal induced effort e_t^* can be interior. Using the envelope theorem, we can easily show that e_t^* is continuously *decreasing* in t when it is interior: an agent who is more career-concerned is optimally induced to put in a lower effort level. The logic of this seemingly unintuitive result is as follows. For an agent with any fixed t, in raising the induced equilibrium effort level $\hat{e}$, the principal faces the trade-off between the marginal benefit from a greater probability of success and the marginal cost from a higher expectation of explicit incentive provided to the agent. The marginal benefit is constant in $\hat{e}$, while the marginal cost is increasing in $\hat{e}$ because a higher $\hat{e}$ not only raises the probability that the bonus is paid out but also a reduced implicit incentive due to the fact that the reputation gap $\Delta(\cdot)$ is decreasing. An increase in the agent's career concerns type t raises the marginal cost because it amplifies the reduction in the implicit incentive, without changing the marginal benefit. Thus, the optimal induced effort e_t^* decreases with the agent's career concerns type t.

As in the case of complements, the optimal bonus b_t^* is decreasing with t. However, unlike in the case of complements where a more career-concerned agent faces the same maximal implicit incentives, in the case of substitutes he is optimally induced to put in a lower effort, which results in a larger reputation gap and a greater implicit incentive. This allows the principal to further reduce the explicit incentive. The resulting optimal profit π_t^* is increasing in t in spite of the probability of success decreasing in t.

4 Optimal Pooling

A novel theoretical feature of this model is that the value of a contract to an agent is endogenous. The principal forms rational expectations of the agent's effort for each contract, and the agent chooses his effort level optimally against such expectations. In essence, the agent plays a game against the market: each type of agent's equilibrium payoff and his deviation payoffs are all determined through the market forming the correct expectations for each contract.[9]

The natural starting point of our analysis of private career concerns is to find the principal's optimal pooling contract. Fix a pooling contract (w, b). Abusing notation slightly, we denote as e_t as the effort level chosen by each type $t = m, n$ under this contract. Then the average expected effort is

$$\bar{e} \equiv \eta e_n + (1 - \eta) e_m.$$

Due to the linearity in our model, the reputation gap Δ depends on e_n and e_m only through $\bar{e}$. Given $\bar{e}$, type $t = m, n$ weakly prefers working to shirking if

$$w + bp_1 - k + t(p_1 R_s(\bar{e}) + (1 - p_1) R_f(\bar{e})) \geq w + bp_0 + t(p_0 R_s(\bar{e}) + (1 - p_0) R_f(\bar{e})).$$

It follows that (e_m, e_n) is a rational expectations equilibrium if for each $t = m, n$ we have

$$\lambda(b + t\Delta(\bar{e})) \geq k \text{ if } e_t > 0 \text{ and } \lambda(b + t\Delta(\bar{e})) \leq k \text{ if } e_t < 1. \tag{10}$$

Since both career-concerns types of the agent face the same explicit incentive under a pooling contract and the same reputation gap, and have the same cost function, in any rational expectations equilibrium under (w, b) we must have

$$e_n \leq \bar{e} \leq e_m \text{ and } e_n(1 - e_m) = 0. \tag{11}$$

That is, type m always puts at least as much effort as type n, and furthermore, there can be at most one type that chooses an interior effort level, with $e_n \in (0, 1)$ implying $e_m = 1$ and $e_m \in (0, 1)$ implying $e_n = 0$. Type m generally works harder because he values the

[9] Thus any equilibrium must be determined by a fixed point argument, and we cannot replace the agent's optimal effort problem by standard optimization with IC and IR constraints.

future more than type n for any given expected effort. In equilibrium, the market's expected effort is just the average effort $\bar{e}$. When $e_m > e_n$, we have $e_m > \bar{e} > e_n$, so that type m outperforms, and type n underperforms, the market's expectation. For each type $t = m, n$,

$$\begin{aligned}
&(e_t p_1 + (1 - e_t)p_0)R_s(\bar{e}) + (e_t(1 - p_1) + (1 - e_t)(1 - p_0))R_f(\bar{e}) \\
=&(\bar{e}p_1 + (1 - \bar{e})p_0)R_s(\bar{e}) + (\bar{e}(1 - p_1) + (1 - \bar{e})(1 - p_0))R_f(\bar{e}) + \lambda(e_t - \bar{e})\Delta(\bar{e}) \\
=&r + \lambda(e_t - \bar{e})\Delta(\bar{e}),
\end{aligned} \tag{12}$$

where the second equality follows from the the law of iterated expectations (3). Thus, type m gets a better reputation because his effort is judged against a lower standard, while type n gets a worse reputation.

A pooling contract (w, b) can be optimal for the principal only if she uses the lowest bonus to induce a given equilibrium (e_m, e_n). If $e_m = 0$, then we have $e_n = 0$, and the lowest bonus $b = 0$. Since the reputation gap depends on (e_m, e_n) only through $\bar{e}$, if $e_m > e_n = 0$, the principal must induce type m to work and so the lowest bonus is

$$b = k/\lambda - m\Delta(\bar{e}).$$

If $e_n > 0$, the principal only needs to induce type n to work, and so the lowest bonus is

$$b = k/\lambda - n\Delta(\bar{e}).$$

Since $e_n \in (0, 1)$ implies $e_m = 1$ and $e_m \in (0, 1)$ implies $e_n = 0$, the value of equilibrium $\bar{e}$ goes from 0, when $e_m = 0$, to $1 - \eta$, when $e_m = 1$ and $e_n = 0$, to 1, when $e_n = 1$. Conversely, given any $\bar{e} \in [0, 1]$, there is a unique equilibrium pair (e_m, e_n). Since the probability of success depends only on the average effort $\bar{e}$, we can then write the principal's candidate optimal profit as

$$\overline{\Pi}(\bar{e}) = \begin{cases} p_0 & \text{if} \quad \bar{e} = 0 \\ \Pi_m(\bar{e}) & \text{if} \quad \bar{e} \in (0, 1 - \eta] \\ \Pi_n(\bar{e}) & \text{if} \quad \bar{e} \in (1 - \eta, 1], \end{cases} \tag{13}$$

where $\Pi_t(e)$ is the profit function under public career concerns for each $t = m, n$, given by (8). Given the profit function (13), finding the optimal pooling contract reduces to identifying the optimal induced effort, by solving $\max_{\bar{e}} \overline{\Pi}(\bar{e})$.

The function $\overline{\Pi}(\bar{e})$ is discontinuous at $\bar{e} = 0$. No bonus is needed if the principal intends to induce shirking by both career concerns types. For $\bar{e}$ just above 0, the principal has to use explicit incentive to motivate type m, and so the profit jumps down. As we have seen under public career concerns, small bonuses are never optimal for the principal. Since e_t^* maximizes $\max\{p_0, \Pi_t(e)\}$ for each $t = m, n$, and since $\pi_m^* \geq \pi_n^*$, if $e_m^* = 0$ then (13) implies that the optimal pooling contract has $b = 0$, with the resulting profit p_0. That is, if it is not profitable to induce any effort by the more career-concerned type m under public career concerns, then it is optimal for the principal to give up on inducing any effort under private career concerns, regardless of whether ability and effort are complements or substitutes.

