Econ 421
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Problem Set 9

1. Exercise 331.1 (Selten's horse)

2. Exercise 331.2 (Two similar games with different equilibria)

3. Exercise 335.1 (Pooling and separating equilibria in a signaling game)

4. Exercise 335.2 (Sir Philip Sydney game)

5. (*Bargaining with incomplete information*) Two players bargain over a dollar. The timing of the game is: In the first period player 1 makes an offer with player 2's share equal to $s \in [0,1]$; player 2 then decides either to accept the offer, in which event the game ends with player 1 getting $1-s$ and player 2 getting s as their payoffs, or to reject the offer, in which event a binding arbitration takes place in the second period and the arbitrator gives $\beta \in [0,1]$ to player 2 and $1-\beta$ to player 1. The share β represents the arbitrator's "bias" toward player 2, and it is not controlled by either player. Suppose that both players are impatient, so that if the arbitration takes place, player 1's payoff is given by $\delta(1-\beta)$ and player 2's payoff is $\delta\beta$, where $\delta \in (0,1)$ is the discount factor. Except for question (a), suppose that the arbitrator's bias toward player 2 is β_H with probability p_H and $\beta_L < \beta_H$ with probability $p_L = 1 - p_H$. Player 1 knows whether arbitrator's bias is β_H or β_L, while player 2 is uncertain about it.

(a) Suppose that β is common knowledge between 1 and 2. Find the unique subgame perfect equilibrium of this game. Show that the outcome of this equilibrium is that player 1 makes an offer $s = \delta\beta$, which player 2 accepts immediately.

(b) Show that in the signaling game if there is a pure strategy separating equilibrium, then the equilibrium probability of arbitration is positive. [Hint: If two different offers made by the two types of player 1 are accepted by player 2, then the type that is making a greater offer is not playing a best response. Formally, if (s^*, r^*, μ) is a separating equilibrium, then $s^*(\beta_L) \neq s^*(\beta_H)$. Show that it is impossible that $r^*(s^*(\beta_L)) = r^*(s^*(\beta_H)) = \textit{accept}$.]

(c) Show that if there is a separating equilibrium, then the offer made by type β_L is rejected by player 2. [Hint: Prove this by contradiction. Suppose that (s^*, r^*, μ) is a separating equilibrium where $r^*(s^*(\beta_L)) = \textit{accept}$. Show that since type β_L can instead make an unacceptable offer (such as 0) to player 2 and get $\delta(1-\beta_L)$ from the arbitrator, $1 - s^*(\beta_L) \geq \delta(1 - \beta_L)$. Since $r^*(s^*(\beta_H)) = \textit{reject}$ by question (d) above, and since type β_H can imitate type β_L by offering $s^*(\beta_L)$, $\delta(1 - \beta_H) \geq 1 - s^*(\beta_L)$, a contradiction.]

(d) Show that if there is a separating equilibrium, then $s^*(\beta_H) = \delta\beta_H$. [Hint: First show that type β_H's offer cannot be rejected in equilibrium because he can guarantee himself a greater payoff than $\delta(1 - \beta_H)$ by offering just above $\delta\beta_H$. Then, show that $s^*(\beta_H) \leq \delta\beta_H$ and $s^*(\beta_H) \geq \delta\beta_H$.]

(e) Show that if the two players are sufficiently patient, in particular if the discount factor $\delta \geq 1/(1 + \beta_H - \beta_L)$, there is a separating equilibrium in which player 1 offers $\delta\beta_H$ if the arbitrator's bias is known to him as β_H and $\delta\beta_L$ if it is β_L.

6. (Easy course or hard course) A student is privately informed of his type T, which is either "good" ($T = G$), or "bad" ($T = B$). His advisor knows only that $T = G$ with probability $p \in (0, 1)$. The student can choose an "easy" course ($c = e$), or a "hard" one ($c = h$). For both courses, the grade is pass or fail. Denote by q_c^T the probability that

a type T passes course c. Assume that $0 < q_e^B = q_e^G < 1$ and $0 < q_h^B < q_h^G < 1$. (This means that the hard course is informative about the student's type but the easy one is not. By Bayes' rule, for any belief β that T is G that the advisor holds after observing his course choice c, the posterior belief that T is G after observing he passes the course is $\beta q_c^G/(\beta q_c^G + (1-\beta)q_c^B)$.) The posterior belief remains β if $c = e$ but it is greater than β if $c = h$ and $0 < \beta < 1$.) After the advisor observes the course the student chooses and the grade he gets from the course, she chooses a recommendation level $r \in [0, 1]$ to minimize the expected value of $(r - t)^2$, where the random variable t takes the value of 1 if $T = G$ and 0 if $T = B$. (Minimizing the expected value of $(r - t)^2$ means she will choose r equal to the posterior belief that the student is of type G.) The student's payoff is r regardless of his type.

(a) Show that there is no separating equilibrium; that is, show that there is no perfect Bayesian equilibrium in which the two types choose different courses.

(b) Find all pooling equilibria. Is there a pooling equilibrium that allows the advisor to partially separate the two types? Explain your answer carefully.

(c) Are there equilibria in which one or both types randomize between the two courses? Explain your answer carefully.