Econ 421
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Midterm Review

We will review three key concepts, and give examples to show how they are applied.

- Strictly and weakly dominated strategies.
- Pure-strategy and mixed-strategy Nash equilibrium.
- Subgame perfect Nash equilibrium.

1. Strictly and weakly dominated strategies

Iterated eliminations of strictly dominated strategies.

- Definition of strictly dominated strategies.
- Conceptual motivation.
- Practical application.

Example: A Cournot duopoly.

- Two firms, 1 and 2, simultaneously choosing $q_i \geq 0$, $i = 1, 2$; payoff for $i = 1, 2$ is $q_i(1 - q_i - q_j)$ for $j \neq i$.

- $q_1^* = q_2^* = \frac{1}{3}$ is the unique strategy profile that survives an infinite number of rounds of elimination of strictly dominated strategies.

Weakly dominated strategies.

- Similarities to and difference with strictly dominated strategies.
- Practical implication.

Example: A second-price sealed-bid auction.

- Bidders $1, \ldots, n$ simultaneously submit bids $b_i \geq 0$, $i = 1, \ldots, n$; payoff for each bidder $i = 1, \ldots, n$ is given by $v_i - \max_{j \neq i} b_j$ if $b_i > \max_{j \neq i} b_j$, by $(v_i - \max_{j \neq i} b_j)/(1 + \#\{j \neq i : b_j = b_i\})$ if $b_i = \max_{j \neq i} b_j$, and by 0 otherwise, with $v_1 > v_2 > \ldots > v_n > 0$.

- For each bidder i, bidding $b_i = v_i$ weakly dominates all other bids.

2. Nash equilibrium

Pure-strategy Nash equilibria.

- Verifying Nash equilibrium.
- Finding all Nash equilibria by exhausting all strategy profiles.
- Finding all Nash equilibria by constructing best response functions and identifying all intersections.

Example: Home-court advantage.

- Two teams $i = 1, 2$ each choose effort $a_i \geq 0$ to win a game; payoff to team 1 is $a_1\beta/(a_1\beta + a_2) - a_1$, and payoff to team 2 is $a_2/(a_1\beta + a_2) - a_2$, where $\beta > 1$ is team 1's home-court advantage.

- Find the unique Nash equilibrium $a_1^* = a_2^* = \beta/(1+\beta)^2$ by solving for best responses: team 1's best response is $\sqrt{a_2/\beta} - a_2/\beta$ if $a_2 < \beta$, and 0 otherwise, and team 2's best response is $\sqrt{a_1\beta} - a_1\beta$ if $a_1 < 1/\beta$ and 0 otherwise.

Mixed-strategy Nash equilibria.

- Verifying mixed-strategy Nash equilibrium using the generalized indifference principle.
- Finding mixed-strategy Nash equilibrium in 2-by-2 games.

Example: Sun Bin's wisdom.

- A 6-by-6 game table.
- Sun Bin's wisdom as best response against a naive King.
- Verifying a symmetric mixed-strategy Nash equilibrium with equal probability for each of the 6 strategies.

3. Subgame perfect equilibrium

Key concepts.

- Verifying Nash equilibrium in extensive-form games, including games with simultaneous moves and moves by Nature.
- Verifying subgame perfect equilibrium.
- Finding all subgame perfect equilibria by backward induction.

Example: First mover advantage in patent race.

- Two firms $i = 1, 2$ each choose investment $a_i \geq 0$ to win a patent race; payoff to firm 1 is $a_1\beta/(a_1\beta + a_2) - a_1$, and payoff to 2 is $a_2/(a_1\beta + a_2) - a_2$, with $\beta > 0$.

- We have found unique Nash equilibrium $a_1^{NE} = a_2^{NE} = \beta/(1+\beta)^2$.

- Find unique subgame perfect equilibrium outcome by backward induction: $a_1^{SPE} = \beta/4$ and $a_2^{SPE} = \sqrt{a_1\beta} - a_1\beta$ for $\beta < 2$; $a_1^{SPE} = 1/\beta$ and $a_2^{SPE} = 0$ for $\beta \geq 2$.

4. Further questions

- Strictly dominated strategies and never best responses.
- Iterated eliminations of strictly dominated strategies and mixed-strategy Nash equilibrium.
- Weakly dominated strategies and subgame perfect equilibrium.