In the more interesting case, we have $e_m^* > 0$. It is optimal for the principal to induce type m to put in at least some effort, if η is sufficiently close to 0 and thus the average effort $\bar{e}$ is close to the effort e_m by type m. Indeed, if $\eta = 0$, the model of private career concerns collapses to complete information with type m alone, and the optimal pooling contract is the same as what we have characterized in section 3. For fixed $\eta > 0$, inducing the optimal average effort $\bar{e}$ to maximize $\overline{\Pi}(\bar{e})$ may require the principal to motivate the less career-concerned type n. Importantly, the function $\overline{\Pi}(\bar{e})$ is discontinuous at $\bar{e} = 1 - \eta$. To achieve the profit $\overline{\Pi}(1 - \eta)$, which by (13) is equal to $\Pi_m(1 - \eta)$, the principal only has to apply explicit incentives to type m, but for any effort slightly above $1 - \eta$, she has to motivate type n, which is more costly. Thus, $\overline{\Pi}(\bar{e})$ jumps down at $\bar{e} = 1 - \eta$. Let $\hat{\eta}$ be such that

$$\Pi_m(1 - \hat{\eta}) = \pi_n^*. \tag{14}$$

From (13), the above defines a prior belief η of $t = n$ such that the profit from pooling together type m that puts in full effort and type n that puts in no effort is equal to the optimal profit under public career concerns with type n alone. This belief is well-defined when $e_m^* > 0$, because $\Pi_m(e_m^*) = \pi_m^* > \pi_n^*$ and $\Pi_m(0) < p_0 \leq \pi_n^*$. Further, $\hat{\eta} \in (1 - e_m^*, 1)$ is unique, because in the case of complements $\Pi_m(e)$ is strictly increasing in e, while in the case of substitutes $\Pi_m(e)$ is strictly concave and maximized at $e = e_m^*$.

The complete characterization of the optimal pooling contract depends on whether effort and ability are complements or substitutes. In the case of complements, under complete information the principal always wants to induce working if she wants to induce any effort at all.[10] Partial effort is never optimal under private career concerns either. Under any pooling contract, if any type t weakly prefers to work, then there is an equilibrium in which both types put in full effort, because the reputation gap is increasing as the average expected effort increases, motivating each type to raise their effort still further. The proof of the following result is straightforward and relegated to the appendix.

Proposition 1 *Suppose that ability and effort are complements. In any optimal pooling contract,* $w = 0$*. If* $e_m^* = 0$*, then* $b = 0$*, with* $e_m = e_n = 0$*. If* $e_m^* = 1$*, then* $b = k/\lambda - m\Delta(1-\eta)$ *for* $\eta \leq \hat{\eta}$*, with* $e_m = 1$ *and* $e_n = 0$*; and* $b = b_n^*$ *for* $\eta > \hat{\eta}$*, with* $e_m = e_n = e_n^*$*.*

When $e_m^* > 0$, the discontinuity of $\overline{\Pi}(\overline{e})$ at $\overline{e} = 1 - \eta$ implies that when we characterize the optimal pooling contract in the case of $e_m^* = 1$ for different values of η, the equilibrium average expected effort $\overline{e}$ changes discontinuously with η. For values of η lower than $\hat{\eta}$, the principal pays just enough for type m (and only type m) to be indifferent between working and shirking. As η increases, the principal is more likely over-paying a type-n agent who is shirking. Her profit from this contract is decreasing in η, because the decrease in the average expected effort makes it difficult for type m to signal his ability through success, who then in turn demands more bonus. In contrast, both type m and type n become better off as η increases. To see this, note that since type m is indifferent between working and shirking, his payoff can be obtained by exerting the average effort $\overline{e}$ and using (3):

$$\overline{U}_m = b(p_1\overline{e} + p_0(1-\overline{e})) - k\overline{e} + mr = kp_0/\lambda + mR_f(\overline{e}). \tag{15}$$

By Lemma 1, type m is better off as η increases and $\overline{e}$ decreases.[11] Intuitively, if type m faces no implicit incentive, then being indifferent would mean that his payoff is independent of the effort that the principal induces using only the explicit incentive; with part of his payoff coming from meeting the expectation of $\overline{e}$, his payoff is greater when the reputation

[10] As under complete information, there can be multiple rational expectations equilibria for any given pooling contract (w, b), and when this happens, we select the most profitable equilibrium.

[11] Since at $\eta = 0$ the optimal pooling contract is the same as the optimal contract for type m under public career-concerns give by Lemma 2, type m benefits from the optimal pooling with type n.

gap $\Delta(\overline{e})$ is smaller for a lower $\overline{e}$, given that ability and effort are complements. Similarly, type n's payoff is

$$\overline{U}_n = bp_0 + n(p_0 R_s(\overline{e}) + (1-p_0)R_f(\overline{e})) = kp_0/\lambda + nR_f(\overline{e}) - p_0(m-n)\Delta(\overline{e}), \qquad (16)$$

which by Lemma 1 increases as η increases and $\overline{e}$ decreases, so that type n benefits not only from meeting a lower expectation $\overline{e}$ as type m does, but also from free-riding on higher explicit incentives provided to type m.

When the value of η exceeds $\hat{\eta}$, it becomes optimal for the principal to switch the target of explicit incentives to type n. Since ability and effort are complements, from section 3 we know that it is optimal for the principal to induce full effort for type n if (9) with $t = n$ holds and zero effort otherwise. Thus, e_n^* is discontinuous in the parameters of the model. An implication is that the optimal pooling contract when $\eta > \hat{\eta}$ is also discontinuous. Under any pooling contract less career-motivated agents impose an externality on more motivated ones by pulling down the average expected effort level and reducing the implicit incentives provided by the second-period market. There are two cases. First, when it is not profitable to use explicit incentives for the less career-motivated agents under complete information, this externality leads the principal to forgo explicit incentives even for the more motivated agents and results in no effort by the latter. Second, when the principal applies explicit incentives to the less career-motivated under complete information, the same externality allows the more motivated agents to enjoy a higher bonus than what the principal would have provided to them on their own. In either case, the optimal contract and the resulting profit are independent of η. In the first case, type m's payoff $\overline{U}_n$ is lower than his payoff U_m^* under public career-concerns derived from Lemma 2 if $e_m^* = 1$, while in the second case $\overline{U}_m > U_m^*$ because $b_n^* > b_m^*$. Type n's payoff $\overline{U}_n$ in either case is the same as his payoff U_n^* under public career-concerns derived from Lemma 2.

In Proposition 1, the principal always wants to induce full effort if she wants to induce any effort at all. Thus, the equilibrium effort of each career concerns type under the optimal pooling contract is independent of the principal's belief about the type, except at cutoff beliefs when a type switches between working and shirking. In contrast, it is possible for a contract inducing partial effort to be optimal when ability and effort are substitutes. As a result, the equilibrium effort of each type under the optimal pooling contract generally depends on the principle's belief about the type. The proof of the following result can be found in the appendix.

