Q5

Philip's Lagrangian is

$$L: 4q_1^{0.5} + q_2 + \lambda(Y - p_1q_1 - q_2). \text{ (Hint: } p_2=1)$$

The first order conditions are

$$L_1: 2q_1^{-0.5} - p_1\lambda = 0,$$

$$L_2: 1 - \lambda = 0, \text{ and}$$

$$L_3: Y - p_1q_1 - q_2 = 0.$$

Dividing the first condition by the second,

$$2q_1^{-0.5} = p_1.$$

Solving for q_1, Philip's demand for good q_1 is

$$q_1 = 4/p_1^2.$$

Substituting this into the third condition and solving for q_2, Philip's demand for good q_2 is

$$Y - p_1(4/p_1^2) - q_2 = 0$$

$$Y - (4/p_1) - q_2 = 0$$

$$q_2 = Y - (4/p_1).$$

Philip's demand for good 2 is a function of income (Y) but his demand for good 1 is not. Therefore, income affects the demand curve for good 2 but not the demand curve for good 1 when at an interior solution.

The effect of a change in p on the demand for good 1 is

$$\frac{dq_1}{dp} = -8p_1^{-3}$$

The total effect of a price change (ε) equals the substitution effect (ε^*) plus the income effect ($-\theta\xi$):

$$\varepsilon = \varepsilon^* - \theta\xi.$$

Since the demand for good 1 does not depend on income when at an interior optimum, the income effect is zero, and the total effect equals the substitution effect:

$$\varepsilon = \varepsilon^* = -8p_1^{-3}.$$

Q6

The expenditure function (E) is the relationship showing the minimal expenditures necessary to achieve a specific utility level for a given set of prices. Since q_1 and q_2 are perfect complements, Sylvia will consume them in fixed proportions. So, her uncompensated demand functions are

$$q_1 = \frac{Y}{p_1 + \frac{p_2}{j}} \text{ and } q_2 = \frac{Y}{p_1 + \frac{p_2}{2}}.$$

Her indirect utility function (V) is

$$V = \min\left(\frac{Y}{p_1 + \frac{p_2}{j}}, j\frac{Y}{jp_1 + p_2}\right) = \frac{Y}{p_1 + \frac{p_2}{j}} \text{ for } \mathbf{0 < j < 1.}$$

And

$$V = \frac{Y}{jp_1 + p_2} \text{ for } j > 1$$

The expenditure function and indirect utility function are inverses:

$$E = \left(p_1 + \frac{p_2}{j}\right)\bar{U}.$$

Therefore, the compensated demand functions for $0 < j < 1$ are

$$q_1 = \frac{\partial E}{\partial p_1} = U \quad \text{and} \quad q_2 = \frac{\partial E}{\partial p_2} = \frac{U}{j}.$$

And for $j > 1$, the compensated demand functions are

$$q_1 = jU, \text{ and } q_2 = U.$$

Q7

After the income and price change, Felix spends \$1,050 on food (from income of \$1,200 minus expenditures of \$150 on clothing). At a price of \$10 per unit of food, this is 105 units of food. Felix's initial consumption bundle (2 units of clothing and 80 units of food) would cost \$1,100 with the new price of clothing (from a new price of clothing of \$150 multiplied by 2 units of clothing plus the same \$10 price of food multiplied by 80 units of food). Because this bundle is attainable with Felix's higher income but he does not choose it (he instead chooses q_1=1 and q_2=105), he must prefer his new bundle to his initial bundle. (Note that Felix's new bundle would have cost \$1,150 at original prices, so it had been unattainable.)

Q8.

a. Hong and Wouk (2008) estimate that the loss of postal revenue is -215 million.

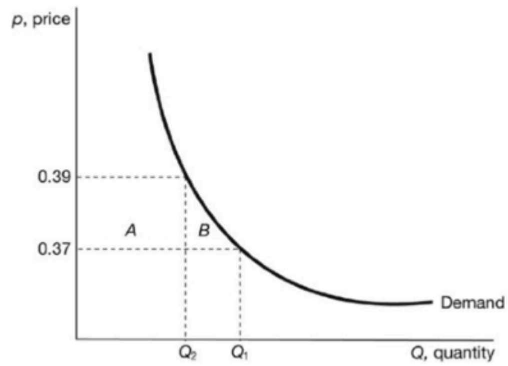
Revenue = p^*Q, or $Xp^{-1.6}$

Since Δ revenue = 215 = $X(37^{0.6} - 39^{0.6})$, X = 60,353

NOTE: The problem does not specify whether to use prices in cents (37, 39) or decimals. This answer uses cents. If you use decimals (.37, .39), X = 3,808.

b. When the price is 39, quantity demanded is 171.79 million (q = 60,353(39)$^{-1.6}$). Therefore, the area of rectangle A is equal to 2(171.79), or 343.58. Given that the change in consumer surplus is 358.33 (= $\int_{37}^{39} Xp^{-1.6}\,dp$), area B is equal to 358.33 - 343.581, or \$14.75 million. (The change in CS is slightly different from the results in Hong and Wouk since they used a different demand function.)

NOTE: if the decimal version (.37, .39, .02) is used, final answers will be the same.



Graph should look like Figure 5.2, except that the prices are not given in this part of the problem. Area A + B = 333 million. The loss of postal revenue is the difference between the revenue increase (A) less the original price times the change in the number of units sold, which is area A minus the rectangle below area B on the graph. B cannot be found from the information given in this problem.