Econ 421
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Lecture 5. Applications of SPE

1. The ultimatum game

- The ultimatum game is the simplest bargaining game.

- We have analyzed the discrete version of the game.

Player 1 offers an amount $x \in [0, 1]$ to player 2, who can either accept or reject it (A or R). If accepted, 2 gets x and 1 gets $1 - x$. If rejected, both get 0.

- Each offer x starts a smallest subgame.
- Player 2's best response is A if $x > 0$, and $\{A, R\}$ if $x = 0$.

- There are two possible SPE strategies for 2: choose A for all $x \in [0, 1]$, or choose A for all $x \in (0, 1]$ and R for $x = 0$.

- In the first case, 1's best response is to choose $x = 0$, and in the second case, 1 has no best response.

- A unique SPE: $x = 0$ and A for all $x \in [0, 1]$.

- The outcome is that player 1 offers 0 and player 2 accepts it; there are many NE that are not SPE.

2. Stackelberg duopoly

Consider the following duopoly quantity competition (called Stackelberg duopoly).

- First Firm 1 produces a quantity $q_1 \geq 0$, at a constant marginal cost c.
- After observing Firm 1's choice, Firm 2 chooses $q_2 \geq 0$, at the same c.
- Market-clearing price is given by $a - Q$, with $Q = q_1 + q_2$ and $a > c$.

Firm 1 is the market leader, and 2 is the follower.

- How does the outcome differ from the Nash equilibrium outcome in Cournot duopoly?
- Does Firm 1 necessarily have a first-mover advantage?
- Do the conclusions depend on the assumption of linear demand function?

Find SPE using backward induction.

- Each choice q_1 by Firm 1 corresponds to a smallest subgame.

- Firm 2's best response is $b_2(q_1) = \frac{1}{2}(a - c - q_1)$ if $q_1 \leq a - c$, and 0 otherwise, which is the same as in Cournot duopoly.

- Firm 1 chooses q_1 to maximize $q_1(a - q_1 - b_2(q_1) - c)$: assuming $q_1 \leq a - c$ and hence $b_2(q_1) = \frac{1}{2}(a - c - q_1)$, we have $q_1 = \frac{1}{2}(a - c)$, which is indeed smaller than $a - c$.

- Unique SPE: $\left(\frac{1}{2}(a - c), b_2(q_1)\right)$, with outcome $q_1 = \frac{1}{2}(a - c)$ and $q_2 = \frac{1}{4}(a - c)$.

Comparing SPE in Stackelberg duopoly and NE in Cournot duopoly.

- SPE quantiteis in Stackelberg duopoly: $\left(\frac{1}{2}(a-c), \frac{1}{4}(a-c)\right)$ vs NE quantities in Cournot duopoly: $\left(\frac{1}{3}(a-c), \frac{1}{3}(a-c)\right)$.

- Firm 1 produces more and makes a greater profit as the Stackelberg leader than as a Cournot duopolist.

- Firm 2 produces less and makes a smaller profit as the Stackelberg follower than as a Cournot duopolist.

First-mover advantage of Stackelberg leader holds generally.

- Leader cannot do worse than in Cournot duopoly.

- Leader can do strictly better: by increasing marginally quantity from its Nash equilibrium quantity, Leader gets a strictly greater profit than in Nash equilibrium.

- Stackelberg leader produces more than follower, so long as best response of follower is downward sloping.

Understanding first-mover advantage in Stackelberg duopoly.

- What's important is making quantity commitment known to the opponent, not the timing of move per se.
- Commitment power is necessary to realize first-mover advantage, because leader's equilibrium quantity is not a best response to follower's equilibrium quantity.

Source of first-mover advantage in Stackelberg duopoly.

- Ex ante commitment to action is not beneficial for a decision maker playing against nature.
- Commitment to price has no value in Bertrand duopoly.
- In Cournot duopoly, commitment to quantity yields a first-mover advantage, because appropriate commitment leads to change in opponent's quantity that's beneficial to the firm that makes the commitment.

3. Exiting a declining industry

- At beginning of each period $t = 0, 1, \ldots$, two identical firms, A and B, simultaneously decide whether to exit or stay.

- If a firm exits in period t, its payoff from period t onward is 0; if it is the only one that stays in t, its payoff is μ_t from period t; if both firms stay in t, each gets δ_t; for each terminal history, each firm's payoff is the sum of the payoffs over all periods.

- Declining industry: both μ_t and δ_t are decreasing in t, satisfying $t_1 = \max\{t : \mu_t \geq 0\} > t_2 = \max\{t : \delta_t \geq 0\}$, and $\delta_{t_1-1}+\mu_{t_1} > 0$.

Extensive form.

- Players are A and B.

- A non-terminal history h^t at the beginning of period t is sequence of action profiles $((a_0, b_0), \ldots, (a_{t-1}, b_{t-1}))$, such that: (i) at least one of a_{t-1} and b_{t-1} is *Stay*; (ii) if $a_s = \mathit{Exit}$ for $s = 0, \ldots, t-2$, then $a_{s'} = \mathit{Exit}$ for all $s' = s+1, \ldots, t-1$, and the same holds for firm B.

- A terminal history η^t at the end of period t is $(h^t, (a_t, b_t))$, where h^t is a non-terminal history, and $a_t = b_t = \mathit{Exit}$.