Proposition 2 *Suppose that ability and effort are substitutes. In any optimal pooling contract, $w = 0$. If $e^*_m = 0$, then $b = 0$, with $e_m = e_n = 0$. If $e^*_m > 0$, then $b = b^*_m$ for $\eta \leq 1 - e^*_m$, with $e_m = e^*_m/(1-\eta)$ and $e_n = 0$; $b = k/\lambda - m\Delta_m(1-\eta)$ for $1 - e^*_m < \eta \leq \hat{\eta}$, with $e_m = 1$ and $e_n = 0$; and $b = b^*_n$ for $\eta > \hat{\eta}$, with $e_m = 1$ and $e_n = (e^*_n - (1-\eta))/\eta$ when $e^*_n > 0$ and $e_m = e_n = 0$ otherwise.*

If the principal finds it optimal to induce partial effort for type m under complete information, by continuity she continues to induce equilibrium $e_m < 1$ and $e_n = 0$ under the optimal pooling when η is just above 0. Unlike in Proposition 1, when ability and effort are substitutes, the presence of the less career-concerned type n drives up the reputation gap for the more concerned type m. The increased implicit incentive results in type m putting a greater effort than under complete information to "pick up the slack" for the shirking type n. This allows the principal to achieve the complete-information profit π^*_m by using the same bonus b^*_m as the explicit incentive. The optimal pooling profit remains π^*_m for all $\eta \leq 1 - e^*_m$, and the payoff $\overline{U}_t$ for each type $t = m, n$ is also independent of η. In particular, $\overline{U}_m$ is the same as type m's payoff U^*_m under public career concerns derived from Lemma 2, despite putting in a greater effort as η increases, because the principal implements the same average effort e^*_m.

For values of η above $1 - e^*_m$, it is no longer possible for type m to pick up the slack. The optimal pooling profit is given by $\overline{\Pi}(1-\eta)$, from inducing full effort by type m and no effort by type n. By the definition of $\hat{\eta}$ in (14), the optimal pooling profit is strictly decreasing in η. Both type m and type n are also worse off as η increases. Type m's payoff $\overline{U}_m$ and type n's payoff $\overline{U}_n$ are given by (15) and (16) respectively, but unlike in Proposition 1, by Lemma 1, both payoffs decrease as η increases and $\overline{e}$ decreases. The reason is that part of the agent's payoff is the implicit incentive, and so when the reputation gap $\Delta(\overline{e})$ becomes larger as $\overline{e}$ decreases, given that ability and effort are substitutes, the agent is worse off from reduced explicit incentive. For type n, an additional adverse effect is that there is less free-riding.

As in Proposition 1, it becomes optimal for the principal to switch the target of explicit incentives to type n when η is sufficiently close to 1. Unlike in the case of complements, however, the optimal contract under complete information for type n is continuous in the parameters of the model, because partial effort can be optimal when ability and effort are substitutes. In particular, if $e_n^* \in (0,1)$, then the optimal pooling contract continues to induce partial effort for type n when η is just below 1. The presence of type m, who strictly prefers to put in full effort because type n is indifferent, drives up the average expected effort and, since ability and effort are substitutes, results in a smaller reputation gap compared to public career concerns with type n alone. Type n is thus less motivated and reduces the equilibrium effort e_n to below e_n^*, while type m again picks up the slack and allows the principal to adopt the complete-information optimal bonus b_n^* for type n to achieve the corresponding optimal profit π_n^* as long as η is greater than $\hat{\eta}$. The optimal contract and the resulting profit remain the same for all $\eta > \hat{\eta}$, and so are the payoffs $\overline{U}_m$ and $\overline{U}_n$ for the two types. In particular, type n is just as well as under public career-concerns, with $\overline{U}_n = U_n^*$, despite having to work harder when η increases and the agent is less likely to be the more motivate type to pick up the slack.

5 Profitable Separation

Can our principal improve on the optimal pooling characterized by the previous section by separating the two career-concerns types to some extent through different explicit incentives? When career commitment is public information, in section 3 we have shown that the principal should pay less to those who care more about their reputation of career commitment. That is, the agent in this model may be strongly motivated by his future career prospects and thus willing to accept less explicit pay. The principal naturally wants to design a menu of contracts optimally to take the best advantage of such an agent.

We denote the two distinct contracts offered by the principal as (w^C, b^C), $C = A, B$, with $(w^A, b^A) \neq (w^B, b^B)$. Let type m and n use contract (w^A, b^A) with probability z_m and z_n respectively. Since we are dealing with separation in this section, we assume that each contract is chosen with a positive probability on the equilibrium path. Bayes' rule applies, and the probability the agent is type n given his contract choice is, respectively, (w^A, b^A) and (w^B, b^B), is given by:

$$q^A = \frac{\eta z_n}{\eta z_n + (1-\eta) z_m}, \; q^B = \frac{\eta(1-z_n)}{\eta(1-z_n) + (1-\eta)(1-z_m)}.$$

Let the induced efforts of type m and type n in contract $C = A, B$ be (e_m^C, e_n^C). Each of these efforts is either part of the continuation equilibrium after the contract choice by the corresponding type, or the best response to the continuation equilibrium by a type who makes an out-of-equilibrium contract choice. Let $\overline{e}^C$ be the average equilibrium expected effort in contract $C = A, B$. We have:

$$\overline{e}^C = (1 - q^C) e_m^C + q^C e_n^C.$$

The above is well-defined because each contract is chosen with a positive probability on the equilibrium path. Further, if an effort level, say e_t^C, is the best response of type $t = m, n$ out of the equilibrium path in contract $C = A, B$, then it does not contribute to the corresponding average effort $\overline{e}^C$, as $q^C = 1$ in equilibrium. Given $\overline{e}^C$ in contract $C = A, B$, the efforts e_m^C and e_n^C of the two types satisfy the characterization given by (10) regardless of whether they are on the equilibrium path and off the path.

Due to the linearity in how we model the effort choice, given $\overline{e}^C$ in each contract $C = A, B$, the expected payoff of type $t = m, n$ from making the effort choice e_t^C after choosing (w^C, b^C) is given by

$$U_t^C = w^C + b^C(e_t^C p_1 + (1 - e_t^C) p_0) - k e_t^C + t(R_f(\overline{e}^B) + (e_t^C p_1 + (1 - e_t^C) p_0) \Delta(\overline{e}^C)). \quad (17)$$

The incentive condition for type m to make the equilibrium contract choice is that

$$U_m^A \geq U_m^B \text{ if } z_m > 0 \text{ and } U_m^A \leq U_m^B \text{ if } z_m < 1.$$

The above implies that $U_m^A = U_m^B$ if $z_m \in (0, 1)$. The incentive condition for type n's contract choice is similarly given.

The principal's expected profit is given by

$$(\eta z_n + (1-\eta) z_m) \pi^A + (\eta(1 - z_n) + (1-\eta)(1 - z_m)) \pi^B, \quad (18)$$

where π^C, $C = A, B$, is the profit from contract (w^C, b^C), given by

$$\pi^C = (1 - b^C)(\overline{e}^C p_1 + (1 - \overline{e}^C)p_0) - w^C.$$

Following the definition of candidate optimal pooling contract (13), we have

$$\pi^C \leq \begin{cases} p_0 & \text{if} \quad \overline{e}^C = 0 \\ \Pi_m(\overline{e}^C) & \text{if} \quad \overline{e}^C \in (0, 1 - q^C] \\ \Pi_n(\overline{e}^C) & \text{if} \quad \overline{e}^C \in (1 - q^C, 1]. \end{cases} \tag{19}$$

The above imposes an upper-bound on the profit π^C from contract (w^C, b^C), $C = A, B$.

