Econ 421
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Lecture 2. Applications of Nash Equilibrium

1. Cournot oligopoly

- Oligopoly: A fixed number of producers of a homogeneous good compete strategically.
  - Cournot oligopoly: quantity competition.
  - Bertrand oligopoly: price competition.

- Relate Nash equilibrium to competitive equilibrium and monopoly.
  - Competitive equilibrium: firms are price-taking.
  - Monopoly: firms collude.
- In the same oligopoly environment, quantity competition and price competition lead to drastically different equilibrium outcomes.

A formal model of Cournot Oligopoly

- Players: N firms.

- Strategies: Each firm j chooses $q_j \geq 0$.

- Payoff to each firm j is its profit $\pi_j(q_1, \ldots, q_N) = q_j\,(P(Q) - c)$, where $Q = \sum_{j=1}^{N} q_j$ is the total quantity, inverse demand function $P(Q)$ is downward-sloping and continuously differentiable, and c is the constant marginal cost.

- Assume that $P(Q) > c$ for Q sufficiently small and $P(Q) < c$ for Q sufficiently large.

Find Nash equilibrium using best responses.

- The best response for firm j to q_{-j} satisfies the first-order condition $P(q_j + Q_{-j}) + q_j P'(q_j + Q_{-j}) = c$, where $Q_{-j} = \sum_{i \neq j} q_i$.

- Each firm j in Nash equilibrium produces the same amount q^*, implicitly given by the intersection of best response functions: $P(Nq^*) - c = -q^* P'(Nq^*)$.

- Equilibrium q^* depends on N, denoted as $q^*(N)$.

- Equilibrium price is $P(Nq^*(N))$.

Nash equilibrium, monopoly, and competitive equilibrium.

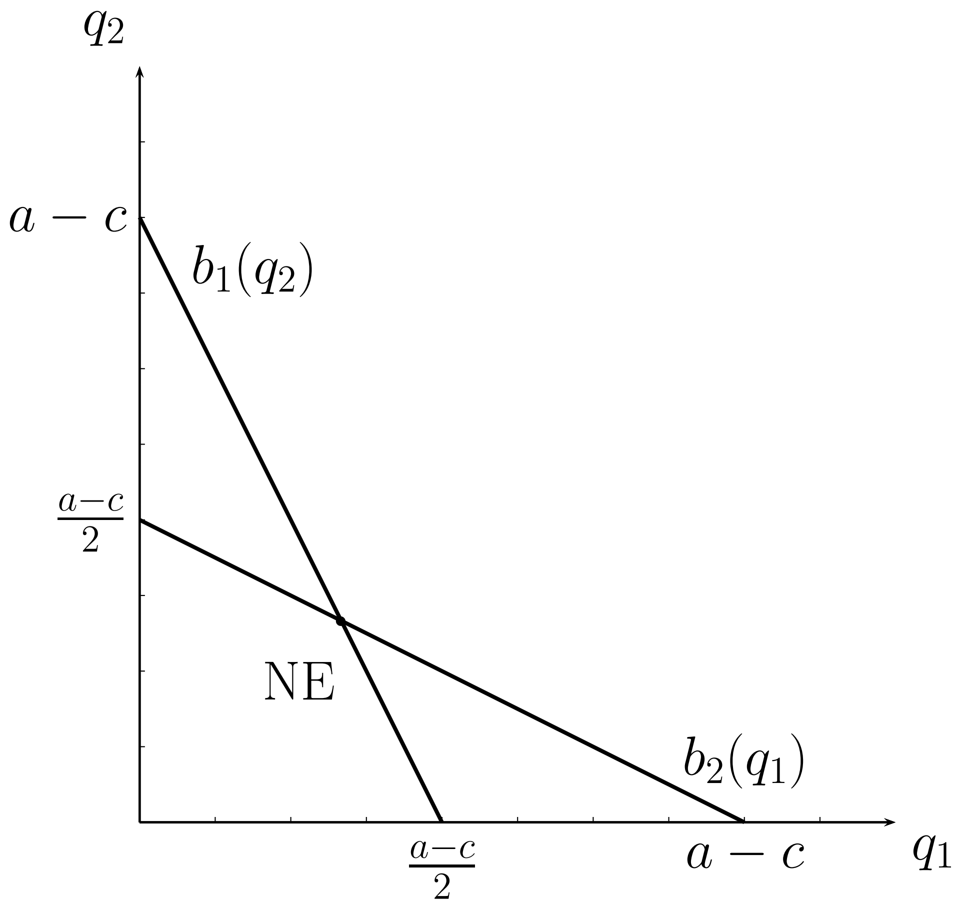
- Since $P' < 0$, in Nash equilibrium $P(Nq^*(N)) > c$.
- When $N = 1$, Nash equilibrium coincides with monopoly.
- When $N \to \infty$, Nash equilibrium converges to the competitive equilibrium, with $P(Nq^*(N)) \to c$ and $q^*(N) \to 0$.

