Econ 421
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Problem Set 10

1. Exercise 340.1 (Pooling equilibria of game in which expenditure signals quality)

2. Exercise 342.1 (Pooling equilibria of game in which education signals ability)

3. (*Entry deterrence under incomplete information*) There are two firms and two periods. In period 1, both firms are in the market. Only firm 1, the "incumbent," takes an action. Firm 1 can choose either P, to prey P on firm 2 (the "entrant"), or A, to accommodate firm 2. There are two types of firm 1, "sane" and "crazy"; firm 2 knows only that the probability that firm 1 is sane is l, a number between 0 and 1. A sane firm 1 gets -1 if it chooses P and 1 if it chooses A. A crazy firm 1's payoffs are such that it always chooses P. In period 2, only firm 2 takes an action. Firm 2 can either choose E, to exit the market, or S, to stay. If firm 2 chooses E, its payoff is 0. If firm 2 chooses S, it gets payoff 1 if firm 1 is actually sane, and -1 if firm 1 is crazy. The sane firm 1 gets payoff 1 if firm 2 chooses S and a monopoly payoff π if firm 2 chooses E. Firm 1 cares only about the sum of its payoffs in the two periods (with no discounting); firm 2 cares only about its payoff in the period 2.

The idea of this problem is the following. We presumed that the crazy type always preys in the first period. The interesting thing to study is the sane type's behavior. From a static point of view, the sane type should choose A. But by choosing P instead in the first period, the sane type might convince firm 2 that it is crazy, and therefore induce firm 2 to exist so that the sane type gets the monopoly profit in the second period.

(a) Show that if $\pi \leq 3$, there is a separating equilibrium, where the sane type chooses A in the first period.

(b) Show that if $\pi \geq 3$ and $l \leq \frac{1}{2}$, there is a pooling equilibrium, where the sane type chooses P in the first period.

(c) Describe what will happen if $\pi > 3$ and $l > \frac{1}{2}$. (Hint: find an equilibrium where the sane firm 1 randomizes between P and A, and firm 2 randomizes between S and E after observing P.)

4. (*Money-back guarantee*) Half of the products in a market are good and the other half are bad. Buyers are willing to pay up to g dollars for a good product and up to $b < g$ dollars for a bad product. Sellers know whether their products are good or bad. Sellers of bad products can spend c dollars to make them indistinguishable from good products. They can charge either a high price h dollars or a low price l dollars. Suppose that $g - h > b - l > 0 > b - h$, so that buyers prefer a good product at the high price to a bad product at the low price, and prefer no purchase to purchasing a bad product at the high price.

(a) Suppose $c > h - l$. Give a separating equilibrium where all good products are offered at h and bad products are offered at l.

(b) Suppose $c < h - l$. Does there exist the same kind of separating equilibrium as in (a)? Explain.

(c) What happens when $c < h - l$ and $l < \frac{1}{2}(g + b) < h$? Explain.

(d) Suppose that $c < h - l$ and bad products and good products can be identified by buyers after purchase (even if sellers have spent c on bad products to make them look good). Give a separating equilibrium where bad products are offered at l, and good products are offered at h with a money-back guarantee that buyers can get $g-b$ from the seller if they find that the product is bad.

5. (*Capital market signaling*) In a capital market, each borrower (entrepreneur) has a project and needs one dollar of capital (cash) to carry it out. If the project succeeds, it yields $y > 1$ dollars; if it fails, it yields 0. Borrowers have exclusive knowledge about the success rate of their projects. The potential lenders (banks) know only that the success rate σ of a project is either σ_H, with probability p_H, or $\sigma_L < \sigma_H$ with probability $p_L = 1 - p_H$. Denote $\overline{\sigma} = p_H\sigma_H + p_L\sigma_L$. Each borrower has some non-liquid wealth (assets that cannot be converted into liquid capital at the time when the project begins, such houses, but which are just as valuable as capital to the lenders and to the borrower), which he may use as collateral in his contract with a lender. Let $\bar{c}$ be the maximal dollar amount of non-liquid wealth that each borrower can put up as collateral in his contract with the lender. The contract between a borrower and a lender takes the following form (called risky debt): The borrower puts down $c \leq \bar{c}$ dollars of non-liquid wealth as collateral and receives one dollar of capital from the lender to begin his project; if the project succeeds, the borrower pays the lender x dollars as interests and gets his collateral back; if it fails, the lender keeps the collateral c. At the time of signing of the contract, the expected payoff of the borrower, whose project has a success rate σ, is given by $\sigma(y-x) + (1-\sigma)(-c)$, and the expected payoff of the lender is given by $\sigma x + (1-\sigma)c$. Of course, the market observes the borrower's choice of c, but not the success rate of his project.

The capital market is assumed to be competitive for the lenders so that in any perfect Bayesian equilibrium the lender earns a constant expected payoff equal to γ (think of γ as the deposit rate that banks must pay back to the depositors from whom banks obtain their capital.) Formally, let $\beta(\sigma_H|c)$ and $\beta(\sigma_L|c)$ be the beliefs of the market when the borrower chooses c dollars of collateral. Then the best-response requirement for the market in the definition of perfect Bayesian equilibrium is satisfied, if for each c that the borrower chooses, the interest rate x satisfies

$$\beta(\sigma_H|c)(\sigma_H x + (1-\sigma_H)c) + \beta(\sigma_L|c)(\sigma_L x + (1-\sigma_L)c) = \gamma.$$

Note that $\beta(\sigma_H|c)$ is part of the perfect Bayesian equilibrium to be determined.