Throughout this section, we restrict our analysis to the economically interesting case in which the principal wants to induce some type to work under complete information: $p_0 < \pi_m^*$. Otherwise, the pooling contract with $w = b = 0$ is clearly optimal, and there is no profitable separation.

5.1 Full separation

We first consider the possibility of screening contracts in which the two types of agent self-select into different contracts.[12] That is, whether it is possible to have $z_m = 0$ and $z_n = 1$, or $z_m = 1$ and $z_n = 0$. As in any screening problem, we start by finding out which types of agents have incentives to imitate others. From the optimal contract under complete information characterized in section 3, under both complements and substitutes, we have $\pi_m^* > \pi_n^*$. An agent who cares less about his career is less profitable for the principal, so it may be natural to think that the principal needs to offer information rent to type m. Then type n is is tempted to imitate one who cares more about his career. In fact the opposite is true. As a thought experiment, consider the optimal contract under complete information characterized in section 3. Under both complements and substitutes, the optimal bonus satisfies $b_m^* < b_n^*$. If a menu of two complete-information contracts is offered, a more career-concerned agent

[12]Because the second period is perfectly competitive in this model, the principal is *not* trying to screen individuals for hire in the next period. Any rent from improved screening is driven to zero as firms compete for workers in the second period. Thus, the principal is concerned *exclusively* about her expected profit in the first period.

would want to pretend to be one who is less concerned. Doing so leads to more bonus in the first period, and he can appear more high-ability by beating the market's expectation of how hard he has worked.

We argue by contradiction that there is no *strictly separating* menu such that two types choose different contracts with probability one and at least one type strictly prefers his contract to the other one. Without loss, suppose that type m chooses (w^B, b^B) and type n chooses (w^A, b^A); that is, $z_m = 0$ and $z_n = 1$. The key is (3): by following the recommended effort, the expected implicit incentive received by each type $t = m, n$ from his own contract is equal to mr. Since under a separating menu, type m always has the option of following the recommended effort e_n^A after deviating and choosing type n's contract (w^A, b^A), we must have

$$U_m^A \geq w^A + b^A(e_n^A p_1 + (1 - e_n^A)p_0) - ke_n^A + mr.$$

Since type m's incentive condition is $U_m^B \geq U_m^A$, cancelling mr from both sides gives

$$w^B + b^B(e_m^B p_1 + (1 - e_m^B)p_0) - ke_m^B \geq w^A + b^A(e_n^A p_1 + (1 - e_n^A)p_0) - ke_n^A.$$

The above no longer depends on the implicit incentives provided by the respective contracts; thus contract (w^B, b^B) provides a weakly better explicit incentive for effort than contract (w^A, b^A) does. By exchanging the roles of m and n, we see that the two contracts must provide the same explicit incentive for effort. As a result, we have both $U_m^B = U_m^A$ and $U_n^A = U_n^B$, and the menu cannot strictly separating.

The above argument does not rule out the possibility that the principal could offer menus that fully separate the two types of career concerns. For example, full separation occurs if b^A and b^B are such that each type prefers full effort to no effort under (w^A, b^A) and the opposite happens under (w^B, b^B), and both types are indifferent between the two contracts. However, we show that full separation is never strictly more profitable than the optimal pooling for the principal. The proof of this part of the following proposition is in the appendix. We use the result that there is no strict separation to show that under any full separation the profit π^A from type n's contract (w^A, b^A) and π^B from type m's contract (w^B, b^B) are both at most π_n^*, which by Proposition 1 and Proposition 2 is a lower bound of the optimal pooling profit when ability and effort are complements and substitutes respectively.

Proposition 3 *There is no equilibrium with strict separation. Further, full separation is not profitable.*

The impossibility of strict separation holds generally. In particular, it does not rely on our modeling assumption that there are only two types of career concerns, or even there is a finite number of such types. What is driving the result is that explicit and implicit incentives are perfect substitutes in the agent's utility function. From now on, we say that a menu of two contracts is "profitable" if induces an equilibrium with a strictly higher profit than the optimal pooling contract.

5.2 Partial separation

Since strict separation is impossible in equilibrium and full separation is not profitable, we turn to "partial separation" in which at least one of two career-concerns types randomizes between the two contracts in the menu offered by the first-period principal, and the principal updates her belief about the type of the agent after his contract choice. In this subsection we consider "semi-separation," where one type randomizes between the two contracts while the other type chooses one of them with probability one. For convenience, let (w^B, b^B) be the separating contract used by only one type and (w^A, b^A) be the other, mixing contract used by both, with $z_m, z_n > 0$. If it is type m that chooses (w^B, b^B), with $z_m < 1$ and $z_n = 1$, we say it is "m-separation;" if it is type n with $z_n < 1$ and $z_m = 1$, we say it is "n-separation."

We first characterize the continuation equilibrium under any profitable semi separation. In any semi separation there is at least one mixing contract on the menu chosen by both career-concerns types with positive probabilities. For semi separation to be profitable, the two types must have different preferences between working and shirking under the mixing contract, type m preferring working and type n preferring shirking. We show that at least one of the two types' preference between working and shirking must be strict. The proof is in the appendix.

Lemma 4 *In any profitable m-separation equilibrium, type n strictly prefers shirking under the mixing contract (w^A, b^A) and $e_m^A > 0$; in any profitable n-separation equilibrium, type m strictly prefers working under the mixing contract (w^A, b^A) and $e_n^A < 1$.*

Under semi-separation, when the more career-concerned type m chooses the separating contract, he can only meet the market expectations in equilibrium. That is, his payoff from the second period market is mr. By choosing the mixing contract, type m will always weakly outperform the market's expectations because he works at least as hard as type n. Indifference between the two contracts therefore implies that type m gets higher implicit incentives and lower explicit incentives in the mixing contract than in the separating contract. If the less career-concerned type n weakly prefers working to shirking in the mixing contract, then type m strictly prefers working under the same contract, and the two types have the same explicit payoff. But the difference in explicit incentives between the two contracts is just enough to compensate type m for the lower implicit incentives in the separating contract, and type n cares less about implicit incentives. Thus, type n would prefer to deviate to the separating contract (w^B, b^B) for its higher explicit incentives, unless type n makes the same effort choice as type m both under the mixing contract and in deviation under the separating contract. Thus, such semi-separation is either not an equilibrium, or as in Proposition 3, no more profitable than the optimal pooling contract. It follows that in any m-separation equilibrium, type n strictly prefers shirking under the mixing contract. Further, type m must put in at least some effort under the mixing contract in order for m-separation equilibrium to be profitable. For n-separation equilibrium, a symmetric argument as above establishes that type m strictly prefers working to shirking and type n does not exert full effort under the mixing contract if the semi-separation is to be more profitable than the optimal pooling contract.

Lemma 4 provides a characterization of profitable menus that achieve semi separation by exploiting the incentive conditions of the two career-concerns types m and n. Now we use this characterization to address the question of whether such profitable menus exist. By (18), the principal's profit under semi separation is just a weighted sum of the profits π^A and π^B from the two contracts in the menu, and by (19), both π^A and π^B are in turn bounded from above. As we have shown in section 4, the bounds depend on their respective average effort across the two types. We want to compare the profit under semi separation to the profit from pooling with the average effort across the two types and across the two contracts into a single contract.