Example (Cournot Duopoly with linear demand). Suppose $N = 2$ and $P(Q) = a - Q$, where $a > c$.

- Firm 1's best response function is given by $b_1(q_2) = \frac{1}{2}(a - c - q_2)$ if $q_2 \leq a - c$, and 0 otherwise; Firm 2's best response function is $b_2(q_1) = \frac{1}{2}(a - c - q_1)$ if $q_1 \leq a - c$, and 0 otherwise.

- The Nash equilibrium is the intersection of the two best response functions: $q_1^* = q_2^* = \frac{1}{3}(a - c)$.

- Illustration: a unique Nash equilibrium $\left(\frac{1}{3}(a-c), \frac{1}{3}(a-c)\right)$.



2. Bertrand oligopoly

Price competition versus quantity competition, with the same demand and costs.

- In Cournot Oligopoly, firms choose quantities and then price is determined by demand.

- In Bertrand Oligopoly, firms choose price and supply the quantity demanded from their firm at that price: consumers buy only from the firm offering the lowest price.

A formal model of Bertrand Duopoly.

- Players: Firm 1 and firm 2.
- Strategies: Price $p_j \in [0, \infty)$ for each firm j.
- Payoffs: with c the constant marginal cost for both firms, and $Q(p)$ the total demand decreasing and continuous, satisfying $Q(c) > 0$:

$$\pi_1(p_1, p_2) = \begin{cases} Q(p_1)(p_1 - c) & \text{if } p_1 < p_2, \\ \frac{1}{2}Q(p_1)(p_1 - c) & \text{if } p_1 = p_2, \\ 0 & \text{if } p_1 > p_2. \end{cases}$$

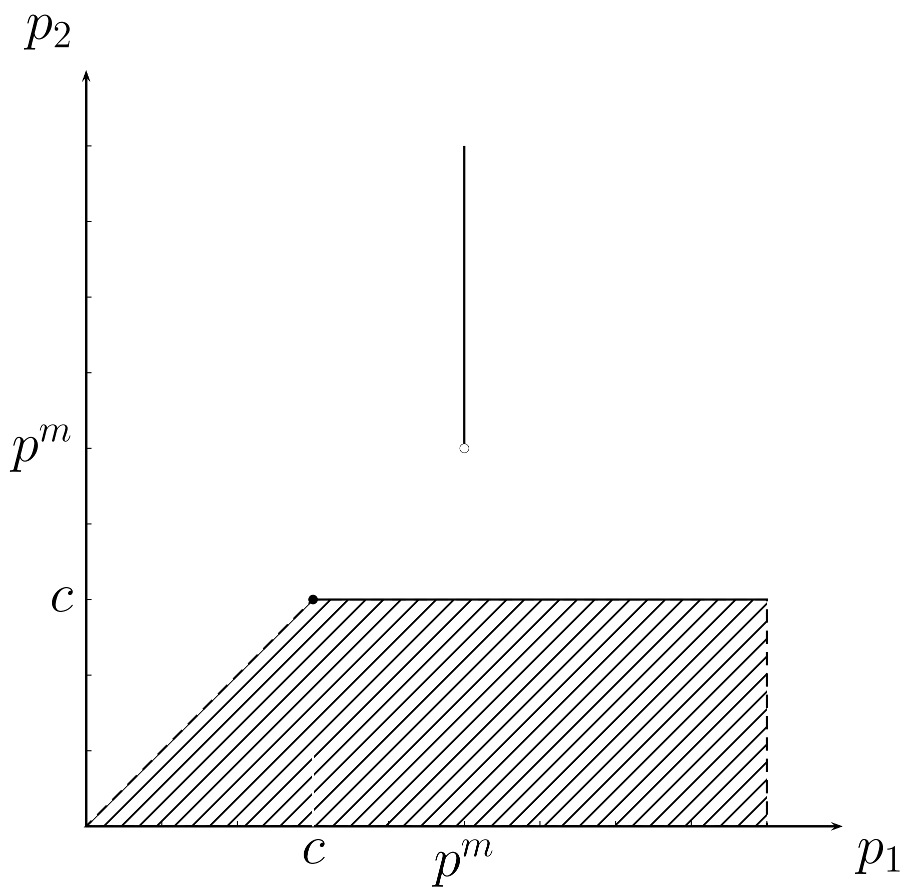
Bertrand Duopoly has a unique Nash equilibrium given by $p_1 = p_2 = c$.

