Econ 420
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Problem Set 6

1. (*Quasi-Convex Programming*) Consider the problem of maximizing px subject to $G(x) \leq c$, where the choice variable x is a n-dimensional column vector, p is a n-dimensional row vector, and G is quasi-convex and differentiable. Show that if there exist a vector x and a positive number λ such that $p - \lambda G_x(\overline{x}) = 0$ where $G_x(\overline{x})$ is the row vector of derivatives of G with respect to x, then $\overline{x}$ is a solution to the maximization problem. (Hint: you first need to use quasi-convexity of G to establish that $G_x(\overline{x})(x' - \overline{x}) \leq 0$ for any x' such that $G(x') \leq G(\overline{x})$.)

2. (*Convexity of Maximum Value Function*) Let θ be a vector of parameters, and consider the problem of choosing a vector x to maximize $F(x, \theta)$ subject to a vector of constraints $G(x) \leq c$. Let $V(\theta)$ denote the maximum value function.

(a) Show that if F is convex in θ for any x, then V is convex in θ.

(b) Use your result in (a) to establish the concavity of the minimum cost function in θ, defined as $\min_x\{\theta x | G(x) \geq c\}$, where θ is a row vector of input prices, the choice variable x is a column vector of inputs, G is the production function and c is an output target.

3. (*More on Linear Programming*) Consider a linear programming problem of choosing an n-dimensional column vector x to maximize ax subject to $Bx \leq c$ and $x \geq 0$, where a is an n-dimensional row vector, B an m-by-n matrix, and c an m-dimensional row vector.

Let the Lagrangian be $L(x, \lambda) = ax + \lambda(c - Bx)$, where λ is an m-dimensional vector of multipliers.

(a) Show that if $\overline{x}$ is a solution to the linear programming problem, with the corresponding vector of multipliers $\overline{\lambda}$, then $L(x, \overline{\lambda}) \leq L(\overline{x}, \overline{\lambda}) \leq L(\overline{x}, \lambda)$ for all $x \geq 0$ and $\lambda \geq 0$.

(b) Let $V(a, c)$ be the maximum value function of the linear programming problem. Show that V is convex in a for each c, and concave in c for each a.