We first consider m-separation equilibrium. By our analysis in sections 3 and 4, the principal's profit from the optimal pooling is generally lower than the profit when the agent is known to be the more concerned type, regardless of whether ability and effort are complements or substitutes. Since in any m-separation equilibrium only type m chooses the separating contract (w^B, b^B), it can yield a higher profit than the the optimal pooling profit. However, the maximum profit from the mixing contract (w^A, b^A) is always lower than the optimal pooling profit. To see this, note that by Lemma 4, in any profitable m-separation, the mixing contract must induce some positive effort from type m but zero effort from type n; that is $e_m^A > e_n^A = 0$. As a result, the average effort across the two types in the mixing contract is $(1-q^A)e_m^A$. Since $z_m < 1$ and $z_n = 1$, we have $q^A > \eta$ and thus the average effort in the mixing contract is strictly less than $1-\eta$, which is the highest effort that the principal can induce with a pooling contract.

For an m-separation equilibrium to be profitable, the principal has to not only obtain a higher profit from type m in the separating contract than the optimal pooling, but also compensate for the loss of profit from the mixing contract relative to the optimal pooling. When ability and effort are substitutes, the concavity of the profit in the effort level under public career concerns given in Lemma 3 directly implies that this is not possible. The average profit from the separating contract and the mixing contract is always lower than what the principal can obtain by pooling the two career-concerns types together, inducing more concerned type to work and the less concerned type to shirk. The same conclusion holds when ability and effort are complements, even though the profit is convex in the effort level under public career concerns as given in Lemma 2. A cost of separation to the menu arises from the fact that in the mixing contract the presence of the less concerned type allows type m to enjoy a higher reputation payoff, compared to the separating contract. For type m to be indifferent between the two contracts, he has to be compensated by the explicit incentives in the separating contract, which limits how much more profitable the separating contract can be than the mixing contract. In turn, this limit on the profitability of the separating contract establishes that the average profit from the separating contract and the mixing contract is always lower than the optimal pooling profit. The proof of the following proposition is in the appendix.

Proposition 4 *There does not exist a profitable m-separation equilibrium.*

Next, we examine whether n-separation can be profitable. Since the separating contract (w^B, b^B) is chosen by type n only, by (19), it can yield to the principal at most π_n^*, the profit when the agent is known to be the less concerned type. We know from Proposition 1 and Proposition 2 that π_n^* is a lower bound on the profit from the optimal pooling. This means that in order for an n-separation equilibrium to be profitable, the mixing contract (w^A, b^A) has to do better for the principal than the optimal pooling, and in particular, type n must strictly prefer to shirk in the mixing contract. As a result, in our search for a profitable n-separation equilibrium, we can assume that type m is indifferent between the two contracts. Otherwise, since type n strictly prefers to shirk in the mixing contract (w^A, b^A), and since by Lemma 4 type m strictly prefers to work under the same contract, a marginal decrease in the bonus b^A coupled with an increase in w^A that leaves U_n^A unchanged would increase the profit from the mixing contract without violating type m's incentive condition. Further, from Proposition 1 and Proposition 2 we know that for any initial belief η that the agent is the less concerned type higher than the cutoff belief $\hat{\eta}$, the profit from the optimal pooling is equal to π_n^*, regardless of whether ability and effort are complements or substitutes. Thus, if the initial belief η is just above $\hat{\eta}$, so that the optimal pooling profit is π_n^*, and if there is a separating contract for type n on the menu that also yields π_n^*, all we need for an n-separation equilibrium to be profitable is to have a mixing contract that yields a profit greater than π_n^*. We have the following result.

Proposition 5 *There exist parameters of the model such that an n-separation equilibrium is profitable.*

The details of the construction for the following proposition is in the appendix. A key component is a positive base wage w^A in the mixing contract (w^A, b^A). Of course the base wage plays no role under public career concerns, or in the optimal pooling. In a profitable n-separation, the base wage is used to keep the less concerned type, type n, indifferent between the mixing the contract and the separating contract. This is necessary because, for the separating contract to output perform the optimal pooling contract, it needs to induce type m to work and type n to shirk, by exploiting type m's greater concern for his career than type n, but the reduced explicit incentives make the mixing contract relatively unattractive for the latter.

The contrast between Proposition 5 and Proposition 4 derives from the different roles played by the more concerned type m and the less concerned type n under the optimal pooling. Take the case of substitutes for example. The concavity of the profit function $\Pi_m(e)$ when the agent's career-concerns type is m and is publicly known implies there is no profitable m-separation equilibrium, because the target of explicit incentives in both the separating contract for type m alone and the mixing contract for both m and n is type m, and the average effort across the two contracts is feasible under pooling of the two types. Although the profit function $\Pi_n(e)$ is also concave, it does not imply there is no profitable n-separation equilibrium. Unlike in an m-separation equilibrium, where the separating contract has to be sufficiently more profitable than the optimal pooling contract for the equilibrium to be profitable, in an n-separating equilibrium the mixing contract is more profitable as type m works and type n shirks. The target of the mixing contract is type m, not type n, and so the concavity of $\Pi_n(e)$ has no bite.

6 Concluding remarks

The characterization of the optimal pooling under private career concerns in section 4 (Propositions 1 and 2), and the impossibility of strict separation (Proposition 3), with the consequent examination of the possibility of profitable semi-separation in section 5 (Propositions 4 and 5), raise the question of what is the optimal menu of contracts for the first-period principal. To address this question, we must also consider "weak separation" of the two career-concerns types, who choose with different probabilities both contracts on the menu with positive probabilities. It is straightforward to extend Lemma 4 for a characterization of profitable weak separation; in particular, we can show that in any profitable weak separation, either the more concerned type strictly prefers working in both contracts, or the less concerned type strictly prefers shirking in both contracts. The construction used to establish Proposition 5 shows that weak separation can indeed be profitable. With the large number of cases we need to consider, we have yet to find a systematic approach to compare the maximal profit under n-separation or weak separation to the optimal pooling profit. We leave this research question to future work.

Unlike in a standard optimal contracting problem, our first-period principal can only directly contract on the output in the first period by the agent, who cares also about his reputation for ability in the second-period competitive market. In this sense, there is contract incompleteness in our model, as in the literature on incomplete contracts (Grossman and Hart, 1986; Hart and Moore, 1999) or the literature on limited commitment (Bester and Strausz, 2001; Kolotilin, Li, and Li, 2013). However, unlike those two strands of the literature, our agent derives his payoff from a rational expectations equilibrium with the second-period market, which affects the nature of incentive compatibility constraints with respect to the first-period principal. Our problem thus appears to lie outside of established paradigms, and calls for a more general analysis in future work.

7 Appendix

7.1 Proof of Proposition 1

In the text we have already argued that in any optimal pooling contract, $w = 0$ and b induces an average effort $\bar{e}$ that maximizes $\overline{\Pi}$ as given by (13).

Suppose that $e_m^* = 0$. Since $e_m^* \geq e_n^*$, we have $e_n^* = 0$. It follows that $\Pi_t(e) \leq p_0$ for each $t = m, n$. Then (13) implies that $\overline{\Pi}(\bar{e}) \leq p_0$ for all $\bar{e}$. Thus, the optimal $\bar{e} = 0$, induced by setting $b = 0$.

