Econ 421
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Li, Hao
UBC

Problem Set 6

1. Exercise 468.1 (Two-period bargaining with constant cost of delay)

2. Exercise 473.1 (One-sided offers)

3. (*Alternating offers with three players*) Three players split a dollar. In period 0, player 1 makes a proposal $y^0 = (y_1^0, y_2^0, y_3^0)$, where y_j^0 is the share to player j, $j = 1, 2, 3$. Player 2 first considers whether to accept it or reject it; and then player 3. If both 2 and 3 accept the proposal, the game ends with each player j getting y_j^0 share. Otherwise, bargaining moves on to period 1. Player 2 becomes the proposer, player 3 the first responder and player 1 the second responder. If both 3 and 1 accept 2's proposal $y^1 = (y_1^1, y_2^1, y_3^1)$, the game ends with each player receiving the share proposed by player 2. Otherwise, in period 2 player 3 becomes the proposer, player 1 the first responder and player 2 the second responder, and so on. We require the proposal $y^t = (y_1^t, y_2^t, y_3^t)$ in period t to satisfy $y_j^t \geq 0$, $j = 1, 2, 3$, and $\sum_{j=1}^{3} y_j^t = 1$. Each player cares only about his share: if the game ends in period t, the payoff to each player j is y_j^t. The common discount factor is $\delta \in (0, 1)$.

(a) Show that there is a unique stationary subgame perfect equilibrium, where each player makes the same proposal whenever it is his turn, and uses the same rule to accept proposals whenever he is the first responder and the same rule to accept proposals whenever he is the second responder.

(b) What happens to the equilibrium shares when δ goes to 1? Give an intuitive explanation of your result.

4. (*Finitely repeated Prisoner's Dilemma with Nash punishment*) Consider the following modified Prisoner's Dilemma game:

	D	P	C
D	$1, 1$	$-2, 0$	$3, 0$
P	$0, -2$	$-1, -1$	$0, -3$
C	$0, 3$	$-3, 0$	$2, 2$

Suppose that the above is repeated for a finite number of T periods. For simplicity, we assume that there is no discounting, i.e., each player's payoff is simply the sum of the payoffs in all T periods. Show that the non-Nash equilibrium stage game outcome (C, C) as a subgame perfect equilibrium in every period except for period T.

5. (*Grim-trigger in repeated Prisoner's Dilemma*) Consider an infinitely repeated Prisoner's Dilemma game, given by

	D	C
D	$1, 1$	$3, 0$
C	$0, 3$	$2, 2$

Let $\delta \in (0, 1)$ be the common discount factor. A grim-trigger strategy is defined as follows: start with C, continue with C so long as the opponent played C in the last period, and switch to D permanently otherwise. Use the one-shot deviation principle to argue that a pair of grim-trigger strategies does not form a subgame perfect equilibrium for any discount factor.

6. (*Tit-for-tat in repeated Prisoner's Dilemma*) Consider the same infinitely repeated Prisoner's Dilemma game in Question 5 above. A tit-for-tat strategy is defined as follows: start with C, play the action chosen by the opponent in the previous period. Does a pair of

tit-for-tat strategies form a subgame perfect equilibrium for any discount factor? Explain your answer carefully.

7. (*Nash-trigger in a repeated Cournot game*) Cournot duopolists with the same constant marginal cost c (and no fixed cost) face an inverse demand function given by $P(Q) = a - Q$ with $a > c$. They play this Cournot duopoly game for an infinite number of periods. Each firm evaluates an infinite sequence of profits $(\pi_t)_{t=0}^{\infty}$ according the geometric sum $\sum_{t=0}^{\infty} \delta^t \pi_t$, where $\delta \in (0, 1)$ is the common discount factor.

(a) Show that for δ sufficiently close to 1, by using a trigger strategy that permanently switches to the Nash equilibrium quantity $\frac{1}{3}(a-c)$ after any deviation, the duopolists can support the collusive outcome of each producing half of the monopoly quantity $\frac{1}{4}(a - c)$ in every period as a subgame perfect equilibrium outcome.

(b) What is the most profitable quantity that the Nash-trigger strategy in (a) can support as a subgame perfect equilibrium outcome? Explain your answer.

8. (*Carrot-and-stick in a repeated Cournot game*) Consider the same infinitely repeated Cournot duopoly game in Question 7 above. Fix a fraction $y \in \left[\frac{1}{4}, \frac{1}{3}\right]$. In a carrot-and-stick strategy to support the quantity $y(a-c)$ in a SPE outcome with some fraction $x \geq \frac{1}{3}$, each firm plays $y(a-c)$ if last period outcome is either $(y(a-c), y(a-c))$ or $(x(a-c), x(a-c))$, and plays $x(a-c)$ otherwise. In questions (a) and (b) below, we first identify the necessary and sufficient conditions for a pair of carrot-and-stick strategies to form a subgame perfect equilibrium.

(a) Show that in all subgames where the quantity pair in the previous period is either $(y(a - c), y(a - c))$ or $(x(a - c), x(a - c))$, there is no profitable one-shot deviation from the carrot-and-stick strategy if

$$\delta(y(1-2y) - x(1-2x)) \geq \frac{(1-3y)^2}{4}.$$

(b) Show that in all subgames where the quantity pair in the previous period is neither $(y(a-c), y(a-c))$ nor $(x(a-c), x(a-c))$, there is no profitable one-shot deviation from the carrot-and-stick strategy if

$$\delta(y(1-2y) - x(1-2x)) \geq \frac{(3x-1)^2}{4}.$$

By the one-shot deviation principle, the two inequalities together are both necessary and sufficient conditions for a pair of carrot-and-stick strategies to form a subgame perfect equilibrium. Note that the left-hand side of the two inequalities is the same, and is increasing in δ and x, and deceasing in y. In questions (c) and (d) below, we ask what is the most profitable outcome, i.e., the lowest quantity y that can be supported as a SPE outcome.

(c) Show that the lowest value of δ for which there exists x that supports the collusive outcome of $y = \frac{1}{4}$ is $\frac{9}{32}$. [Hint: At the lowest δ, the inequality in (b) must be binding, i.e., holding as an equality, for otherwise we could increase x to relax the inequality in (a) without violating the inequality in (b), which then would allow us to reduce the value of δ; given this, the inequality in (a) also must bind, for otherwise we could decrease δ while adjusting x to keep the inequality in (b) binding, without violating the inequality in (a); the lowest value of δ, and the supporting x, can then be found from the two binding inequalities.]

(d) Show that if $\delta < \frac{9}{32}$, the most profitable outcome that can be supported by a pair of carrot-and-stick strategies is $y = (9 - 8\delta)/27$. [Hint: At the lowest y, the inequality in (a) must be binding, for otherwise we could decrease y to relax the inequality in (b) without violating the inequality in (a); given this, the inequality in (b) also must bind, for otherwise we could decrease y while adjusting x to keep the inequality in (a) binding, without violating the inequality in (b); the lowest value of y, and the supporting x, can then be found from the two binding inequalities.]