Econ 420
Fall, 2022
Li, Hao
UBC

Problem Set 2

1. (*Rationing*) Suppose a consumer has the utility function $U(x_1, x_2, x_3) = \sum_{j=1}^{3} \alpha_j \ln(x_j)$, where each α_j is a positive constant, and $\sum_{j=1}^{3} \alpha_j = 1$. The budget constraint is given by $\sum_{j=1}^{3} p_j x_j \leq I$, where I and each p_j are positive constants. In addition, the consumer faces a rationing constraint that $x_1 \leq k$, where k is a positive constant.

(a) Solve the consumers optimization problem. Under what condition is the rationing constraint binding?

(b) Show that when the rationing constraint binds, the income that the consumer would have liked to spend on good 1 but can not is split between good 2 and good 3 in the proportions $\alpha_2 : \alpha_3$.

2. (*Distribution Between Envious Consumers*) There is a fixed total Y of goods to be allocated between consumers 1 and 2. If consumer 1 gets Y_1 and consumer 2 gets Y_2, the utility to consumer 1 is $U_1 = Y_1 - kY_2^2$ and the utility to consumer 2 is $U_2 = Y_2 - kY_1^2$, where k is a positive constant. Consider the problem of maximizing $U_1 + U_2$ subject to the resource constraint $Y_1 + Y_2 \leq Y$.

(a) Show that if $Y > 1/k$, the resource constraint is slack at the optimum.

(b) Give an interpretation of the result in (a).

3. (*Investment Allocation*) A positive capital sum C is to be allocated among n investment projects. If a non-negative amount x_j is allocated to project j for $j = 1, \ldots, n$, the expected

return from this allocation is $\sum_{j=1}^{n}\left(\alpha_j x_j - \frac{1}{2}\beta_j x_j^2\right)$, where α_j and β_j are positive. Let $H = \sum_{j=1}^{n}(\alpha_j/\beta_j)$ and $K = \sum_{j=1}^{n}(1/\beta_j)$. Consider the problem of maximizing the expected return by choosing a non-negative funding x_j, $j = 1, \ldots, n$, subject to $\sum_{j=1}^{n} x_j \leq C$.

(a) Show that if $C > H$, then the constraint $\sum_{j=1}^{n} x_j \leq C$ is not binding.

(b) Show that if $\alpha_j > (H - C)/K$ for all j, then each project receives some funding.

(c) Show that if some project j receives zero funding and another project j' receives some funding, then $\alpha_j < \alpha_{j'}$.