Suppose that $e_m^* = 1$. By Lemma 2, $\Pi_t(e)$ is strictly increasing for each $t = m, n$. Thus, $\overline{\Pi}(\bar{e})$ as given by (13) is strictly increasing for $\bar{e} \in (0, 1-\eta]$ and for $\bar{e} \in (1-\eta, 1]$, with $\overline{\Pi}(0) > \lim_{\bar{e}\downarrow 0} \overline{\Pi}(\bar{e})$, and $\overline{\Pi}(1-\eta) > \lim_{\bar{e}\downarrow 1-\eta} \overline{\Pi}(\bar{e})$. The optimal $\bar{e}$ is either 0, $1-\eta$ or 1. Recall that in the case of complements, $\pi_n^* = \max\{p_0, \Pi_n(1)\}$. By the definition of $\hat{\eta}$, we have $\overline{\Pi}(1-\eta) \geq \pi_n^*$ for $\eta \leq \hat{\eta}$, and $\overline{\Pi}(1-\eta) < \pi_n^*$ for $\eta > \hat{\eta}$. The proposition follows immediately.

7.2 Proof of Proposition 2

As in the proof of Proposition 1, in any optimal pooling contract, $w = 0$, and b induces $\overline{e}$ that maximizes $\overline{\Pi}$, with $b = 0$ if $e_m^* = 0$.

Suppose that $e_m^* = 1$. By Lemma 3, $\Pi_m(e)$ is strictly increasing for all e, and $\Pi_m(1) \geq p_0$. From (13) we have that $\overline{\Pi}(\overline{e})$ is strictly increasing for $\overline{e} \in (0, 1-\eta]$. If $\eta \leq \hat{\eta}$, we have $\overline{\Pi}(1-\eta) \geq \overline{\Pi}(\overline{e})$ for all $\overline{e} > 1-\eta$. It follows that $\overline{e} = 1-\eta$ maximizes $\overline{\Pi}(\overline{e})$. The optimal pooling contract has $b = k/\lambda - m\Delta_m(1-\eta)$ that induces $\overline{e} = 1-\eta$ through $e_m = 1$ and $e_n = 0$. If $\eta > \hat{\eta}$, we have $\overline{\Pi}(1-\eta) < \pi_n^*$, and $\overline{e} = e_n^*$ maximizes $\overline{\Pi}(\overline{e})$. The optimal pooling contract has $b = b_n^*$ that induces $\overline{e} = e_n^*$ through $e_m = 1$ and $e_n = (e_n^* - (1-\eta))/\eta$.

Suppose that $e_m^* \in (0,1)$. By Lemma 3, $\Pi_m(e)$ is strictly concave and reaches the maximum at e_m^*, with $\Pi_m(e_m^*) \geq p_0$. Recall that by definition $\hat{\eta} \in (1-e_m^*, 1)$. If $\eta \leq 1-e_m^*$, we have $\overline{\Pi}(e_m^*) \geq \overline{\Pi}(1-\eta) \geq \overline{\Pi}(\overline{e})$ for all $\overline{e} > 1-\eta$. It follows that $\overline{e} = e_m^*$ maximizes $\overline{\Pi}(\overline{e})$. The optimal pooling contract has $b = b_m^*$ that induces $\overline{e} = e_m^*$ through $e_m = e_m^*/(1-\eta)$ and $e_n = 0$. If $\eta \in (1-e_m^*, \hat{\eta}]$, we have $\overline{\Pi}(1-\eta) > \overline{\Pi}(\overline{e})$ for all $\overline{e} < 1-\eta$ because $\Pi_m(e)$ is strictly concave and reaches the maximum at e_m^* by Lemma 3, and $\overline{\Pi}(1-\eta) \geq \overline{\Pi}(\overline{e})$ for all $\overline{e} > 1-\eta$ by the definition $\hat{\eta}$. Since $\pi_n^* \geq p_0$, we also have $\overline{\Pi}(1-\eta) \geq p_0$. It follows that $\overline{e} = 1-\eta$ maximizes $\overline{\Pi}(\overline{e})$. The optimal pooling contract has $b = k/\lambda - m\Delta_m(1-\eta)$ that induces $\overline{e} = 1-\eta$ through $e_m = 1$ and $e_n = 0$. If $\eta > \hat{\eta}$, we still have $\overline{\Pi}(1-\eta) > \overline{\Pi}(\overline{e})$ for all $\overline{e} < 1-\eta$, but now $\overline{\Pi}(1-\eta) < \pi_n^*$. Thus, $\overline{e} = e_n^*$ maximizes $\overline{\Pi}(\overline{e})$. The optimal pooling contract has $b = b_n^*$ that induces $\overline{e} = e_n^*$ through $e_m = 1$ and $e_n = (e_n^* - (1-\eta))/\eta$. The equilibrium effort e_n for type n is strictly positive, because Lemma 3 implies that $e_n^* \geq e_m^* > 1-\hat{\eta} > 1-\eta$.

7.3 Proof of Proposition 3

In the text we have established the first part of the proposition that there cannot be strict separation. For the second part, note that the argument for the first part also shows that type m's deviation payoff from choosing type n's contract (w^A, b^A) is

$$U_m^A = w^A + b^A(e_n^A p_1 + (1-e_n^A)p_0) - ke_n^A + mr.$$

By definition, e_m^A is type m's best response to the same contract (w^A, b^A) under the second period market's expectation of e_n^A. Thus, either $e_m^A = e_n^A$, or type m' is indifferent between e_m^A and e_n^A after deviating to (w^A, b^A). We claim that $e_n^A = 0$ or 1. Suppose instead that $e_n^A \in (0,1)$. Since condition (6) holds with $t = n$ and $\hat{e} = e_n^A$, we have

$$\lambda(b^A + n\Delta(e_n^A)) = k.$$

the above implies that $e_m^A = 1$ and type m strictly prefers full effort to e_n^A after deviating to (w^A, b^A), contradicting the result that type m is indifferent between e_m^A and e_n^A after deviation. Thus, $e_n^A = 0$ or 1, and further, either $e_m^A = e_n^A$, or type m is indifferent between e_m^A and e_n^A after deviating to (w^A, b^A), which implies that $e_n^A = 0$. It follows that the profit π^A from (w^A, b^A) is at most π_n^*, because either $e_m^B = e_n^B = 1$, or $e_m^B = e_n^B = 0$, or $e_n^A = 0$ and zero effort is a best response to (w^A, b^A) by type m. By a similar argument, the profit π^B from (w^B, b^B) is at most π_n^*, because either $e_m^B = e_n^B = 0$ or 1, or if $e_n^B \neq e_m^B$ then $e_m^B = 1$ and full effort is a best response to (w^B, b^B) by type n. By Proposition 1, when ability and effort are complements and $e_m^* = 1$, then optimal pooling profit strictly decreases from π_m^* at $\eta = 0$ to π_n^* at $\eta = \hat{\eta}$, and stays at π_n^* for all $\eta > \hat{\eta}$. By Proposition 2, when ability and effort are substitutes and $e_m^* > 0$, the optimal pooling profit is equal to π_m^* for all $\eta \in [0, 1 - e_m^*]$, strictly decreases to π_n^* at $\eta = \hat{\eta}$, and stays at π_n^* for all $\eta > \hat{\eta}$. In either case, the optimal pooling profit is at least π_n^*. The proposition follows immediately.

