Econ 420
Fall, 2022
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Lecture 3b. Shadow Price in Decentralization

1. Planned economy vs market economy

- A long history of competition for superiority by economic systems.
  - The role of shadow price in parallel intellectual debate.
  - Decentralization of planned economy shows how invisible hand guides allocation of scarce resources with no need for planning or command.

2. Planner and planned economy

- A benevolent social planner's problem.

  - Objective is social welfare function $W(u_1, \ldots, u_C)$, where each u_c, $c = 1, \ldots, C$, is utility level of consumer c, given by

    $$u_c = U^c(x_{c1}, \ldots, x_{cG}),$$

    with U^c utility function of consumer c over c's consumption x_{cg} of good g, $g = 1, \ldots, G$.

- Feasibility constraints.
  - For each good g, $g = 1, \ldots, G$,

    $$\sum_{c=1}^{C} x_{cg} \leq X_g,$$

    where X_g is total amount of good g available for distribution by planner.

- Simplifications.

  - Assume that W is strictly increasing in each u_c, $c = 1, \ldots, C$, each U^c is strictly increasing in each x_{cg}, $g = 1, \ldots, G$, and the marginal utility of each U^c is arbitrarily large when any $x_{cg} = 0$, $g = 1, \ldots, G$.

  - Feasibility constraint for any good g, $g = 1, \ldots, G$, is binding at any solution to the planner's problem.

  - Non-negativity constraint for any consumption x_{cg}, $c = 1, \ldots, C$ and $g = 1, \ldots, G$, is slack at any solution to the planner's problem.

- First order conditions.

  – Lagrangian:

$$W(U^1(x_{11},\dots,x_{1G}),\dots,U^C(x_{C1},\dots,x_{CG}))$$
$$+\sum_{g=1}^{G}\pi_g\left(X_g-\sum_{c=1}^{C}x_{cg}\right),$$

where π_g is the multiplier for feasibility constraint associated with good g, $g=1,\dots,G$.

- First order necessary conditions for $\{\{x_{cg}^*\}_{g=1}^G\}_{c=1}^C$ to solve the planner's problem: for each $c = 1, \ldots, C$ and for each $g = 1, \ldots, G$,

$$\frac{\partial W}{\partial u_c}\frac{\partial U^c}{\partial x_{cg}} = \pi_g,$$

and

$$\sum_{c=1}^{C} x_{cg}^* = X_g,$$

- For any good $g = 1, \ldots, G$, marginal impact on social welfare of increasing consumer c's consumption is equated across all $c = 1, \ldots, C$.

- Interpretation of π_g, $g = 1, \ldots, G$.
  - For each g, π_g is shadow price of good g.
  - Rate of increase in maximized social welfare with respect to increase in X_g.
  - There is no market price in a planned economy.

3. Decentralization

- Market economy as an alternative to planned economy.
  - Consumers make their own consumption decisions.
  - Consumers interact with each other in goods markets through prices.

- Market economy decentralizes solution $\{\{x_{cg}^*\}_{g=1}^G\}_{c=1}^C$ to planner's problem through competitive equilibrium.
  - Competitive: consumers take prices of goods as given, and submit their demand and supply of goods by solving their own utility maximization problem.
  - Equilibrium: prices of goods are such that demand equals supply for each good.

- Individual consumer's utility maximization problem.

  – Each consumer c, $c = 1, \dots, C$, chooses a bundle $(x_{c1}, \dots, x_{cG})$ to maximize $U(x_{c1}, \dots, x_{cG})$ subject to a budget constraint $\sum_{g=1}^{G} p_g x_{cg} \leq I_c$, where p_g is price of g and I_c is c's budget.

  – First order necessary conditions for allocation $\{\{x_{cg}\}_{g=1}^{G}\}_{c=1}^{C}$ to be a solution are, for each $g = 1, \dots, G$,

$$\frac{\partial U^c}{\partial x_{cg}} = \lambda_c p_g,$$

  where λ_c is multiplier associated with c's budget constraint.

- Decentralizing $\{\{x^*_{cg}\}_{g=1}^{G}\}_{c=1}^{C}$ as a competitive equilibrium.
  - For each $g = 1, \ldots, G$, choose price $p_g = \pi_g$, where π_g is multiplier associated with feasibility constraint of good g.
  - For each $c = 1, \ldots, C$, choose I_c such that $I_c = \sum_{g=1}^{G} \pi_g x^*_{cg}$.
  - For each $c = 1, \ldots, C$, choose $\lambda_c = 1/(\partial W / \partial U^c)$, evaluated at $(x^*_{c1}, \ldots, x^*_{cG})$.
  - First order conditions for consumers in the market economy are identical to first order conditions for planner's solution.

- Suppose $\{\{x^*_{cg}\}_{g=1}^G\}_{c=1}^C$ solves planner's problem with multipliers π_g, $g = 1, \ldots, G$.
  - $\{\{x^*_{cg}\}_{g=1}^G\}_{c=1}^C$ is a competitive equilibrium with prices π_g in an exchange economy where each consumer c, $c = 1, \ldots, C$, is endowed with $(e_{c1}, \ldots, e_{cG})$ such that

$$\sum_{g=1}^{G} e_{cg}\pi_g = \sum_{g=1}^{G} x^*_{cg}\pi_g.$$

- Illustration with Edgeworth Box: $C = 2$ and $G = 2$.