- Player function P assigns each non-terminal history h^t to A and B if $a_{t-1} = b_{t-1} = \mathit{Stay}$, to A if $a_{t-1} = \mathit{Stay}$ and $b_{t-1} = \mathit{Exit}$, and to B if $b_{t-1} = \mathit{Stay}$ and $a_{t-1} = \mathit{Exit}$, with the same set of actions $A(h^t) = \{\mathit{Stay}, \mathit{Exit}\}$.

- For each terminal history η^t, A's payoff is sum of the payoffs over all periods $s = 0, \ldots, t$, where the payoff from period s equals δ_s if $a_s = b_s = \mathit{Exit}$, equals μ_s if $a_s = \mathit{Stay}$ and $b_s = \mathit{Exit}$, and equals 0 if $a_s = \mathit{Exit}$, and analogously for B.

Backward induction.

- We can start backward induction at beginning of period t_1, because firms will exit in $t_1 + 1$ even if they are the only one in period t.
- If A or B is the only one left, it should choose *Stay*, followed by exiting in $t_1 + 1$ and onward.
- If both are still in, there are three Nash equilibria in the subgame: $a_{t_1} = \mathit{Stay}$ and $b_{t_1} = \mathit{Exit}$, $a_{t_1} = \mathit{Exit}$ and $b_{t_1} = \mathit{Stay}$, and each choosing *Exit* with probability $-\delta_{t_1}/(\mu_{t_1} - \delta_{t_1})$, all followed by exiting in $t_1 + 1$ and onward.

Take $a_{t_1} = \mathit{Stay}$ and $b_{t_1} = \mathit{Exit}$ in period t_1.

- Go backward to the beginning of period $t_1 - 1$, assuming $t_1 - 1 > t_2$ so that $\delta_{t_1-1} < 0 < \mu_{t_1-1}$.
  - If A or B is the only one left, it should choose *Stay*, followed by strategies specified in backward induction.
  - If both are still in, since $\delta_{t_1-1} + \mu_{t_1} > 0$ by assumption, there is a unique Nash equilibrium with $a_{t_1-1} = \mathit{Stay}$ and $b_{t_1-1} = \mathit{Exit}$, followed by strategies specified in backward induction.

- Go backward to the beginning of period t_1-2, assuming $t_1-2 > t_2$ so that $\delta_{t_1-2} < 0 < \mu_{t_1-2}$.
  - If A or B is the only one left, it should choose *Stay*, followed by strategies specified in backward induction.
  - If both are still in, since $\delta_{t_1-2}+\mu_{t_1-1}+\mu_{t_1} > \delta_{t_1-1}+\mu_{t_1} > 0$, there is a unique Nash equilibrium with $a_{t_1-2} = \textit{Stay}$ and $b_{t_1-2} = \textit{Exit}$, followed by strategies specified in backward induction.

- Backward induction continues until beginning of period t_2.
  - If A or B is the only one left, it should choose *Stay*, followed by strategies specified in backward induction.
  - If both are still in, assuming $\delta_{t_2} > 0$, there is a unique Nash equilibrium with $a_{t_2} = b_{t_2} = Stay$, followed by strategies specified in backward induction.

- We have completed backward induction.
  - The subgame perfect equilibrium is given by: for A, regardless of whether or when B has exited, choose *Stay* for periods from 0 through to t_1 and *Exit* from period $t_1 + 1$ onwards; for B, choose *Stay* for periods from 0 through to t_2 regardless of whether or when A has exited, choose *Stay* if A has already exited and *Exit* otherwise for periods from $t_2 + 1$ through to t_1, choose *Exit* regardless of whether or when A has exited from period $t_1 + 1$ onwards.

Now we take each choosing *Exit* with probability $-\delta_{t_1}/(\mu_{t_1} - \delta_{t_1})$.

- Go backward to the beginning of period t_1-1, assuming $t_1-1 > t_2$ so that $\delta_{t_1-1} < 0 < \mu_{t_1-1}$.
  - If A or B is the only one left, it should choose *Stay*, followed by strategies specified in backward induction.
  - If both are left, there are three Nash equilibria, $a_{t_1-1} = Stay$ and $b_{t_1-1} = Exit$, $a_{t_1-1} = Exit$ and $b_{t_1-1} = Stay$, and each choosing *Exit* with probability $-\delta_{t_1-1}/(\mu_{t_1-1}+\mu_{t_1}-\delta_{t_1-1})$, all followed by strategies already specified in backward induction.

- If we select either pure-strategy Nash equilibrium in $t_1 - 1$, we can go backwards in the same way as in previous case, and so let's select the mixed-strategy Nash equilibrium, and go backward to the beginning of period $t_1 - 2$, assuming $t_1 - 2 > t_2$ so that $\delta_{t_1-2} < 0 < \mu_{t_1-2}$.
  - If both are still in, there are again three Nash equilibria, and we can continue to select mixed-strategy Nash equilibrium, with each firm choosing *Exit* with a probability equal to $-\delta_{t_1-2}/(\mu_{t_1-2} + \mu_{t_1-1} + \mu_{t_1} - \delta_{t_1-2})$, followed by strategies already specified in backward induction.

- We have completed backward induction.
  - The symmetric subgame perfect equilibrium is given by: for periods from 0 through to t_2 choose *Stay* regardless of whether or when the other firm has exited; for each period s from t_2+1 through to t_1, choose *Stay* if the other firm has already exited and otherwise choose *Exit* with a probability equal to $-\delta_s/(\sum_{s'=s}^{t_1} \mu_{s'} - \delta_s)$; from periods $t_1 + 1$ onwards, choose *Exit* regardless of whether the other firm has exited.