7.4 Proof of Lemma 4

In an m-separation equilibrium, we have $U_m^A = U_m^B$ and $U_n^A \geq U_n^B$. Suppose that type n weakly prefers working to shirking under the mixing contract (w^A, b^A). By (11), we have $e_m^A = 1$. From (17), regardless of whether type n strictly prefers working or is indifferent between working and shirking under the mixing contract, we have

$$U_m^A - U_n^A = (m - n)(R_f(\bar{e}^A) + p_1\Delta(\bar{e}^A)) \geq (m - n)r,$$

with equality if and only if $e_n^A = 1$. In deviating to the separating contract, type n could always choose the effort e_m^B as type m. Since only type m chooses the separating contract, by (3) the deviation payoff U_n satisfies

$$U_m^B - U_n^B \leq (m-n)r,$$

with equality if $e_n^B = e_m^B$. Since $U_m^A = U_m^B$ and $U_n^A \geq U_n^B$, we immediately have $U_n^A = U_n^B$. Further, $e_n^A = e_m^A = 1$, and either $e_n^B = e_m^B$, or type n is indifferent between e_n^B and e_m^B after deviating to (w^B, b^B). By the same argument as in Proposition 3, we have that $e_m^B = 0$ or 1, and if $e_n^B \neq e_m^B$, then e_m^B is a best response to (w^B, b^B) by type n after deviation. It follows that the profit π^A from (w^A, b^A) and the profit π^B from (w^B, b^B) are both at most π_n^*, and such equilibrium is never more profitable than the optimal pooling equilibrium. Therefore, for any profitable m-separation equilibrium, type n strictly prefers shirking to working under the mixing contract (w^A, b^A). Furthermore, given that $e_n^A = 0$ we must have $e_m^A > 0$ in order for m-separation equilibrium to be profitable.

The argument for the n-separation equilibrium is symmetric to the above argument for m-separation equilibrium. For such equilibrium to be profitable, type m strictly prefers working to shirking under the mixing contract (w^A, b^A). Further, given that $e_m^A = 1$ we must have $e_n^A < 1$ in order for n-separation equilibrium to be profitable.

For weak separation to be profitable, we must have $e_m^B \neq e_n^B$, or $e_m^A \neq e_n^A$, or both. By (11), either in both contracts type m strictly prefers working and type n strictly prefers shirking, or at least one of the two types is indifferent between working and shirking in at least one contract. Suppose that type m is indifferent under (w^B, b^B). Then, by (11), type n strictly prefers shirking under (w^B, b^B) and $e_n^B = 0$. Similar to the argument above for n-separation equilibrium, we have

$$U_m^B - U_n^B = (m-n)(R_f(\bar{e}^B) + p_0\Delta(\bar{e}^B)) \leq (m-n)r,$$

with equality if and only if $e_m^B = 0$. If type n weakly prefers working to shirking under (w^A, b^A), then by (11) we have $e_m^A = 1$. Similar to the argument above for m-separation equilibrium, we have

$$U_m^A - U_n^A = (m-n)(R_f(\bar{e}^A) + p_1\Delta(\bar{e}^A)) \geq (m-n)r,$$

with equality if and only if $e_n^A = 1$. Since $U_m^A - U_n^A = U_m^B - U_n^B$ in weak separation, the above two inequalities imply that $e_m^B = e_n^B = 0$ and $e_m^A = e_n^A = 1$, but we already know that such equilibrium is not profitable. Thus, if type m is indifferent between working and shirking under one contract, type n strictly prefers shirking under both contracts. A symmetric argument establishes that if type n is indifferent under one contract, type m strictly prefers working under both contracts.

7.5 Proof of Proposition 4

Fix an m-separation equilibrium, with a mixing contract (w^A, b^A) and a separating contract (w^B, b^B), and $z_n = 1$ and $z_m \in (0, 1)$. By Lemma 4, in the mixing contract we have $e_m^A > e_n^A = 0$. Using (19), we have

$$\pi^A \leq \Pi_m(\bar{e}^A) = \Pi_m((1 - q^A)e_m^A).$$

Since $q^A > \eta$ in any m-separation equilibrium, we have

$$(1 - q^A)e_m^A < 1 - \eta.$$

From the pooling profit (13) in section 4, the principal can obtain the profit $\Pi_m((1 - q^A)e_m^A)$ by inducing that effort $(1 - q^A)e_m^A$ from type m and zero effort from type n. This implies that π^A cannot exceed the optimal pooling profit.

Suppose that ability and effort are substitutes. First, assume that $e_m^B = 0$. From (19) we have

$$\pi^B \leq p_0 \leq \pi_n^*.$$

By Proposition 2, the profit from optimal pooling is at least π_n^*. This implies that π^B cannot exceed the optimal pooling profit, and thus the m-separation equilibrium is not profitable.

Next, assume that $e_m^B > 0$. Since only type m chooses the separating contract, using (19) we have

$$\pi^B \leq \Pi_m(\bar{e}^B) = \Pi_m(e_m^B).$$

From (18), the total expected profit of the firm is bounded from above by

$$(\eta(1 - z_n) + (1 - \eta)(1 - z_m))\Pi_m(e_m^B) + (\eta z_n + (1 - \eta)z_m)\Pi_m((1 - q^A)e_m^A).$$

Since $\Pi_m(e)$ is strictly concave, the above is bounded from above by $\Pi_m(\overline{\overline{e}})$, where $\overline{\overline{e}}$ is the average effort across the types and the two contracts, given by

$$\overline{\overline{e}} \equiv (\eta(1-z_n)+(1-\eta)(1-z_m))\overline{e}^B+(\eta z_n+(1-\eta)z_m)\overline{e}^A = (1-\eta)((1-z_m)e_m^B+z_m e_m^A) \leq 1-\eta.$$

From the pooling profit (13) in section 4, the principal can obtain the profit $\Pi_m(\overline{\overline{e}})$ by inducing that effort $(1 - z_m)e_m^B + z_m e_m^A$ from type m and zero effort from type n. Thus, m-separation equilibrium with $e_m^B > 0$ is not profitable either.

Now, suppose that ability and effort are complements. First, we argue by contradiction that for the m-separation equilibrium to be profitable, type m strictly prefers to work under the separating contract (w^B, b^B). If type m is instead indifferent with some e_m^B, then upon deviating the separating contract type n would choose $e_n^B = 0$. Thus, the indifference of type m between efforts in the separating contract implies that

$$U_m^B - U_n^B = (m-n)(p_0 R_s(\overline{e}^B) + (1-p_0)R_f(\overline{e}^B)).$$

Since $e_m^A > e_n^A = 0$, and since type m can always choose to shirk, we also have

$$U_m^A - U_n^A \geq (m-n)(p_0 R_s(\overline{e}^A) + (1-p_0)R_f(\overline{e}^A)).$$

Given that $U_m^A = U_m^B$ and $U_n^A \geq U_n^B$, the above two inequalities imply

$$p_0 R_s(\overline{e}^A) + (1-p_0)R_f(\overline{e}^A) \leq p_0 R_s(\overline{e}^B) + (1-p_0)R_f(\overline{e}^B).$$

By (12), given that ability and effort are complements, the above implies that $\overline{e}^A \geq \overline{e}^B$. In the m-separation equilibrium, $\overline{e}^A = (1-q^A)e_m^A$ and $\overline{e}^B = e_m^B$, with $q^A > \eta$. It then follows that $e_m^B < 1-\eta$, and thus $\Pi_m(\overline{e}^B)$ is achievable under pooling. By (19), the profit π^B from the separating contract cannot exceed the optimal pooling profit.

