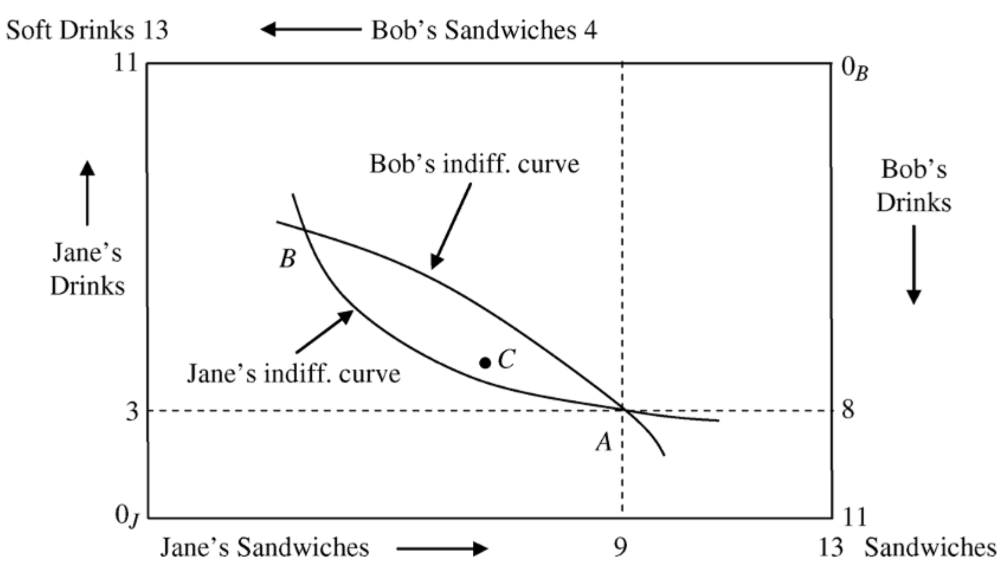
3. Jane has 3 liters of soft drinks and 9 sandwiches. Bob, on the other hand, has 8 liters of soft drinks and 4 sandwiches. With these endowments, Jane's marginal rate of substitution (*MRS*) of soft drinks for sandwiches is 4 and Bob's *MRS* is equal to 2. Draw an Edgeworth box diagram to show whether this allocation of resources is efficient. If it is, explain why. If it is not, what exchanges will make both parties better off?

Given that $MRS_{Bob} \neq MRS_{Jane}$, the current allocation of resources is inefficient. Jane and Bob could trade to make one of them better off without making the other worse off. Although we do not know the exact shape of Jane and Bob's indifference curves, we do know the slope of both indifference curves *at the current allocation*, because we know that $MRS_{Jane} = 4$ and $MRS_{Bob} = 2$. At the current allocation (point *A* in the diagram), Jane is willing to trade 4 sandwiches for 1 drink, or she will give up 1 drink in exchange for 4 sandwiches. Bob is willing to trade 2 sandwiches for 1 drink, or he will give up 1 drink in exchange for 2 sandwiches. Jane will give 4 sandwiches for 1 drink while Bob is willing to accept only 2 sandwiches in exchange for 1 drink.

Any exchange that leaves both parties inside the lens-shaped area between points *A* and *B* will make both better off. For example, if Jane gives Bob 3 sandwiches for 1 drink, they will be at point *C*. Jane is better off because she was willing to give up 4 sandwiches but only had to give up 3. Bob is better off because he was willing to accept 2 sandwiches and actually received 3. Jane ends up with 4 drinks and 6 sandwiches and Bob ends up with 7 drinks and 7 sandwiches, and both are better off than at point *A*.



