Econ 420
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Lecture 4b. Long-run vs Short-run Optimization

1. Comparison of comparative statics

- Comparison of value functions between long run and short run.
  - Short run: some choice variables are fixed.
  - Long trun: all choice variables are variable.

2. Application of envelope theorem

- Long run problem: $\max_{y,z} F(y, z, \theta)$ subject to $G(y, z, \theta) \leq 0$, where y and z are distinct vectors of choice variables, θ is a scalar parameter, and G is a vector of constraint functions.
  - Maximum value function $V(\theta)$.
  - Solution functions $Y(\theta)$, $Z(\theta)$.

- Short run problem: $\max_y F(y, z, \theta)$ subject to $G(y, z, \theta) \leq 0$, where z is fixed.
  - z is a parameter.
  - Maximum value function $v(z, \theta)$.
  - Solution function $y(z, \theta)$.

- Relation between value functions.

  - $V(\theta) \geq v(z, \theta)$ for all z, with equality if $z = Z(\theta)$.

  - Proof.

- Application of envelope theorem.
  - $V'(\theta) = v_\theta(Z(\theta), \theta)$.
  - Proof: follows from $V(\theta) = \max_z v(z, \theta)$.

- Interpretation and illustration.
  - Long run value function is upper envelope of short run value functions.
  - Illustration.

3. An example

- Consider $\min rK + wL$ subject to $(KL)^{1/\alpha} \geq Q$.
  - K is capital, with interest rate r.
  - L is labor, with wage rate w.
  - Q is output target.
  - $(KL)^{1/\alpha}$ is production function, with $\alpha > 0$

- Long run cost function is defined as

$$C(w, r, Q) = \min_{K,L}\{rK + wL \big| (KL)^{1/\alpha} \geq Q\}$$

  - Solution: $K(w, r, Q) = (wQ^\alpha/r)^{1/2}$, $L(w, r, Q) = (rQ^\alpha/w)^{1/2}$.
  - Minimum cost: $C(w, r, Q) = 2(wrQ^\alpha)^{1/2}$.
  - Marginal cost $C_Q(w, r, Q) = \alpha(wr)^{1/2}Q^{\alpha/2-1}$.

- Suppose in short run, K is fixed and L is variable.
  - Short run cost function is defined as

$$c(w, r, Q, K) = \min_L \{rK + wL \big| (KL)^{1/\alpha} \geq Q\}.$$

  - Solution: $L = Q^\alpha / K$.
  - Minimum cost: $c(w, r, Q, K) = rK + wQ^\alpha / K$.
  - Marginal cost $c_Q(w, r, Q, K) = wQ^{\alpha-1} / K$.

- Relationships between value functions.

  - $\min_K c(w, r, Q, K) = C(w, r, Q)$, with solution $K = K(w, r, Q)$.

  - $c_Q(w, r, Q, K(w, r, Q)) = C_Q(w, r, Q)$.