Econ 301
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Final Practice Questions

1. (*Firm Types*) There are two types of firms in the market, type A and type B. Each type A firm has a short-run cost function given by $C^A(q) = q^2 + 100$, and each type B firm has a short-run cost function given by $C^B(q) = q^2/2 + q + 100$.

(a) Find the individual short-run supply curve $q^A(p)$ of type A firms.

(b) Find the individual short-run supply curve $q^B(p)$ of type B firms.

(c) Suppose there are 2000 type A firms and 1000 type B firms in the market. What is the short-run market supply $Q(p)$ from all firms?

2. (*Labor Law*) Consider a competitive market with market demand $Q^D(p) = 50000 - 2000p$. The number of firms in the market is n. The production function of each firm is given by

$$f(L, K) = \min\{10, \min\{L, K\}\}.$$

That is, all firms' productions are capped above by 10. Firms can hire labor and capital from competitive markets at price $w = 3$ and $r = 2$.

(a) What's the long-run conditional cost function for each firm? In the long-run competitive equilibrium, what is the equilibrium price p? What is the minimal equilibrium number of firms in the market?

(b) Now suppose the government passes a minimum wage law so that the price for labor increases from 3 to 4. What is the new long-run equilibrium price? Compute the loss of consumer surplus in the output market due to the increase in the equilibrium price.

3. (*Regional Trade*) Suppose there are two competitive regional markets: region A and region B, for a homogeneous good. The demand and supply curves in region A are

$$Q_A^D(p) = 10 - p, \quad Q_A^S(p) = p,$$

respectively, while the demand and supply curves in region B are

$$Q_B^D(p) = 10 - p, \quad Q_B^S(p) = p - 4,$$

respectively.

(a) Suppose there is no trade between the two regions. Solve the equilibrium prices for each region: p_A and p_B.

(b) Now suppose that there is free trade and there is no transportation cost. Let p denote the equilibrium price in both regions. Solve p. [Hint: At the equilibrium price p, the excess supply in one region must be equal to the excess demand in the other region.]

(c) What is the total welfare gain from trade in region A, and what is the total welfare gain from trade in region B? It is not necessary to draw a diagram to get full credit, but a clearly drawn and labeled diagram can get partial credit.

(d) Now suppose region B imposes a tariff $t = 1$ per unit on the good imported from region A. Let p_A^* and p_B^* denote the equilibrium prices after tariff in region A and region B, respectively. Solve p_A^* and p_B^*.

4. (*2-by-2*) Consider an exchange economy with two people (A and B), two goods (1 and 2) and endowment $(e_1^A, e_2^A) = (1,1)$ and $(e_1^B, e_2^B) = (1,0)$.

(a) Find all Pareto efficient allocations (i.e., contract curve) and the core, when A and B have the following preferences:

$$U^A(x_1^A, x_2^A) = \min\{2x_1^A, x_2^A\},$$

$$U^B(x_1^B, x_2^B) = x_1^B + x_2^B.$$

To answer this question, draw a clearly labeled Edgeworth box. Label precisely the contract curve and the core. Math calculation is not necessary.

(b) Now suppose A and B have the following utility functions:

$$U^A(x_1^A, x_2^A) = (x_1^A)^{1/3}(x_2^A)^{2/3},$$

$$U^B(x_1^B, x_2^B) = (x_1^B)^{2/3}(x_2^B)^{1/3}.$$

Let's normalize the price of good 2 to be 1, and let p denote the price of good 1 in the competitive equilibrium. Solve p. [Hint: For a Cobb-Douglas utility function $x_1^a x_2^{1-a}$ with income I, the demand functions are $x_1 = aI/p_1$ and $x_1 = (1-a)I/p_2$.]

5. (*Long-run Equilibrium*) Each firm in a competitive market has a long run cost function of $C(q) = 10q - 4q^2 + q^3$. There are an unlimited number of potential firms in this market. The market demand function is $Q = 30 - p$.

(a) What are the long-run equilibrium price, quantity per firm, market quantity, and the number of firms?

(b) How do these variables change if a tax of $2 per unit is collected from each firm?

(c) If that the demand curve for wheat is $Q = 100 - 10p$ and the supply curve is $Q = 10p$. The government imposes a specific tax of $\tau = 1$ per unit. Compute the changes in CS and PS due to the government tax. Compute the deadweight loss due to this tax.

6. (*No Calculations*) Consider an economy with two people, A and B, with endowment $(e_1^A, e_2^A) = (1, 1)$ and $(e_1^B, e_2^B) = (0, 1)$. No calculation is required for this question. You are asked to carefully draw Edgeworth box to answer this question.

(a) Find all Pareto efficient allocations (i.e., contract curve) and the core if A and B have utility functions:

$$U^A(x_1^A, x_2^A) = min\{x_1^A, x_2^A\},$$

$$U^B(x_1^B, x_2^B) = min\{x_1^B, x_2^B\}.$$

(b) Find all Pareto efficient allocations (i.e., contract curve) and the core if A and B have utility functions:

$$U^A(x_1^A, x_2^A) = min\{x_1^A, x_2^A\},$$

$$U^B(x_1^B, x_2^B) = x_1^B + x_2^B.$$

7. (*Competitive Equilibrium with Production*) Consider an economy with two consumer-producers, Jane and Denise. Both Jane and Denise have the same utility function over quantities x_c of candy bars and x_w of firewood:

$$U(x_c, x_w) = \ln x_c + \ln x_w.$$

They each have one unit of labor: if Jane spends fraction ϕ_j to produce candy then she can produce $3\phi_j$ units of candies and $6(1-\phi_j)$ units of wood; if Denise spends fraction ϕ_d to produce candy then she can produce $6\phi_d$ units of candies and $3(1-\phi_d)$ units of wood.