(a) Explain that if the success rate of each project σ were known by the market (i.e. under complete information), the payoff to a borrower who has a project with a success rate σ would be $\sigma y - \gamma$, regardless of the amount of non-liquid wealth he puts up as collateral. (It is assumed that $\sigma_L y > \gamma$ so that both types of borrowers would be willing to borrow under complete information.)

Consider the strategy profile and system of beliefs where both types of borrowers choose $c = 0$ and where the market believes that any borrower that chooses $c > 0$ has a project with success rate σ_L. Formally, consider

$$s^*(\sigma_H) = s^*(\sigma_L) = 0;$$

$$r^*(c) = \begin{cases} \gamma/\overline{\sigma} & \text{if } c = 0 \\ (\gamma - (1-\sigma_L)c)/\sigma_L & \text{if } c > 0; \end{cases}$$

$$\beta(\sigma_H|c) = \begin{cases} p_H & \text{if } c = 0 \\ 0 & \text{if } c > 0 \end{cases}$$

(b) Show that if $\overline{c} < \gamma p_H \sigma_H / \overline{\sigma}$, the above strategy profile and system of beliefs constitute a pooling perfect Bayesian equilibrium. (Hint: A crucial step is to show that type σ_L borrowers never have incentives to choose a positive level of collateral, while each type

σ_H borrower obtains his maximal deviation profits by putting up all his non-liquid wealth, i.e. $\bar{c}$, as collateral.)

(c) Assume that $\gamma p_L \sigma_L/\bar{\sigma} < \bar{c} < \gamma p_H \sigma_H/\bar{\sigma}$. Show that in the pooling equilibrium given above, type σ_L will never choose to offer more than c_s of his non-liquid wealth as collateral, where $c_s = \gamma p_L \sigma_L/\bar{\sigma}$, *even if* the market believes that any borrower who chooses $c \geq c_s$ has a project with success rate σ_H. Use this to argue that the equilibrium given in (b) is not reasonable.

(d) Assume that $\bar{c} > \gamma$. Show that the following strategy profile and system of beliefs constitute a separating perfect Bayesian equilibrium:

$$s^*(\sigma_L) = 0, s^*(\sigma_H) = \gamma;$$

$$r^*(c) = \begin{cases} (\gamma - (1-\sigma_L)c)/\sigma_L & \text{if } c < \gamma \\ (\gamma - (1-\sigma_H)c)/\sigma_H & \text{if } c \geq \gamma; \end{cases}$$

$$\beta(\sigma_H|c) = \begin{cases} 0 & \text{if } c < \gamma \\ 1 & \text{if } c \geq \gamma \end{cases}$$

6. (*Seller versus buyer certification*) Consider a one-time relationship between a seller and a buyer. The quality of the seller's good represents the buyer's value for the good and is either $q_l > 0$ or $q_h = q_l + \delta$, with $\delta > 0$. High quality q_h has production cost $c_h > 0$, while low quality has zero cost. The quality level is known only to the seller, while the buyer only knows that the quality is q_h with probability λ and q_l with $1 - \lambda$, where $0 < \lambda < 1$. High quality delivers a greater surplus, i.e. $\delta > c_h$, but its production cost exceeds the quality expected by the buyer, i.e. $c_h > \lambda q_h + (1-\lambda)q_l$. By incurring a cost of k, with $k \in (0, q_h - c_h)$, the seller or the buyer can certify the actual quality of the seller's good.

You are asked to compare the "seller-certify" game with the "buyer-certify" game, together with the benchmark game with no certification. All three games have the following identical timing except in (iii) below: (i) the seller privately learns the quality of his good; (ii) the seller decides whether to produce and, if he does, sets a price p for the good; (iii) the seller/buyer decides whether to certify; (iv) the buyer decides whether or not to buy. The payoffs in all three games are specified in the same way, except for the possible certification cost: the seller's payoff is the difference between the price and the production cost if the good is produced and traded, with the price set to zero if the good is produced but not traded, minus any certification cost he incurs in the seller-certify game, and is otherwise zero; the buyer's payoff is the difference between the actual quality of the good and the price if she buys the good, minus any certification cost she incurs in the buyer-certification game, and is otherwise zero.

(a) Show that in the benchmark game where certification is unavailable, in any perfect Bayesian Nash equilibrium, the high-quality seller does not produce.

(b) Show that in the seller-certify game, there is a perfect Bayesian equilibrium in which the high-quality seller certifies but the low-quality seller does not, and the good is always produced and traded. Be sure to include in your argument the complete equilibrium strategy of the seller and the buyer and the belief system.

(c) Are there other perfect Bayesian equilibrium outcomes in the seller-certify game? Explain your answer carefully.

(d) For this question and the next question, assume that $\delta > 4k$, and define $p_* = \frac{1}{2}\left(q_h + q_l + \sqrt{(\delta - 4k)\delta}\right)$ and $\beta_* = \frac{1}{2}\left(1 + \sqrt{(\delta - 4k)/\delta}\right)$. Show that in the buyer-certify game, if the buyer faces the price p_* and holds the belief β_* that the good is of high quality, she has three undominated actions, not buy, buy without certification and buy only after certifying the good as high quality, and is indifferent among them.

(e) Show that if $p_* > c_h$ and $\beta_* > \lambda$, in the buyer-certify game there is a perfect Bayesian equilibrium in which the high-quality seller sets the price p_* with probability one, the low-quality seller randomizes between p_* and q_l, and upon observing the good being offered at p_* the buyer randomizes between buying without certification and buying only after certifying the good as high quality.

(f) Are there perfect Bayesian equilibrium outcomes in the buyer-certify game in which the good is always produced and traded? Explain your answers carefully.