

Final Practice Questions

1. (*Firm Types*) There are two types of firms in the market, type A and type B. Each type A firm has a short-run cost function given by $C^A(q) = q^2 + 100$, and each type B firm has a short-run cost function given by $C^B(q) = q^2/2 + q + 100$.

- (a) Find the individual short-run supply curve $q^A(p)$ of type A firms.
- (b) Find the individual short-run supply curve $q^B(p)$ of type B firms.
- (c) Suppose there are 2000 type A firms and 1000 type B firms in the market. What is the short-run market supply $Q(p)$ from all firms?

2. (*Labor Law*) Consider a competitive market with market demand $Q^D(p) = 50000 - 2000p$. The number of firms in the market is n . The production function of each firm is given by

$$f(L, K) = \min\{10, \min\{L, K\}\}.$$

That is, all firms' productions are capped above by 10. Firms can hire labor and capital from competitive markets at price $w = 3$ and $r = 2$.

- (a) What's the long-run conditional cost function for each firm? In the long-run competitive equilibrium, what is the equilibrium price p ? What is the minimal equilibrium number of firms in the market?
- (b) Now suppose the government passes a minimum wage law so that the price for labor increases from 3 to 4. What is the new long-run equilibrium price? Compute the loss

of consumer surplus in the output market due to the increase in the equilibrium price.

3. (*Regional Trade*) Suppose there are two competitive regional markets: region A and region B, for a homogeneous good. The demand and supply curves in region A are

$$Q_A^D(p) = 10 - p, \quad Q_A^S(p) = p,$$

respectively, while the demand and supply curves in region B are

$$Q_B^D(p) = 10 - p, \quad Q_B^S(p) = p - 4,$$

respectively.

- (a) Suppose there is no trade between the two regions. Solve the equilibrium prices for each region: p_A and p_B .
- (b) Now suppose that there is free trade and there is no transportation cost. Let p denote the equilibrium price in both regions. Solve p . [Hint: At the equilibrium price p , the excess supply in one region must be equal to the excess demand in the other region.]
- (c) What is the total welfare gain from trade in region A, and what is the total welfare gain from trade in region B? It is not necessary to draw a diagram to get full credit, but a clearly drawn and labeled diagram can get partial credit.
- (d) Now suppose region B imposes a tariff $t = 1$ per unit on the good imported from region A. Let p_A^* and p_B^* denote the equilibrium prices after tariff in region A and region B, respectively. Solve p_A^* and p_B^* .

4. (2-by-2) Consider an exchange economy with two people (A and B), two goods (1 and 2) and endowment $(e_1^A, e_2^A) = (1, 1)$ and $(e_1^B, e_2^B) = (1, 0)$. Suppose A and B have the following utility functions:

$$U^A(x_1^A, x_2^A) = (x_1^A)^{1/3}(x_2^A)^{2/3},$$

$$U^B(x_1^B, x_2^B) = (x_1^B)^{2/3}(x_2^B)^{1/3}.$$

- (a) Find all Pareto efficient allocations (i.e., contract curve) and the core.
- (b) Let's normalize the price of good 2 to be 1, and let p denote the price of good 1 in the competitive equilibrium. Solve p . [Hint: For a Cobb-Douglas utility function $x_1^a x_2^{1-a}$ with income I , the demand functions are $x_1 = aI/p_1$ and $x_2 = (1-a)I/p_2$.]

5. (Long-run Equilibrium and Taxation) Each firm in a competitive market has a long run cost function of $C(q) = \frac{q^2}{5} + 5$. There are an unlimited number of potential firms in this market. The market demand function is $p = 12 - \frac{Q}{100}$.

- (a) What are the long-run equilibrium price, quantity per firm, market quantity, and the number of firms?
- (b) How do these variables change if a tax of \$2 per unit is collected from each firm? Compute the changes in CS and PS due to the government tax. What is the dead-weight loss?
- (c) How do your answers in (a) change if the tax is collected from consumers?

6. (No Calculations) Consider an economy with two people, A and B, with endowment $(e_1^A, e_2^A) = (1, 1)$ and $(e_1^B, e_2^B) = (0, 1)$. No calculation is required for this question.

