Econ 421
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Problem Set 7

1. Exercise 282.1 (Fighting an opponent of unknown strength)

2. Exercise 282.2 (An exchange game)

3. Exercise 282.3 (Adverse selection)

4. Exercise 284.1 (Infection)

5. (*Absent-minded player*)

Find *all* the pure-strategy Bayesian Nash equilibria in the following static Bayesian game. Nature determines whether the payoffs are as in Game 1 or as in Game 2, each game being equally likely; Player 1 learns whether Nature has drawn Game 1,

	L	R
T	$1,1$	$0,0$
B	$0,0$	$0,0$

or Game 2,

	L	R
T	$0,0$	$0,0$
B	$0,0$	$2,2$

but Player 2 does not; Player 1 chooses wither T or B; Player 2 simultaneously chooses either L or R.

6. (Incomplete-information interpretation of mixed strategy Nash equilibrium) Consider the following *Hawk and Dove* game with payoff matrix:

	H	D
H	$0, 0$	$4, 1$
D	$1, 4$	$3, 3$

(a) Show that the above game (of complete information) has a mixed strategy Nash equilibrium where each player plays H with $\frac{1}{2}$ probability.

Consider the following incomplete information variant of the above game. As before, there are two players and each can choose either H or D, but imagine that each player can be in different mood: If he is in better mood, he obtains a greater payoff from the outcome where he plays H and the other plays D (gets a greater payoff from beating his opponent). Formally, we imagine that nature decides randomly the mood of each player. If Nature "chooses" m_1 and m_2, the moods of the two players, the payoff matrix of the game is

	H	D
H	$0, 0$	$4 + m_1, 1$
D	$1, 4 + m_2$	$3, 3$

Each player knows his mood, but not his opponent's. Moreover, m_1 and m_2 are independently distributed so that for each player i, knowing his own mood m_i does not give any more information regarding the mood of his opponent (player j) than what he already knows before he learns m_i, namely $m_j = \epsilon$ with probability $\frac{1}{2}$ and $m_j = -\epsilon$ with probability $\frac{1}{2}$, where ϵ is a small positive number.

(b) Set up the incomplete information game as a static Bayesian game for any fixed ϵ.

Note that when ϵ is very small, the above incomplete information game is very "close" to the original *Hawk and Dove* game. The next question asks you to verify that, no matter how small ϵ is (as long as it is not zero), there is a Bayesian Nash equilibrium where each player uses a *pure* strategy that mimics the mixed strategy Nash equilibrium found in (a).

(c) Show that the following strategy profile is a Bayesian Nash equilibrium for any $\epsilon > 0$: Each player i plays H if he is in good mood ($m_i = \epsilon$) and plays D if he is in bad mood. What is the probability that each player plays H?

(d) Find two other Bayesian Nash equilibria for any fixed ϵ. (Hint: What are the other Nash equilibria of the original hawk and dove game besides the mixed strategy Nash equilibrium in (a)?)

7. (*Cournot duopoly with private demand information*) Consider a Cournot duopoly operating in a market with inverse demand function $p(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c_i(q_i) = cq_i$, but demand is uncertain: it is high ($a = a_H$) with probability θ and low ($a = a_L$) with probability $1 - \theta$. Furthermore, information is asymmetric: firm 1 knows whether demand is high or low, but firm 2 does not. The two firms simultaneously choose quantities. What are the strategy spaces for the two firms? Make assumptions concerning a_H, a_L, θ and c such that all equilibrium quantities are positive. What is the Bayesian Nash equilibrium of this game?