Econ 420
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Lecture 5b. Maximization by separation

1. Applying separating hyperplane theorem

- Consider $\max_x F(x)$ subject to $G(x) \leq c$.
  - Suppose F is quasi-concave.
  - Suppose G is quasi-convex.
  - Consider common boundary points of $\{x : F(x) \geq v\}$ and $\{x : G(x) \leq c\}$.

2. Necessary and sufficient for constrained optimum

- $\overline{x}$ solves $\max_x F(x)$ subject to $G(x) \leq c$ if and only if there is a non-zero vector p such that:
  - $\overline{x}$ maximizes px subject to $G(x) \leq c$.
  - $\overline{x}$ minimizes px such that $F(x) \geq F(\overline{x})$.

- Proof of “only if.”

- Proof of “if.”

3. Economic interpretation: decentralization

- Planner's problem: choose a production-consumption vector x to maximize consumer's utility function $F(x)$ subject to a resource constraint $G(x) \leq c$.

  – Illustration.

- Decentralization through market prices p.
  - Firm chooses output x to maximize profit px subject to a resource constraint $G(x) \leq c$.
  - Consumer chooses consumption x to minimize expenditure px subject to achieving utility target $F(x) \geq F(\overline{x})$.
  - illustration.

4. Second-order properties of maximum value functions

- Expenditure function $E(p, u) = \min_x\{px|U(x) \geq u\}$ is concave in p.
  - Proof.
  - Illustration.

- Indirect utility function $V(p, I) = \max_x\{U(x)|px \leq I\}$ is quasi-convex in (p, I).
  - Proof.
  - Illustration.