Econ 420
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Li, Hao
UBC

Problem Set 7

1. (*Production Theory*) A firm chooses vectors x of inputs and y of outputs subject to a production possibility constraint $G(x, y) \leq 0$, to maximize profit $qy - px$, where q is the row vector of output prices and p is the row vector of the input prices. Let $\Pi(q, p) = \max_{x,y}\{qy - px \mid G(x, y) \leq 0\}$ be the maximized profit function.

   (a) Show that $\Pi(q, p)$ is convex in (q, p). [This is the same as Question 2 in Problem Set 5; answer it if you got it wrong.]

   (b) Use the value function Π to derive the optimum output supplies y and the optimum input demands x, both as functions of q and p.

   (c) Show that the optimum supply of each output is non-decreasing in the price of that output, and the optimum demand for each input is non-increasing in the price of that input. [For this question you are free to use the result that if a function is convex, then the matrix of the second derivatives evaluated at any point is positive semi-definite.]

2. (*Properties of Minimum Cost Function*) The production function is given by $f(x)$ where x is the vector of inputs. Let w be the row vector of input prices. Consider the problem of choosing x to minimize the total production cost wx subject to meeting to the output target y, i.e., $f(x) \geq y$. Define $C(w, y) = \min_x\{wx \mid f(x) \geq y\}$ as the resulting minimum cost function.

(a) Fix an arbitrary y. Prove that, as a function of w, $C(w, y)$ is increasing, concave, and homogeneous of degree 1 (i.e., $C(\alpha w, y) = \alpha C(w, y)$ for all non-negative scalar α).

(b) Suppose that the production function f exhibits constant returns to scale, i.e., $f(\alpha x) = \alpha f(x)$ for all non-negative scalar α. Show that there exists a function $c(w)$ such that $C(w, y) = yc(w)$.

3. (*More on Derived Demand*) Let $C(w, y)$ be the minimum cost function, given by $\min_x\{wx \mid f(x) \geq y\}$, where x is an n-dimensional column vector of inputs, w the corresponding n-dimensional row vector of input prices, y a scalar output, and f the production function. Assume that $C_{yy} > 0$. Consider the problem of choosing the output y to maximize the profit $py - C(w, y)$, where p is the output price. The solution to the profit maximization problem, denoted as y, is a function of p and w. The derived demand for inputs, denoted as x, is the solution to the cost minimization problem of $\min_x\{wx \mid f(x) \geq y\}$, and is therefore also a function of p and w.

(a) Use the minimum cost function C to derive a set of equations that jointly determines x and y as functions of p and w.

(b) Use your answer to (a) to derive an expression for $\partial x_j / \partial w_k$ for any $j, k = 1, ..., n$. Why does the sign of $\partial x_j / \partial w_k$ depend on the sign of $\partial^2 C / \partial y \partial w_k$? Interpret your result.