Econ 420
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Problem Set 3

1. (*The Invisible Hand — Production*) Consider an economy with C consumers, G goods, and F factors of production. The utility u_c of each consumer c, $c = 1, \ldots, C$, is given by a strictly increasing function $U^c(x_{c1}, ..., x_{cG})$, where each x_{cg}, $g = 1, \ldots, G$, is consumer c's consumption of good g. The output X_g of each good g, $g = 1, \ldots, G$, is given by a strictly increasing production function $\Phi^g(z_{1g}, \ldots, z_{Fg})$, where each z_{fg}, $f = 1, \ldots, F$, is the input of factor f. The total amount of inputs Z_f of each factor f, $f = 1, \ldots, F$, is fixed. A social planner chooses factor inputs z_{fg} for each $f = 1, \ldots, F$ and each $g = 1, \ldots, G$, and consumption levels x_{cg} for each $c = 1, \ldots, C$ and each $g = 1, \ldots, G$, to maximize a strictly increasing social welfare function $W(u_1, \ldots, u_C)$, subject to the production and distribution constraints $\sum_{c=1}^{C} x_{cg} = \Phi^g(z_{1g}, \ldots, z_{Fg})$ for each $g = 1, \ldots, G$, and the resource constraints $\sum_{g=1}^{G} z_{fg} = Z_f$ for each $f = 1, \ldots, F$.

(a) Write down the first-order conditions for the planners problem, and give an interpretation of the multipliers.

Now imagine that there is no social planner to direct production and distribution. Instead, each consumer c, $c = 1, \ldots, C$, chooses his own consumption level x_{cg}, $g = 1, \ldots, G$, to maximize his utility $u_c = U^c(x_{c1}, \ldots, x_{cG})$, subject to a budget constraint $\sum_{g=1}^{G} x_{cg}\pi_g = I_c$, where each π_g is the Lagrange multiplier associated with the constraint $\sum_{c=1}^{C} x_{cg} = \Phi^g(z_{1g}, \ldots, z_{Fg})$ in the planners solution, and I_c is consumer c's fixed income. For each good g, $g = 1, \ldots, G$, there is a single firm that chooses factor inputs z_{fg}, $f = 1, \ldots, F$, to maximize its profit $\pi_g \Phi^g(z_{1g}, \ldots, z_{Fg}) - \sum_{f=1}^{F} z_{fg}\mu_f$, where each μ_f is the Lagrange multiplier associated with the constraint $\sum_{g=1}^{G} z_{fg} = Z_f$ in the planners solution.

(b) Show that with appropriate choices of I_c, $c = 1, \ldots, C$, the first order conditions for the utility maximization problems of the consumers and the profit maximization problems of the firms are identical to the first-order conditions for the planners problem.

(c) Prove that the sum $\sum_{c=1}^{C} I_c$ is equal to the value of the aggregate output. Give an economic interpretation of this result.

2. (*The Invisible Hand — Factor Supplies*) Consider an economy with C consumers, G goods, and F factors of production. The utility u_c of each consumer c, $c = 1, \ldots, C$, is given by a function $U^c(x_{c1}, \ldots, x_{cG}, z_{c1}, \ldots, z_{cF})$, where each x_{cg}, $g = 1, \ldots, G$, is consumer c's consumption of good g, and each z_{cf}, $f = 1, \ldots, F$, is consumer c's supply of factor f. The function U^c is strictly increasing in each x_{cg} and strictly decreasing in each z_{cf}. The output X_g of each good g, $g = 1, \ldots, G$, is given by a strictly increasing production function $\Phi^g(z_{1g}, \ldots, z_{Fg})$, where each z_{fg}, $f = 1, \ldots, F$, is the input of factor f. A social planner chooses factor inputs z_{fg} for each $f = 1, \ldots, F$ and each $g = 1, \ldots, G$, factor supplies z_{cf} and consumption levels x_{cg} for each $c = 1, \ldots, C$, each $f = 1, \ldots, F$ and each $g = 1, \ldots, G$, to maximize a strictly increasing social welfare function $W(u_1, \ldots, u_C)$, subject to the production and distribution constraints $\sum_{c=1}^{C} x_{cg} = \Phi^g(z_{1g}, \ldots, z_{Fg})$ for each $g = 1, \ldots, G$, and the factor constraints $\sum_{g=1}^{G} z_{fg} = \sum_{c=1}^{C} z_{cf}$ for each $f = 1, \ldots, F$.

(a) Write down the first-order conditions for the planners problem, and give an interpretation of the multipliers.

Now imagine each consumer c, $c = 1, \ldots, C$, chooses his own consumption level x_{cg}, $g = 1, \ldots, G$, and his own factor supply z_{cf}, $f = 1, \ldots, F$ to maximize his utility

$u_c = U^c(x_{c1}, \ldots, x_{cG}, z_{c1}, \ldots, z_{cF})$, subject to a budget constraint $\sum_{g=1}^{G} x_{cg}\pi_g = I_c + \sum_{f=1}^{F} z_{cf}\mu_f$, where each π_g is the Lagrange multiplier associated with the constraint $\sum_{c=1}^{C} x_{cg} = \Phi^g(z_{1g}, \ldots, z_{Fg})$ in the planners solution, I_c is consumer c's fixed income, and each μ_f is Lagrange multiplier associated with the constraint $\sum_{g=1}^{G} z_{fg} = \sum_{c=1}^{C} z_{cf}$ in the planners solution. Also, for each good g, $g = 1, \ldots, G$, there is a single firm that chooses factor inputs z_{fg}, $f = 1, \ldots, F$, to maximize its profit $\pi_g\Phi^g(z_{1g}, \ldots, z_{Fg}) - \sum_{f=1}^{F} z_{fg}\mu_f$.

(b) Show that with appropriate choices of I_c, $c = 1, \ldots, C$, the first order conditions for the utility maximization problems of the consumers and the profit maximization problems of the firms are identical to the first-order conditions for the planners problem.

(c) Prove that the sum $\sum_{c=1}^{C} I_c$ is equal to the total profit in production.

3. (*Borrowing and Lending*) Consider a consumer planning his consumption over two years. In each year $i = 1, 2$, there are two goods to consume, with prices p_i and q_i, and quantities denoted as x_i and y_i. The budget constraint in each year is $p_i x_i + q_i y_i = I_i$, where I_i is the fixed positive income in year i. The utility function over the two years is $\sum_{i=1}^{2} \alpha_i \ln(x_i) + \beta_i \ln(y_i)$, where α_i and β_i are positive parameters. The consumer chooses x_i and y_i, $i = 1, 2$, to maximize the above utility function subject to the two budget constraints.

(a) Find the multipliers associated with the two budget constraints. Discuss how they depend on the parameters α_i, β_i and I_i, $i = 1, 2$.

(b) Give an interpretation of the multipliers that relates to borrowing and lending, and the interest rate.