For questions (a)-(c), suppose that Jane and Denise live on their own.

(a) Write down Jane's optimality condition for choosing ϕ_j to maximize her utility.

(b) Use the optimality condition from (a) to show that Jane will produce and consume more firewood than candies.

(c) Write down Denise's optimality condition for choosing ϕ_d to maximize her utility, and use it to argue that she will produce and consume more candies than wood.

For the remaining questions, suppose that Jane and Denise make their own production decisions as before, but can now trade with each other. We construct a competitive equilibrium with equal prices for candies and wood $p_c = p_w = 1$.

(d) Show that given $p_c = p_w = 1$, Jane's optimal production decision is $\phi_j = 0$, that is, Jane optimally specializes in producing wood.

(e) Show that given $p_c = p_w = 1$, Denise's optimal production decision is $\phi_d = 1$, that is, Denise optimally specializes in producing candies.

(f) Using her optimal profit from (d) to show that Jane's optimal consumption bundle is 3 units of candies and 3 units of wood, that is, she trades 3 units of wood she produced for 3 units of candies Denise produced.

(g) Using her optimal profit from (e) to show that Denise's optimal consumption bundle is also 3 units of candies and 3 units of wood.

(h) Are Jane and Denise better off from trade compared to no trade? Explain your answer.

8. (*Separable Production Functions*) There are 10 firms currently operating in some industry. They all use a separable technology, with production function

$$q = \sqrt{k} + \sqrt{l},$$

where k is units of capital and l is units of labor.

(a) Does the marginal product of capital depend on a firm's labor input? Does the marginal product of labor depend on the capital input? Explain why or why not capital and labor perfect substitutes.

In the short run, the 10 existing firms may only adjust their input of capital; their input of labor is fixed at 25 units. The price of capital and the price of labor are both 1 per unit. Besides capital and labor, to produce any positive quantity, each firm must pay 50 to the government for quality control. This fixed cost is not avoidable even in the long run. The aggregate demand is given by $200 - 10p$, where p is the unit price of the output.

(b) What is the short-run cost function? What is the short-run supply curve? Explain your answer.

(c) What is the short-run equilibrium price? What is the total equilibrium quantity and quantity per firm?

In the long run, firms can adjust their input of labor as well as the input of capital, and can enter the industry with the same technology or exit the industry.

(d) Show that the long run cost function is $q^2/2 + 50$. What is the long run supply curve? [Hint: you may use the fact that $q/2 + 50/q$ is minimized at $q = 10$.]

(e) What is the long-run equilibrium price? What is the long-run total equilibrium quantity? What is the number of firms in the industry?

9. (*Producer-Trader-Consumer*) Amy can produce 45 units of food or 90 units of clothes, or combinations of food and clothes, with a linear production possibility frontier given by

$$2x_f + x_c = 90,$$

where x_f is units of food and x_c is units of clothes. She has a Cobb-Douglas utility function over food and clothes, given by

$$U^A(x_f, x_c) = (x_f)^2 x_c.$$

If Amy lives on her own as a producer-consumer, she chooses x_f and x_c to maximize $U^A(x_f, x_c)$ subject to $2x_f + x_c = 90$.

(a) Make an argument for why the optimality condition of Amy's maximization problem is equating her marginal rate of substitution to the marginal rate of transformation:

$$\frac{2x_f x_c}{(x_f)^2} = \frac{2}{1}.$$

(b) Use the above condition to show that Amy's optimal choice as a live-alone producer-consumer is $x_f = x_c = 30$.

Benny can produce 90 units of food or 45 units of clothes, with a frontier

$$y_f + 2y_c = 90,$$

where y_f is units of food and y_c is units of clothes. His utility over food and clothes is

$$U^B(y_f, y_c) = y_f(y_c)^2,$$

(c) Between the two producer-consumers, Amy and Benny, who has a comparative advantage in producing clothes? Explain your answer.

(d) Make an argument for why, compared to Amy, Benny has a relative preference for clothes.

(e) Show that if Benny lives alone, his optimal choice for production/consumption is identical to Amy's: $y_f = y_c = 30$.

After Amy and Benny completed their identical, optimal production of 30 units of food and 30 units of clothes, they contemplate whether they should trade with each other.

(f) Use your answer to question (d) to argue that, in spite of having the same quantities of food and clothes, they will find it beneficial to trade with each other. Who should trade food for clothes? Explain your answer.

(g) Show that, if they can trade food and clothes at the relative price of 1, starting from 30 units of each good, Amy's demands for food and clothes are

$$x_f^* = 40,\ x_c^* = 20;$$

and Benny's demands for food and for clothes are

$$y_f^* = 20,\ y_c^* = 40.$$

(h) Use (g) above to identify a competitive equilibrium in the exchange economy consisting of Amy and Benny after they each have produced 30 units of food and 30 units of clothes.

Anticipating that they can trade with each other after their production, Amy and Benny may want to change their production to exploit their comparative advantage.

(i) Show that there is a competitive equilibrium with same relative price of 1 that makes both Amy and Benny better off than in (h), if they each produce only the good that is in their respective comparative advantage.