Econ 301
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Li, Hao
UBC

Midterm Practice Questions

1. (*Optimality Condition*) Mark consumes only cookies and books. At his current consumption bundle, his marginal utility from books is 10 and from cookies is 5. Each book costs $10.00 and each cookie costs $2.00. Is he maximizing his utility? Explain. If he is not, then how can he increase his utility while keeping his total expenditure constant?

1. (*Minimum Expenditure for Complements*) Sally's utility function is $U(x_1, x_2) = \min\{x_1, 2x_2\}$. Let p_1, p_2 be the prices for good 1 and 2, respectively. Let u denote the level of utility Sally wants to maintain. Derive her compensated demand functions, and expenditure function $E(p_1, p_2, u)$.

3. (*Luxury Good*) Consider a consumer who consumes two goods x_1 and x_2. The consumer's utility function is by $U(x_1, x_2) = (x_1 + 1)x_2$. The budget constraint is $p_1x_1 + p_2x_2 = I$.

(a) Write down the optimality condition for demand x_1 and x_2.

(b) Use the optimality condition and the budget constraint to find demand x_1 and x_2 as functions of p_1, p_2 and I.

(c) Under what condition is your answer to (b) valid? What happens if the condition is not satisfied?

4. (*Childcare Subsidy*) Miguel's preferences over childcare (x) and the composite good (c) is captured by $u(x,c) = x^{0.2}c^{0.8}$ and his income is \$4000. The price of childcare is denoted as p, and the price of the composite good is normalized to 1.

(a) Derive the demand for childcare and the composite good as functions of p.

Suppose that the current price of childcare p is \$40 per unit, and the government plans to subsidize childcare so the price will become \$20 per unit for Miguel.

(b) Use your answer to (a) to compute the percentage increase in Miguel's utility under the government plan. Is this a good measure of the welfare change for Miguel? Explain your answer.

(c) How much is the government is actually spending on Miguel's childcare? If instead the government gives Miguel this amount as an income subsidy that allows Miguel to spend on childcare at the old price of \$40 per unit and the composite good anyway he wishes to, would Miguel be better off? Explain your answer.

5. (*Job Relocation*) Fangwen's utility function is $U(x_1, x_2) = x_1 + x_2$. The price of each good is 1, and his monthly salary is \$4000. His firm wants him to relocate to another city where the price of good 1 is \$2, but the price of good 1 and his income remain the same as before the move. How much does the firm need to raise his salary by for Fangwen to be willing to relocate? Explain your answer.

6. (*Insuring Valuables*) Maya possesses \$250,000 worth of valuables. She faces a 0.2 probability of a burglary, resulting in her loss of jewelry worth \$160,000. She can buy, at a price of \$36,000, an insurance policy that would fully reimburse the loss of \$160,000 in case of burglary. Her utility function is $u(x) = 5\sqrt{x}$.

(a) Is the price \$36,000 actuarially fair? If not, what is the actuarially fair price for the insurance policy?

(b) Should Maya buy this insurance policy? Why?

7. (*Choosing Lotteries*) There are two lotteries, A and B. The payout of both lotteries depends on the weather. If the weather is sunny, lottery A pays 70 and lottery B pays 10. If the weather is not sunny, lottery A pays 10 and lottery B pays 70. The probability of a sunny weather is 0.6. Suppose your utility function for money is $u(x) = \sqrt{x}$ and you are expected utility maximizer. Your initial wealth is 100.

(a) If only lottery A is available at price 45, should you buy it? If only lottery B is available at price 34, should you buy it?

(b) If both lottery A and lottery B are available at prices 45 and 34, respectively, should you buy the lotteries?

8. (*Water Restrictions*) Homeowner Ailing's utility function over quantity of daily water consumption x_1, and quantity of daily consumption of a composite good x_2 (think of it as expenditure on all other goods), is given by

$$u(x_1, x_2) = 4\sqrt{x_1} + 2x_2.$$

The price per unit of water is \$0.1, and the price per unit of the composite good is \$1. Ailing's daily income is \$200, so the budget constraint is

$$0.1 \cdot x_1 + x_2 = 200.$$

(a) Write down the optimality condition for Ailing's demands for water and the composite good.

(b) Use the optimality condition in (a) and the budget constraint to show that Ailing's demand for water is 100 units. What is the utility u^0 she achieves?

Now suppose that the city government restricts daily water consumption per household to at most 25 units.

(c) Use the optimality condition in (a) to argue that under the water restriction Ailing's daily demand for water is exactly 25 units, instead of a lower quantity. What is the utility u^1 she achieves? Combine with (b) to show that Ailing is worse off with the water restriction.

The difference between the utilities achieved with and without the water restriction, $u^0 - u^1$, or the percentage change, $(u^0 - u^1)/u^0$, is not a good quantitative measure of the welfare loss due to the restriction. In the remaining questions, we quantity the welfare impact by giving monetary measures. Recall that the consumer surplus (CS) is the monetary difference between a consumer's maximal willingness to pay for the quantity purchased and what the good actually costs. Here, the price of water does not change but there is a quota on how much Ailing can consume.

(d) Provide a verbal or graphical argument for why the loss in Ailing's consumer surplus (CS) with the water restriction is

$$\Delta \text{CS} = \int_{0.1}^{p_1^1} x_1(p_1) dp_1 - 25(p_1^1 - 0.1),$$

where, without the water restriction, $x_1(p_1)$ is Ailing's demand for water as a function of its price p_1, and p_1^1 is the price for which Ailing's demand is equal to 25.

(e) Derive Ailing's demand x_1 without the water restriction as a function of p_1. What is the value of p_1^1? Compute ΔCS as the result of the water restriction.

Recall that the equivalent variation (EV) of a change to a consumer's choice problem (such as a price increase or a quota) is the minimum income that needs to be taken away from the consumer's income before the change that would leave the consumer at the same utility level achieved after the change.

(f) Argue that EV of the water restriction is given by

$$\text{EV} = 200 - E(u^1),$$

where u^1 is the utility achieved after the water restriction is imposed, and E is the minimum expenditure as a function of a target utility level.

(g) Derive $E(u^1)$ by solving $\min 0.1 \cdot x_1 + x_2$ subject to $4\sqrt{x_1} + 2x_2 = u^1$. Compute EV as the result of water restriction.

9. (*Escalation and Sanction*) The King of Qin must decide the level of escalation x between 0 and 1 with a small neighboring kingdom in face of possible sanctions by the coalition of the neighbor and the other five kingdoms. Qin's initial wealth is 2. Choosing $x = 0$ is the same as no escalation; $x = 1$ means full escalation. For any choice x by the King of Qin, with probability p it is sanctioned by the coalition and loses x, and with the remaining probability $1 - p$ there is no sanction and Qin gains x. The expected utility of the King of Qin from any choice of x between 0 and 1 is

$$p \cdot u(2 - x) + (1 - p) \cdot u(2 + x),$$

where u is the utility function. For questions (a) and (b) below, p is a known parameter between 0 and 1 that your answers may depend on. You may want to take derivative of the expected utility with respect to x, and evaluate the sign of the derivative at $x = 0$ and at $x = 1$, and consider how it changes when x varies between 0 and 1.

(a) If the King of Qin is risk neutral, would he optimally choose any escalation level other than 0 or 1? For this question, you may assume the utility function is given by $u(w) = w$.

(b) If the King of Qin is risk averse, would he ever optimally choose no escalation or full escalation? For this question, you may assume the utility function is given by $u(w) = \sqrt{w}$.