Econ 420
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Lecture 6c. Linear Programming

1. Application of concave programming

- Consider $\max_x ax$ subject to $x \geq 0$ and $Bx \leq c$.
  - Choice variable x is n-dimensional column vector.
  - a is n-dimensional row vector, B is $m \times n$ matrix, and c is m-dimensional column vector.

- Special case of concave programming.
  - Kuhn-Tucker conditions are both necessary and sufficient.
  - Constraint qualification is not needed, either by general no profitable arbitrage argument, or by concave programming argument.

- Lagrangian $L(x, \lambda) = ax + \lambda(c - Bx)$, where λ is m-dimensional row vector.

  - $a - \overline{\lambda} B \leq 0$, $\overline{x} \geq 0$, with complementary slackness.

  - $c - B\overline{x} \geq 0$, $\overline{\lambda} \geq 0$, with complementary slackness.

2. Dual problem

- Consider $\min_y yc$ subject to $y \geq 0$ and $yB \geq a$.
  - Choice variable y is m-dimensional row vector.
  - a, B and c are the same as in primal problem.
  - Kuhn-Tucker conditions are also necessary and sufficient.

- Lagrangian $M(y, \mu) = yc + (a - yB)\mu$, where μ is n dimensional column vector.
  - $c - B\overline{\mu} \geq 0$, $\overline{y} \geq 0$, with complementary slackness.
  - $a - yB \leq 0$, $\overline{\mu} \geq 0$, with complementary slackness.

- Dual problem has same set of Kuhn-Tucker conditions as primal problem.
  - Choice variables and multipliers are switched.
  - $\overline{y}$ corresponds to $\overline{\lambda}$.
  - $\overline{\mu}$ corresponds to $\overline{x}$.

- Necessary and sufficient condition for existence of solution to a linear programming problem is feasibility set of its dual problem is non-empty.
  - An example of non-existence of solution.
  - Proof of “only if.”
  - Proof of “if.”

- $\overline{x}$ solves primal if and only if corresponding multiplier $\overline{\lambda}$ solves dual and $a\overline{x} = \overline{\lambda}c$.
  - Proof of necessity.
  - Proof of sufficiency.

- Economic interpretations.

  - x outputs; a output prices; c input supplies; λ shadow input prices; $B = \{b_{ij}\}$ input requirement matrix.
  - Primal: maximize output revenue subject to input supplies.
  - Dual: minimize input cost subject to output price targets.
  - $\overline{x}_j = 0$ if $a_j < \sum_{i=1}^{m} \overline{\lambda}_i b_{ij}$.
  - $\overline{\lambda}_i = 0$ if $c_i > \sum_{j=1}^{n} b_{ij} \overline{x}_j$.
  - $a\overline{x} = \overline{\lambda} c$.