Econ 420
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Problem Set 1

1. (*The Cobb-Douglas Utility Function*) Consider a consumer choosing between two goods x and y, with prices p and q respectively. The consumers utility function is $x^\alpha y^\beta$, where α and β are positive constants. The budget constraint is $px + qy = I$, where I is the consumers income.

   (a) Use Lagranges method to derive the constant-budget-share demand functions $x = (\alpha/(\alpha+\beta))(I/p)$ and $y = (\beta/(\alpha+\beta))(I/q)$.

   (b) Derive an expression for the Lagrange multiplier and show that it is given by $(\alpha/p)^\alpha(\beta/q)^\beta$ if $\alpha + \beta = 1$.

2. (*The Linear Expenditure System*) Consider the same consumer problem as in question 1 above, but suppose that the consumers utility function is $\alpha \ln(x - x_0) + \beta \ln(y - y_0)$, where ln is the natural log function, x_0 and y_0 are positive constants, and $\alpha + \beta = 1$.

   (a) Show that the optimal expenditure on the two goods are linear functions of incomes and prices:

$$px = \alpha I + \beta p x_0 - \alpha q y_0,$$
$$qy = \beta I - \beta p x_0 + \alpha q y_0.$$

   (b) Suggest an economic interpretation of x_0 and y_0.

3. (*Production and Cost-Minimization*) Consider a producer who rents machines K at r per year and hires labour L at wage w per year to produce output Q, where $Q = \sqrt{K} + \sqrt{L}$. Suppose that he wishes to produce a fixed quantity Q at minimum cost.

(a) Find his factor demand functions.

(b) Show that the Lagrange multiplier is give by $\lambda = 2wrQ/(w+r)$.

Now let p denote the price of output. Suppose that the producer can vary the quantity of the output, and seeks to maximize profit.

(c) Show that his optimum output supply is $Q = p(w+r)/(2wr)$. Relate this to the multiplier in (b).

4. (*Optimal ATM Trips*) Consider a student who spends the same amount of cash per day. If every time he makes a trip to an ATM he withdraws x dollars, the average amount of cash in his wallet is $x/2$ dollars between two trips. The total expenditure in a semester is A dollars, so the total number of trips the student makes satisfies $A = nx$ (we treat n as a real number rather than an integer). The total cost to the student is $C_h x/2 + C_0 n$, where C_h is the per-dollar daily cost of keeping cash in his wallet, and C_0 is the cost of making one trip.

(a) Use the Lagrange multiplier method to minimize the total cost by choice of x and n subject to $A = nx$.

Suppose that the student does not need to make an ATM trip when his wallet is empty, because he can borrow from his friends. Let U denote the total dollar amount of borrowing between two ATM trips. On each ATM trip, the student withdraws $x+U$, repays his friends U immediately, and holds x in his wallet. When the cash runs out, he keeps spending at the same rate by borrowing even though his wallet remains empty, until the total borrowing reaches U and he makes the next trip to an ATM. Let C_p denote the per-dollar daily cost to the student of carrying the debt.

(b) Find the optimal levels x and U.