(a) Find all Pareto efficient allocations (i.e., contract curve) and the core if A and B have utility functions:

$$U^A(x_1^A, x_2^A) = \min\{x_1^A, x_2^A\},$$

$$U^B(x_1^B, x_2^B) = \min\{x_1^B, x_2^B\}.$$

(b) Find all Pareto efficient allocations (i.e., contract curve) and the core if A and B have utility functions:

$$U^A(x_1^A, x_2^A) = \min\{x_1^A, x_2^A\},$$

$$U^B(x_1^B, x_2^B) = x_1^B + x_2^B.$$

7. (*Separable Production Functions*) There are 10 firms currently operating in some industry. They all use a separable technology, with production function

$$q = \sqrt{k} + \sqrt{l},$$

where k is units of capital and l is units of labor.

(a) Does the marginal product of capital depend on a firm's labor input? Does the marginal product of labor depend on the capital input? Explain why or why not capital and labor are perfect substitutes.

In the short run, the 10 existing firms may only adjust their input of capital; their input of labor is fixed at 25 units. The price of capital and the price of labor are both 1 per unit. Besides capital and labor, to produce any positive quantity, each firm must pay 50 to the government for quality control. This fixed cost is not avoidable even in the long run. The aggregate demand is given by $200 - 10p$, where p is the unit price of the output.

- (b) What is the short-run cost function? What is the short-run supply curve? Explain your answer.
- (c) What is the short-run equilibrium price? What is the total equilibrium quantity and quantity per firm?

In the long run, firms can adjust their input of labor as well as the input of capital, and can enter the industry with the same technology or exit the industry.

- (d) Show that the long run cost function is $q^2/2 + 50$. What is the long run supply curve? [Hint: you may use the fact that $q/2 + 50/q$ is minimized at $q = 10$.]
- (e) What is the long-run equilibrium price? What is the long-run total equilibrium quantity? What is the number of firms in the industry?

8. (*Producer-Trader-Consumer*) Amy can produce 45 units of food or 90 units of clothes, or combinations of food and clothes, with a linear production possibility frontier given by

$$2x_f + x_c = 90,$$

where x_f is units of food and x_c is units of clothes. She has a Cobb-Douglas utility function over food and clothes, given by

$$U^A(x_f, x_c) = (x_f)^2 x_c.$$

If Amy lives on her own as a producer-consumer, she chooses x_f and x_c to maximize $U^A(x_f, x_c)$ subject to $2x_f + x_c = 90$.

- (a) Make an argument for why the optimality condition of Amy's maximization problem

is equating her marginal rate of substitution to the marginal rate of transformation:

$$\frac{2x_f x_c}{(x_f)^2} = \frac{2}{1}.$$

- (b) Use the above condition to show that Amy's optimal choice as a live-alone producer-consumer is $x_f = x_c = 30$.

Benny can produce 90 units of food or 45 units of clothes, with a frontier

$$y_f + 2y_c = 90,$$

where y_f is units of food and y_c is units of clothes. His utility over food and clothes is

$$U^B(y_f, y_c) = y_f(y_c)^2,$$

- (c) Between the two producer-consumers, Amy and Benny, who has a comparative advantage in producing clothes? Explain your answer.
- (d) Make an argument for why, compared to Amy, Benny has a relative preference for clothes.
- (e) Show that if Benny lives alone, his optimal choice for production/consumption is identical to Amy's: $y_f = y_c = 30$.

After Amy and Benny completed their identical, optimal production of 30 units of food and 30 units of clothes, they contemplate whether they should trade with each other.

- (f) Use your answer to question (d) to argue that, in spite of having the same quantities of food and clothes, they will find it beneficial to trade with each other. Who should trade food for clothes? Explain your answer.

(g) Show that, if they can trade food and clothes at the relative price of 1, starting from 30 units of each good, Amy's demands for food and clothes are

$$x_f^* = 40, x_c^* = 20;$$

and Benny's demands for food and for clothes are

$$y_f^* = 20, y_c^* = 40.$$

(h) Use (g) above to identify a competitive equilibrium in the exchange economy consisting of Amy and Benny after they each have produced 30 units of food and 30 units of clothes.

Anticipating that they can trade with each other after their production, Amy and Benny may want to change their production to exploit their comparative advantage.

(i) Show that there is a competitive equilibrium with same relative price of 1 that makes both Amy and Benny better off than in (h), if they each produce only the good that is in their respective comparative advantage.