- Verify that (c, c) is a Nash equilibrium.
- Rule out all other strategy profiles:
  - $p_i < c$ for $i = 1$ or 2;
  - $p_i = c$ and $p_j > c$ for $i \neq j$;
  - $p_1 = p_2 > c$;
  - $p_i > c$, $p_j > c$ but $p_i \neq p_j$.

For $Q(p) = a - p$, we can find the unique Nash equilibria using best response functions (where p^m is the monopoly price $\frac{1}{2}(a+c)$):

$$B_1(p_2) = \begin{cases} (p_2, \infty) & \text{if } p_2 < c, \\ [c, \infty) & \text{if } p_2 = c, \\ \emptyset & \text{if } c < p_2 \leq p^m, \\ \{p^m\} & \text{if } p_2 > p^m. \end{cases}$$

- Illustration: $B_1(p_2)$ and a unique Nash equilibrium (c, c).



3. The Hotelling-Downs model

- Consider two politicians competing for office.
  - They simultaneously commit to policy on left-right spectrum.
  - Both politicians care only about winning.
  - Voters' preferred policies are uniformly distributed, with each voting for whoever offers a closer policy.
  - The winner is the one with more than half of the votes.
- Related game: firms compete for customers by choosing location.

A formal model:

- Two players 1,2.
- Strategies $s_1, s_2 \in [0,1]$.
- Payoffs: $u_1(s_1, s_2) = 1, \frac{1}{2}, 0$ if 1's vote share $r(s_1, s_2) >, =, < \frac{1}{2}$, with $u_2(s_1, s_2) = 1 - u_1(s_1, s_2)$, and $r(s_1, s_2)$ given by

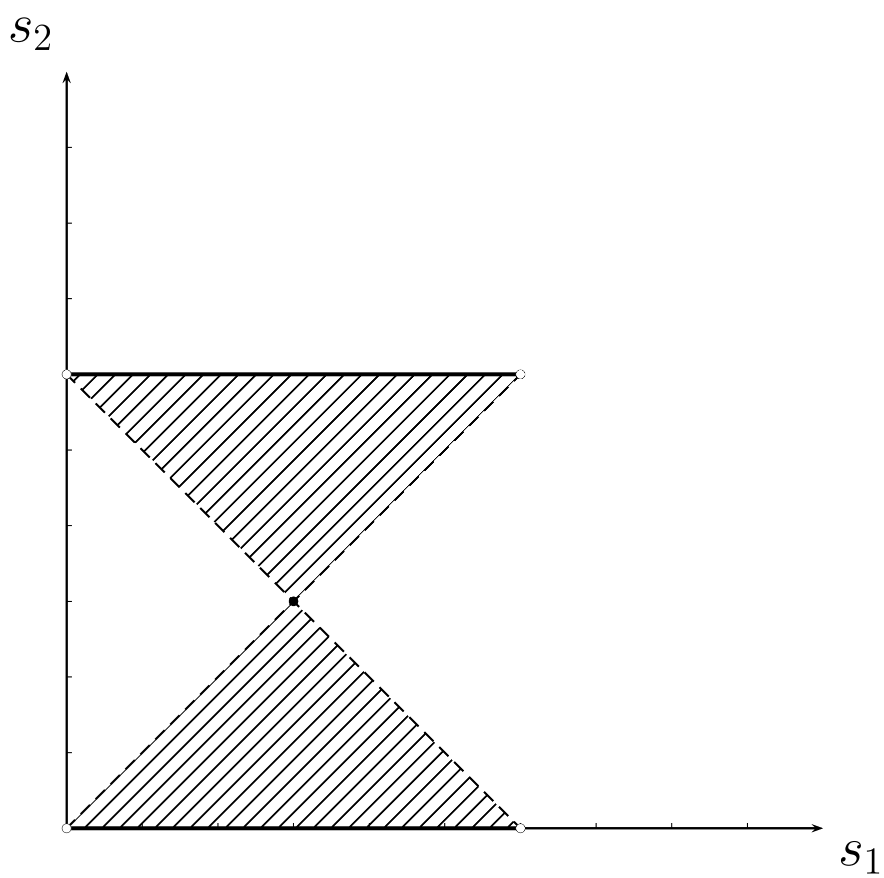
$$r(s_1, s_2) = \begin{cases} \frac{1}{2}(s_1 + s_2) & \text{if } s_1 < s_2, \\ \frac{1}{2} & \text{if } s_1 = s_2, \\ 1 - \frac{1}{2}(s_1 + s_2) & \text{if } s_1 > s_2. \end{cases}$$