Given that we have established $e_m^B = 1$, the incentive condition for type m to be indifferent between the two contracts implies that

$$\begin{aligned}
0 =& U_m^A - U_m^B \\
\geq& w^A + b^A(\overline{e}^A p_1 + (1-\overline{e}^A)p_0) - k\overline{e}^A + mr - (w^B + b^B p_1 - k + mr) \\
=& -(\lambda - k)(1-\overline{e}^A) + \pi^B - \pi^A,
\end{aligned}$$

where the inequality follows from the fact that type m can always choose the average effort $\overline{e}^A$ under the mixing contract. The above impose an upper-bound on $\pi^B - \pi^A$. Meanwhile, since $w^A \geq 0$ and $b^A \geq k/\lambda - m\Delta(\overline{e}^A)$, we have

$$\pi^A \leq (1 - k/\lambda + m\Delta(\overline{e}^A))(\overline{e}^A p_1 + (1 - \overline{e}^A)p_0).$$

By the definition of Π_m (equation 8),

$$\Pi_m(1-\eta) = (1 - k/\lambda + m\Delta(1-\eta))((1-\eta)p_1 + \eta p_0).$$

Since $1 - \eta > \overline{e}^A$, and $\Delta(e)$ increases with e when ability and effort are complements, we have

$$\pi^A \leq \Pi_m(1-\eta) - (\lambda - k)(1 - \eta - \overline{e}^A).$$

The above imposes an upper-bound on π^A. Then, given that $z_n = 1$ in the m-separation equilibrium, (18) implies that the equilibrium profit of the principal is bounded from above by

$$\pi^A + (1-\eta)(1-z_m)(\pi^B - \pi^A) \leq \Pi_m(1-\eta) - (\lambda - k)(1-\eta-\overline{e}^A) + (1-\eta/q^A)(\lambda - k)(1-\overline{e}^A).$$

Since $\overline{e}^A \leq 1 - q^A$, the above is less than or equal to $\Pi_m(1-\eta)$, which is achievable under pooling by inducing full effort by type m and no effort by type n. It follows that the m-separation equilibrium can not be more profitable than the optimal pooling.

7.6 Proof of Proposition 5

Suppose that $e_n^* > 0$ and $\eta \geq \hat{\eta}$. Consider an n-separation equilibrium, with $z_m = 1$ and a fixed $z_n \in (0,1)$, where the separating contract is given by $w^B = 0$, $b^B = b_n^*$, with $e_n^B = e_n^*$ and $e_m^B = 1$, and the mixing contract is given by some w^A and b^A to be determined below, with $e_n^A = 0$, $e_m^A = 1$ and $\overline{e}^A = 1 - q^A$, where $q^A = \eta z_n/(\eta z_n + 1 - \eta)$.

We will construct an n-separation equilibrium where type m, as well as type n, is indifferent between the two contracts. Using (12), we have

$$U_n^B = b_n^*(e_n^* p_1 + (1 - e_n^*)p_0) - k e_n^* + nr$$
$$U_m^B = b_n^* p_1 - k + m(r + \lambda(1 - e_n^*)\Delta(e_n^*))$$

in the separating contract, and

$$U_n^A = w^A + b^A p_0 + n(r - \lambda \overline{e}^A \Delta(\overline{e}^A))$$
$$U_m^A = w^A + b^A p_1 - k + m(r + \lambda(1 - \overline{e}^A)\Delta(\overline{e}^A))$$

in the mixing contract. Solving for w^A and b^A from $U_m^A = U_m^B$ and $U_n^A = U_n^B$, we have

$$w^A = mp_0((1 - \overline{e}^A)\Delta(\overline{e}^A) - (1 - e_n^*)\Delta(e_n^*)) + np_1(\overline{e}^A\Delta(\overline{e}^A) - e_n^*\Delta(e_n^*)),$$

and

$$b^A = \frac{k}{\lambda} - m(1 - \overline{e}^A)\Delta(\overline{e}^A) - n\overline{e}^A\Delta(\overline{e}^A) + (m - n)(1 - e_n^*)\Delta(e_n^*).$$

We have an n-separation equilibrium if w^A and b^A specified above satisfy the following two conditions. First, to induce $e_n^A = 0$ and $e_m^A = 1$, we need b^A to satisfy

$$k - m\lambda\Delta(\overline{e}^A) \leq b^A\lambda \leq k - n\lambda\Delta(\overline{e}^A).$$

Second, for w^A to be feasible, we need $w^A \geq 0$. The difference between the profit from the mixing contract and the optimal pooling profit is given by

$$\pi^A - \pi_*^n = (m - n)\lambda((1 - \overline{e}^A)\Delta(\overline{e}^A) - (1 - e_n^*)\Delta(e_n^*)) - (\lambda - k)(\overline{e}^A - e_n^*).$$

Then, by (18), the n-separation equilibrium is profitable if $\pi^A - \pi_n^* > 0$.

To give an example of a profitable n-separation equilibrium, assume that $n = 0$. By (7), we require $\lambda > k$. Assume that

$$\left(1 - \frac{k}{\lambda}\right) p_1 > p_0.$$

This imposes an upper-bound on k in terms of p_0 and p_1, and ensures that $e_n^* = 1$. From the formulas of w^A and b^A, we have $w^A > 0$, and b^A induces $e_m^A = 1$ and $e_n^A = 0$. For fixed η, by varying z_n, we can choose any $\overline{e}^A$ on the interval $(1 - \eta, 1)$; by choosing η arbitrarily close to 1, we are free to choose any $\overline{e}^A \in (0, 1)$. By (7), we have an upper-bound on m. Putting all these bounds together, we have a profitable n-separation equilibrium if there exists $\overline{e}^A \in (0, 1)$ such that

$$\frac{\Delta(\overline{e}^A)\overline{e}^A}{\overline{\Delta}} > \frac{p_0}{\lambda}.$$

We distinguish two cases. First, suppose that ability and effort are complements. We have $\overline{\Delta} = \Delta(1)$. Since we can choose $\overline{e}^A$ arbitrarily close to 1, the above holds so long as

$p_1 > 2p_0$. This is true regardless of the value of r, if, for example, $p_{h0} = p_{l0} = 0$, which always satisfies (4).

Second, suppose that ability and effort are substitutes. Whenever $p_{h1} = p_{l1}$, condition (5) is satisfied so long as $p_{h0} > p_{l0}$, and

$$\overline{\Delta} = \Delta(0) = \frac{r(1-r)(p_{h0} - p_{l0})}{p_0(1-p_0)}.$$

The desired condition is satisfied if there exist $\overline{e}^A \in (0,1)$ such that

$$\Delta(\overline{e}^A)\overline{e}^A > \frac{r(1-r)(p_{h0} - p_{l0})}{1-p_0}.$$

Since we can choose p_{h0} arbitrarily close to p_{l0}, the above can be satisfied regardless of the value of r.

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