Econ 301
Fall, 2025
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Midterm

This is an open-book, 80-minute exam. There are 100 points. Answer all questions.

1. (*Student Loans*) Summer is currently in her last year of nursing school with no income. She plans to borrow from the government and pay it back next year from her nurse's income, which is $I > 0$ units of consumption good. Her utility function over units of consumption good c_1 this year and units of consumption good c_2 next year is

$$U(c_1, c_2) = V(c_1) + V(c_2),$$

where V is an increasing and concave function. The first L units of consumption good in the government's student loan program are interest free, and for each additional unit of borrowing, the gross interest rate is $R > 1$. This means that if Summer borrows $b < L$ units, the repayment amount is b units, resulting in a consumption bundle of $c_1 = b$ units this year and $c_2 = I - b$ units next year. If she borrows $b > L$, the repayment amount is $L + (b - L)R$, resulting in a consumption bundle of $c_1 = b$ units this year and $c_2 = I - (L + (b - L)R)$ units next year. Summer's problem is to maximize her utility $U(c_1, c_2)$ by choosing the amount of borrowing b.

(a) [5 points] Show that Summer has convex preferences over consumption bundles this year and next year. [Hint: For this and all questions below, you get partial credit by assuming $V(c) = \ln c$ or $V(c) = \sqrt{c}$.]

(b) [10 points] Explain that Summer faces a budget line with a kink at the bundle of $c_1 = L$ and $c_2 = I - L$, where the budget line becomes steeper (c_1 is on the horizontal axis and c_2 is on the vertical axis).

(c) [20 points] Write down the optimality condition for a solution $b_* < L$ to Summer's problem. Explain that this condition can be satisfied only when $L > \frac{I}{2}$. [Hint: Show that Summer will optimally consume an equal amount this year and next year.]

(d) [20 points] Write down the optimality condition for a solution $b_* > L$ to Summer's problem. Can this condition be satisfied if $L > \frac{I}{2}$? Explain your answer. [Hint: Show that Summer will optimally consume less this year than next year.]

(e) [5 points] Is it possible that the solution b_* to Summer's problem satisfies neither the optimality condition in (c) nor the condition in (d)? Explain your answer.

The government considers two proposals to improve the student loan program. The "expansion proposal" is to increase interest-free amount from L to some L^x, without changing interest rate R on borrowing over L^x; and the "reduction proposal" is to decrease interest rate from R to some $R^d > 1$, without changing interest-free amount L.

(f) [10 points] Explain that the two proposals do not affect Summer's welfare in terms of the maximum utility she can obtain if the solution b_* to Summer's problem under the old program with L and R satisfies $b_* < L$, but will make her better off if $b_* > L$. [Hint: For the first part, use the optimality condition from question (c); for the second part, consider how the two proposals affect the budget set.]

(g) [5 points] Suppose that the solution b_* to Summer's problem under the old program with L and R satisfies $b_* > L$. Use the idea of revealed preference to explain that Summer prefers the expansion proposal to the reduction proposal, if the amount b_*^d she optimally borrows under the reduction proposal satisfies

$$(b_*^d - L)(R^d - 1) \geq (b_*^d - L^x)(R - 1),$$

that is, by borrowing b_*^d she makes a greater or equal amount of interest payment under the reduction proposal than under the expansion proposal.

(h) [5 points] Summer's friend Mac, who has a different utility function and a different income next year, is also applying for a student loan. Can you quantify the welfare effect of each proposal on Summer and on Mac and compare them? If no, explain why not; if yes, define a quantitative measure as precisely as you can in this context.

2. (*Bankruptcy Risk*) A loan officer at a commercial bank must decide whether or not to approve an application to fund an investment project. If the loan application is approved, with probability p the project will succeed and yield a gross rate of return $S > 1$, but with probability $1 - p$ the project will fail and can only offer a liquidated fraction L between 0 and 1 as repayment to the bank. The bank's risk-free gross interest rate is $R > 1$. The loan officer has an increasing utility function u, and will approve the loan if and only if

$$pu(X) + (1 - p)u(L) \geq u(R),$$

where $X \leq S$ is the repayment rate to the bank when the project succeeds.

(a) [10 points] Show that if the loan officer is risk-neutral, any projects with an expected rate of return $pS + (1 - p)L$ that is at least R can be funded. [Hint: X can be equal to S; for this question, you get partial credit by assuming that $u(c) = c$.]

(b) [10 points] Show that if the loan offer is risk-averse, projects may not be funded even though the expected rate of return is strictly higher than R. [Hint: X can be at most S; use the concept of risk premium.]