9. (*Temporary and Permanent Tariffs*) Identical firms in BC have been producing lumber for the US market. Each firm's production function is given by

$$q = 4\sqrt{l} + \sqrt{k},$$

where q is units of lumber output, k is units of capital input and l is units of labour input. The price of labour is 4 per unit and the price of capital is 1 per unit. Each firm also pays 20 for an export license regardless of how much it produces. The cost minimization problem

for each firm is choosing input quantities k and l that minimize the total cost $4l + k + 20$ required to produce q units of lumber.

- (a) Write down the optimality condition for the cost minimization problem and show that it implies that each firm uses an equal amount of labour and capital. [Hint: You can also use the Lagrange method.]
- (b) Use (a) to show that the demand for labour and for capital by each firm is $q^2/25$, and the cost function is $20 + q^2/5$.
- (c) Use the cost function in (b) to show that under perfect competition, each firm produces 10 units with 4 units of labour and 4 units of capital, at the average cost of 4. [Hint: For any $A, B > 0$, the expression $(A + Bx^2)/x$ over $x > 0$ is minimized at $x = \sqrt{A/B}$.]

The demand in the US market for lumber from BC is given by

$$p = 12 - \frac{Q}{100},$$

where p is the price of lumber and Q is the total imports.

- (d) Use (c) to show that the equilibrium price is 4, and 80 lumber firms in BC in the export industry producing 800 in total.

For questions (e) and (f), suppose that the US government temporarily imposes a 100% tariff on imported lumber from BC. The demand in the US market becomes

$$2p = 12 - \frac{Q}{100},$$

where p remains the price the lumber firms in BC use to calculate their profits, and $2p$ is the price that determines the total demand Q in the US market. Since the tariff is temporary, the 80 lumber firms cannot exit the export industry and no new firms can enter. However, they can freely vary their labour and capital input, and decide to shut down if they want.

- (e) Explain that the lumber firms will not temporarily shut down. Use the cost function in (b) to show that the temporary supply of each of the 80 firms is $5p/2$, the equilibrium price drops down to 3, and total exports go down to 600.
- (f) Compute the total loss in consumer's surplus in the US market and the total loss in producer's surplus in BC due to the tariff, and the tariff revenue. What is the total deadweight loss – the difference between the sum of the two losses and the tariff revenue? [Hint: For the loss in the US market, use the after-tariff price.]

For questions (g) and (h) below, suppose that the 100% tariff is permanent. Lumber firms can freely enter and exit the export industry.

- (g) Explain that the equilibrium price is 4. What is the number of firms in the market? What is total number of units produced and exported to the US market?
- (h) Is the permanent tariff better or worse than the temporary one in terms of the total deadweight loss as defined in (f)? Explain.

10. (*Identical Preferences*) Consider Edgeworth-box exchange economies with two consumers, Justin and Katie, and two goods, food and clothes. Both Justin and Katie have the same Cobb-Douglas utility function

$$U^J(x_f, x_c) = U^K(x_f, x_c) = x_f x_c,$$

where x_f is units of their individual consumption of food and x_c is units of their consumption of clothes. The total endowment of food between Justin and Katie is 10 units, and the total endowment of clothes is 20 units. [Hint: Drawing the Edgeworth Box will help you answer the questions and get partial credit, but all questions below can also be answered without actually drawing the Edgeworth Box.]

- (a) Explain that no allocation on an edge of the Edgeworth Box – an allocation that gives the total endowment of one good to one consumer and divides the total endowment of the other good between the two consumers – can be on the contract curve. [Hint: What is the marginal rate of substitution of the consumer that has 0 units of one good?]
- (b) Explain that an interior allocation in the Edgeworth Box – an allocation that divides the total endowment of each good between the two consumers – can be on the contract curve only if it is on the main diagonal, which is the straight line from the allocation that gives the total endowment of each good to Justin to the allocation that gives all to Katie.
- (c) Explain that the core of an exchange economy where Justin is endowed with all 20 units of clothes and Katie is endowed with all 10 units of food coincides with the entire contract curve.
- (d) Show that in the competitive equilibrium of the exchange economy in (c) – where Justin has all the clothes endowment and Katie has all the food endowment – Justin and Katie consume an equal amount of clothes and an equal amount of food. [Hint: You may want to use your answer in (b) to figure out what the equilibrium price ratio has to be.]
- (e) Give an example of endowments of Justin and Katie for which the core consists of

a single allocation. What is the competitive equilibrium in this economy? Explain your answers.

(f) Is it possible that for some endowments of Justin and Katie, there are multiple competitive equilibria? If yes, give an example; if no, explain why not.