4. Suppose you are the manager of a watchmaking firm operating in a competitive market. Your cost of production is given by $C = 200 + 2q^2$, where q is the level of output and C is total cost. (The marginal cost of production is $4q$; the fixed cost is \$200.)

a. If the price of watches is \$100, how many watches should you produce to maximize profit?

Profits are maximized where price equals marginal cost. Therefore,

$$100 = 4q, \text{ or } q = 25.$$

b. What will the profit level be?

Profit is equal to total revenue minus total cost: $\pi = Pq - (200 + 2q^2)$. Thus,

$$\pi = (100)(25) - (200 + 2(25)^2) = \$1050.$$

c. At what minimum price will the firm produce a positive output?

The firm will produce in the short run if its revenues are greater than its total variable costs. The firm's short-run supply curve is its MC curve above minimum AVC. Here, $AVC = \frac{VC}{q} = \frac{2q^2}{q} = 2q$.

Also, $MC = 4q$. So, MC is greater than AVC for any quantity greater than 0. This means that the firm produces in the short run as long as price is positive.

7. Suppose the same firm's cost function is $C(q) = 4q^2 + 16$.

a. Find variable cost, fixed cost, average cost, average variable cost, and average fixed cost. (*Hint*: Marginal cost is given by $MC = 8q$.)

Variable cost is that part of total cost that depends on q (so $VC = 4q^2$) and fixed cost is that part of total cost that does not depend on q (so $FC = 16$).

$$VC = 4q^2$$

$$FC = 16$$

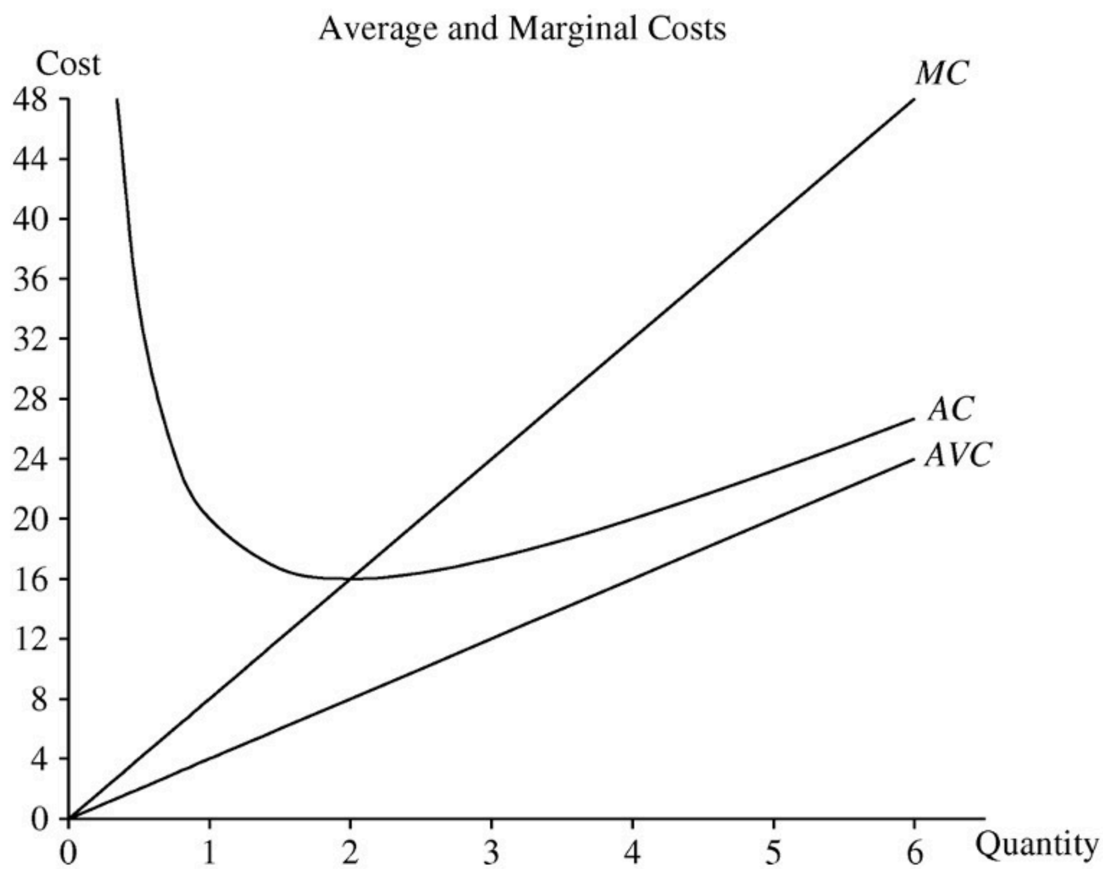
$$AC = \frac{C(q)}{q} = 4q + \frac{16}{q}$$

