Econ 421
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Lecture 10. Applications of PBE

1. Education as costly signal

- Firms pay higher starting wages to workers with higher education.
  - Statistically, worker's productivity is positively correlated with education level.
  - Individual worker's productivity is difficult to measure, as it also depends worker's innate ability.

- Two explanations for higher wages for higher education.
  - Eduction improves productivity given any worker ability level.
  - Education is a costly signal of worker's ability.

Job market signaling game

- Worker learns type (ability), which is $t = L > 0$ with probability p_L or $H > L$ with $p_H = 1 - p_L$, and then chooses education level $e \geq 0$, which is observed by Firm before choosing wage w.
- Payoff to Worker of type t is $w - c_t(e)$, with $c_t(e)$ increasing and convex, satisfying $c'_L(e) > c'_H(e)$: indifference curve of type H can cross that of type L at most once (single-crossing).
- Payoff to Firm is $-(w - t)^2$: maximizing expected payoff requires Firm to choose w equal to expected type.

- Consider any separating equilibrium with strategy profile (s^*, r^*) and belief system β.
  - Since $s^*(L) \neq s^*(H)$, consistency requires $\beta(t|s^*(t)) = 1$, for both $t = H$ and $t = L$.
  - $r^*(s^*(t)) = t$ for both $t = H$ and t_L.
  - $s^*(L) = 0$.
  - $s^*(H)$ satisfies both $H - c_H(s^*(H)) \geq L - c_H(0)$, and also $H - c_L(s^*(H)) \leq L - c_L(0)$.

- There is a unique *least cost* separating level $\underline{e}_s$, which satisfies

  $H - c_L(\underline{e}_s) = L - c_L(0)$.

- The least-cost separating equilibrium (s^*, r^*) with β such that:

  - $s^*(L) = 0$, $s^*(H) = \underline{e}_s$.

  - $r^*(e) = \beta(H|e)H + \beta(L|e)L$.

  - $\beta(H|e) = 0$ for all $e < \underline{e}_s$ and $\beta(H|e) = 1$ for all $e \geq \underline{e}_s$.

- Properties of the least cost separating equilibrium.
  - No education for either type if t were known.
  - Type L gets his complete information payoff.
  - Type H chooses minimum education to differentiate himself from type L.
  - Incomplete information thus makes type H worse off without making type L better off.

2. Out-of-path beliefs in signaling games

Job market signaling game has a continuum of separating equilibria with higher education than the least cost separating level for the high type.

- Let $\overline{e}_s > \underline{e}_s$ be such that $H - c_H(\overline{e}_s) = L - c_H(0)$.
- For each $\hat{e} \in [\underline{e}_s, \overline{e}_s]$, there is a separating equilibrium (s^*, r^*, β):
  - $s^*(L) = 0$, $s^*(H) = \hat{e}$.
  - $r^*(e) = \beta(H|e)H + \beta(L|e)L$.
  - $\beta(H|e) = 0$ for all $e < \hat{e}$ and $\beta(H|e) = 1$ for all $e \geq \hat{e}$.

There are also pooling equilibria in Job market signaling game.

- Let $\overline{e}_p < \underline{e}_s$ be such that $p_H H + p_L L - c_L(\overline{e}_p) = L - c_L(0)$.

- For each $\hat{e} \leq \overline{e}_p$, there is a pooling equilibrium (s^*, r^*, β):

  - $s^*(L) = s^*(H) = \hat{e}$.

  - $r^*(e) = \beta(H|e)H + \beta(L|e)L$.

  - $\beta(H|e) = 0$ for all $e \neq \hat{e}$ and $\beta(H|\hat{e}) = p_H$.

Separating equilibria with higher-than-necessary education and pooling equilibria are constructed with help of out-of-path beliefs.

- The beliefs are not constrained by Bayes' rule.
- But are they "reasonable"?
- What are reasonable restrictions on out-of-path beliefs that allow us to refine perfect Bayesian equilibria?

In economic applications of costly signaling model, "Intuitive Criterion" is used to restrict on out-of-equilibrium beliefs and to refine PBE.

- Fix a PBE (s^*, r^*, β) and let $U_S^*(t)$ be equilibrium payoff of Sender.
- For each message $m \in M$, and for each subset of types $F \subseteq T$, let $B(F, m) \subset A$ be set of best responses of Receiver to m when his belief is restricted to F, and let $D(m)$ be set of types t such that $U_S^*(t) > \max_{a \in B(T,m)} u_S(m, a, t)$.
- PBE (s^*, r^*, β) fails Intuitive Criterion if for some message $m \in M$ there is type $t' \in T$ such that $U_S^*(t') < \min_{a \in B(T \setminus D(m), m)} u_S(m, a, t')$.

Remarks on Intuitive Criterion.

- If m is on the path, $D(m)$ must not contain any t that sends m with positive probability, and thus $r^*(m) \in B(T \setminus D(m), m)$, implying the condition for failing Intuitive Criterion cannot hold for any t, regardless of whether t sends m with positive probability.

- For an out-of-path message m, any type in $D(m)$ in equilibrium is strictly worse off by sending m even under the most favorable belief of Receiver, and once $D(m)$ is excluded from Receiver's belief, type t' will in equilibrium be strictly better off by sending m even under the least favorable belief of Receiver.

Ruling out separating equilibria in Job market signaling with higher than necessary education $\hat{e} \in (\underline{e}_s, \overline{e}_s]$.

- Consider any out-of-path message $\tilde{e} \in (\underline{e}_s, \hat{e})$.
- $D(\tilde{e})$ contains type L but not type H.
- The out-of-equilibrium belief upon $\tilde{e}$ should put all weight on type H, but then the condition for failing Intuitive Criterion holds for type H.

Ruling out pooling equilibria in Job market signaling $\hat{e} \leq \overline{e}_p$.

- Consider out-of-path $\tilde{e}$ such that $H - c_L(\tilde{e}) < p_H H + p_L L - c_L(\hat{e})$ and $H - c_H(\tilde{e}) > p_H H + p_L L - c_H(\hat{e})$: such message $\tilde{e}$ exists by single-crossing.
- $D(\tilde{e})$ contains type L but not type H.
- The out-of-equilibrium belief upon $\tilde{e}$ should put all weight on type H, but then the condition for failing Intuitive Criterion holds for type H.

Least-cost separating equilibrium survives Intuitive Criterion.

- For any $e > \underline{e}_s$, we have $D(e) = T$, and the condition for failing Intuitive Criterion is never satisfied.

- For any $e < \underline{e}_s$, we have $D(e) = \emptyset$, and the condition for failing Intuitive Criterion is never satisfied.