- 1's best response to any s_2:

$$B_1(s_2) = \begin{cases} (s_2, 1 - s_2) & \text{if } s_2 < \frac{1}{2}, \\ \{\frac{1}{2}\} & \text{if } s_2 = \frac{1}{2}, \\ (1 - s_2, s_2) & \text{if } s_2 > \frac{1}{2}. \end{cases}$$

- $B_2(s_1)$ is symmetric.

- Illustration: $B_1(s_2)$ and a unique Nash equilibrium $\left(\frac{1}{2}, \frac{1}{2}\right)$.



4. Public goods

- A *public good*, such as national defence, radio broadcasts, and lighthouses, is
  - non-rival, in that each person's consumption has no effect on the quantity available to others;
  - non-excludable, in that it is impossible to prevent some but not all individuals from consuming the good.

A formal model of private provision of a public good.

- Players: N consumers.

- Strategies: Each i chooses $x_i \geq 0$ units of the public good to provide/purchase.

- Payoffs: $u_i(x_1 + \cdots + x_N) - px_i$ for each i, where $u_i(\cdot)$ is i's utility from consumption of the public good, with $u'(x) > 0 > u''(x)$ for all x and $u_i'(0) > p > u'(\infty)$, and $p > 0$ is the private cost of the public good, is constant p per unit.

Characterizing the best response function $b_i(X_{-i})$, where $X_{-i} = \sum_{j \neq i} x_j$.

- By definition, $b_i(X_{-i})$ solves $\max_{x \geq 0} u_i(x + X_{-i}) - px$, with the first order condition $u_i'(b_i(X_{-i}) + X_{-i}) = p$ if $b_i(X_{-i}) > 0$.

- Then,

$$b_i(X_{-i}) = \begin{cases} x_i^* & \text{if } X_{-i} = 0, \\ x_i^* - X_{-i} & \text{if } 0 < X_{-i} < x_i^*, \\ 0 & \text{if } X_{-i} > x_i^*, \end{cases}$$

where $x_i^* = b_i(0) > 0$ is the private optimal consumption.

Assuming that $x_1^* > x_2^* \geq \ldots \geq x_N^*$, we have a unique Nash equilibrium given by $(x_1, \ldots, x_N) = (x_1^*, 0, \ldots, 0)$.

- Verify this is indeed an equilibrium: for player 1, x_1^* is a best response to $X_{-1} = 0$; for player $i \geq 2$, $x_i = 0$ is a best response to $X_{-i} = x_1^*$.

- Show that it is the only equilibrium: suppose $x_i > 0$ for some $i \geq 2$; from i's best response function, $\sum_j x_j = x_i^* < x_1^*$; from 1's best response function, $\sum_j x_j \geq x_1^*$, a contradiction.

Inefficiency of Nash equilibrium.

- Pareto efficient total quantity x^* solves $\max_{y \geq 0} \sum_{i=1}^{N} u_i(y) - py$, determined by the first order condition $\sum_i u_i'(x^*) = p$.

- Under-provision of public goods in Nash equilibrium: $x_1^* < x^*$.

- Nash equilibrium quantity x_1^* is inefficient low: if any player i other than player 1 contributes a share equal to $u_i'(x_1^*)/p$ of the total infinitesimal amount dx, with player 1 contributing the remaining share, then player i would be just indifferent, but all other players including player 1 are strictly better off.