$$AVC = \frac{VC}{q} = 4q$$

$$AFC = \frac{FC}{q} = \frac{16}{q}$$

b. Show the average cost, marginal cost, and average variable cost curves on a graph.

Average cost is U-shaped. Average cost is relatively large at first because the firm is not able to spread the fixed cost over very many units of output. As output increases, average fixed cost falls quickly, leading to a rapid decline in average cost. Average cost will increase at some point because the average fixed cost will become very small and average variable cost is increasing as q increases. *MC* and *AVC* are linear in this example, and both pass through the origin. Average variable cost is everywhere below average cost. Marginal cost is everywhere above average variable cost. If the average is rising, then the marginal must be above the average. Marginal cost intersects average cost at its minimum point, which occurs at a quantity of 2 where *MC* and *AC* both equal \$16.



c. Find the output that minimizes average cost.

Minimum average cost occurs at the quantity where *MC* is equal to *AC*:

$$AC = 4q + \frac{16}{q} = 8q = MC$$

$$\frac{16}{q} = 4q$$

$$16 = 4q^2$$

$$4 = q^2$$

$$2 = q.$$

d. At what range of prices will the firm produce a positive output?

The firm will supply positive levels of output in the short run as long as $P = MC > AVC$, or as long as the firm is covering its variable costs of production. In this case, marginal cost is above average variable cost at all output levels, so the firm will supply positive output at any positive price.

e. At what range of prices will the firm earn a negative profit?

The firm will earn negative profit when $P = MC < AC$, or at any price below minimum average cost. In part c we found that the minimum average cost occurs where $q = 2$. Plug $q = 2$ into the average cost function to find $AC = 16$. The firm will therefore earn negative profit if price is below 16.

f. At what range of prices will the firm earn a positive profit?

In part e we found that the firm would earn negative profit at any price below 16. The firm therefore earns positive profit as long as price is above 16.

8. A competitive firm has the following short-run cost function:

$$C(q) = q^3 - 8q^2 + 30q + 5.$$

a. Find $MC, AC,$ and AVC and sketch them on a graph.

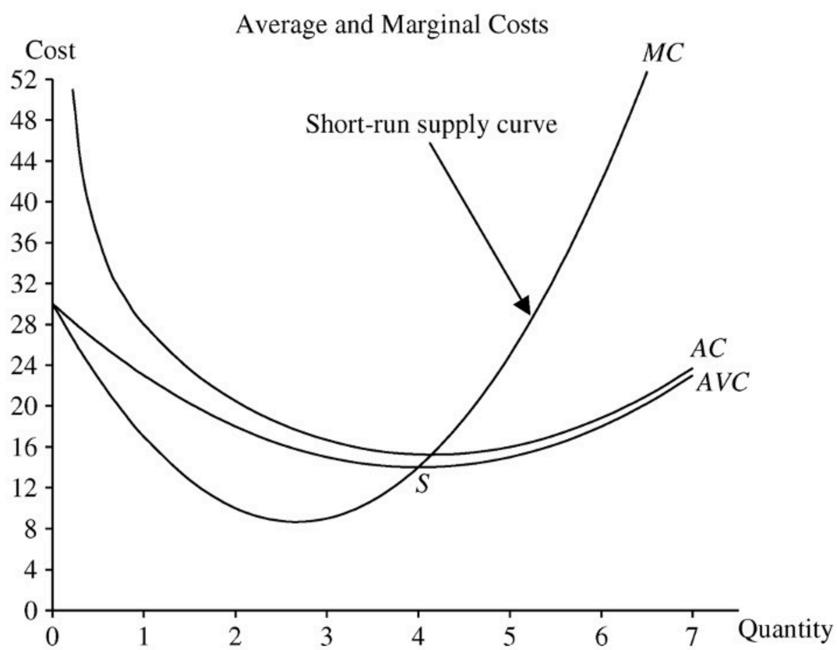
The functions can be calculated as follows:

$$MC = \frac{dC}{dq} = 3q^2 - 16q + 30$$

$$AC = \frac{C}{q} = q^2 - 8q + 30 + \frac{5}{q}$$

$$AVC = \frac{VC}{q} = q^2 - 8q + 30.$$

Graphically, all three cost functions are U-shaped in that cost initially declines as q increases, and then increases as *q* increases. Average variable cost is below average cost everywhere. Marginal cost is initially below *AVC* and then increases to intersect *AVC* at its minimum point, *S*. *MC* is also initially below *AC* and then intersects *AC* at its minimum point.



b. At what range of prices will the firm supply zero output?

The firm will find it profitable to produce in the short run as long as price is greater than or equal to average variable cost. If price is less than average variable cost then the firm will be better off shutting down in the short run, as it will only lose its fixed cost and not fixed plus some variable cost. Here we need to find the minimum average variable cost, which can be done in two different ways. You can either set marginal cost equal to average variable cost, or you can differentiate average variable cost with respect to q and set this equal to zero. In both cases, you can solve for q and then plug into AVC to find the minimum AVC. Here we will set AVC equal to MC:

$$AVC = q^2 - 8q + 30 = 3q^2 - 16q + 30 = MC$$
$$2q^2 = 8q$$
$$q = 4$$
$$AVC(q = 4) = 4^2 - 8 * 4 + 30 = 14.$$

This is point S on the graph. Hence the firm supplies zero output if $P < \$14$.

c. Identify the firm's supply curve on your graph.

The firm's supply curve is the MC curve above the point where $MC = AVC$. The firm will produce at the point where price equals MC as long as MC is greater than or equal to AVC. This point is labeled S on the graph, so the short-run supply curve is the portion of MC that lies above point S.

d. At what price would the firm supply exactly 6 units of output?

The firm maximizes profit by choosing the level of output such that $P = MC$. To find the price where the firm would supply 6 units of output, set q equal to 6 and solve for MC:

$$P = MC = 3q^2 - 16q + 30 = 3(6^2) - 16(6) + 30 = 42.$$

9. a. Suppose that a firm's production function is $q = 9x^{\frac{1}{2}}$ in the short run, where there are fixed costs of \$1000, and x is the variable input whose cost is \$4000 per unit. What is the total cost of producing a level of output q? In other words, identify the total cost function $C(q)$.

Since the variable input costs \$4000 per unit, total variable cost is 4000 times the number of units used, or $4000x$. Therefore, the total cost of the inputs used is $C(x)$ = variable cost + fixed cost = $4000x + 1000$. Now rewrite the production function to express x in terms of q: $x = \frac{q^2}{81}$. We can then substitute this into the above cost function to find $C(q)$:

$$C(q) = \frac{4000q^2}{81} + 1000,$$

or $C(q) = 49.3827q^2 + 1000$.

b. Write down the equation for the supply curve.

The firm supplies output where $P = MC$ as long as $MC > AVC$. In this example, $MC = 98.7654q$ is always greater than $AVC = 49.3827q$, so the entire marginal cost curve is the supply curve. Therefore $P = 98.7654q$, or $q = .010125P$, is the firm's short-run supply curve.

c. If price is \$1000, how many units will the firm produce? What is the level of profit? Illustrate your answer on a cost curve graph.

Use the supply curve from part b: $q = 0.010125(1000) = 10.125$.

Profit $\pi = R - C = 1000(10.125) - [(4000(10.125)^2/81) + 1000] = \$4062.50$. Graphically, the firm produces where the price line hits the MC curve. Since profit is positive, this occurs at a quantity where price is greater than average cost. To find profit on the graph, take the difference between the revenue rectangle (price times quantity) and the cost rectangle (average cost times quantity). The area of the resulting rectangle is the firm's profit.