5. **Fill in the missing information in the following tables. For each table, use the information provided to identify a possible trade. Then identify the final allocation and a possible value for the *MRS* at the efficient solution. (*Note*: There is more than one correct answer.) Illustrate your results using Edgeworth Box diagrams.**

   a. **Norman's *MRS* of food for clothing is 1 and Gina's *MRS* of food for clothing is 4:**

Individual	Initial Allocation	Trade	Final Allocation
Norman	**$6F, 2C$**	$1F$ for $3C$	$5F, 5C$
Gina	**$1F, 8C$**	$3C$ for $1F$	$2F, 5C$

Gina will give 4 clothing for 1 food while Norman is willing to accept 1 clothing for 1 food. If they settle on 2 or 3 units of clothing for one unit of food they will both be better off. Let's say they settle on 3 units of clothing for 1 unit of food. Gina will give up 3 units of clothing and receive 1 unit of food so her final allocation is $2F$ and $5C$. Norman will give up 1 food and gain 3 clothing so his final allocation is $5F$ and $5C$. Gina's *MRS* will decrease and Norman's will increase, so given they must be equal in the end, it will be somewhere between 1 and 4, in absolute value terms. So a possible value for both individual's *MRS* at the efficient solution is 2.5.

   b. **Michael's *MRS* of food for clothing is 1/2 and Kelly's *MRS* of food for clothing is 3.**

Individual	Initial Allocation	Trade	Final Allocation
Michael	**$10F, 3C$**	$3F$ for $3C$	$7F, 6C$
Kelly	**$5F, 15C$**	$3C$ for $3F$	$8F, 12C$

Michael will give 1/2 clothing for 1 food (or 2 food for 1 clothing) while Kelly is willing to trade 3 clothing for 1 food (that is, she will accept 1/3 food for 1 clothing). If they settle on a trading rate of 1 unit of food for 1 unit of clothing they will both be better off. Suppose Michael gives up 3 units of food and receives 3 units of clothing, so his final allocation is $7F$ and $6C$. Kelly will thus give up 3 units of clothing and gain 3 units of food, so her final allocation is $8F$ and $12C$. Kelly's *MRS* will decrease and Michael's will increase, so given they must be equal in equilibrium, their *MRS* value will be somewhere between 3 and 1/2. So a possible value for each person's *MRS* is 2 at the efficient solution.

6. In the analysis of an exchange between two people, suppose both people have identical preferences. Will the contract curve be a straight line? Explain. Can you think of a counterexample?

Given that the contract curve intersects the origin for each individual, a straight line contract curve would be a diagonal line running from one origin to the other. The slope of this line is $\frac{Y}{X}$, where Y is the total amount of the good on the vertical axis and X is the total amount of the good on the horizontal axis. (x_1, y_1) are the amounts of the two goods allocated to one individual and $(x_2, y_2) = (X - x_1, Y - y_1)$ are the amounts of the two goods allocated to the other individual. A straight line contract curve would have the equation

$$y_1 = \left(\frac{Y}{X}\right)x_1.$$

We need to show that when the marginal rates of substitution for the two individuals are equal ($MRS^1 = MRS^2$), the point lies on this linear contract curve.

For example, consider the utility function $U = x_i^2 y_i$. Then

$$MRS^i = \frac{MU_x^i}{MU_y^i} = \frac{2x_i y_i}{x_i^2} = \frac{2y_i}{x_i}.$$

If MRS^1 equals MRS^2, then

$$\left(\frac{2y_1}{x_1}\right) = \left(\frac{2y_2}{x_2}\right).$$

Is this point on the contract curve? Yes, because

$$x_2 = X - x_1 \text{ and } y_2 = Y - y_1, \text{ so}$$

$$2\left(\frac{y_1}{x_1}\right) = 2\left(\frac{Y - y_1}{X - x_1}\right).$$

This means that

$$\frac{y_1(X - x_1)}{x_1} = Y - y_1, \quad \text{or} \quad \frac{y_1X - y_1x_1}{x_1} = Y - y_1, \text{ and}$$

$$\frac{y_1X}{x_1} - y_1 = Y - y_1, \quad \text{or} \quad \frac{y_1X}{x_1} = Y, \quad \text{or} \quad y_1 = \left(\frac{Y}{X}\right)x_1.$$

With this utility function we find $MRS^1 = MRS^2$, and the contract curve is a straight line. This should be the case for utility functions with strictly convex indifference curves. However, if the consumers' preferences are such that the goods are perfect complements or perfect substitutes, the contract curve is not necessarily well defined. For example, with perfect substitutes the indifference curves are straight lines, so every point is a point of tangency between indifference curves, and thus there is no unique contract curve. With perfect complements, there may be a "thick" contract path, not a single line.