Q1

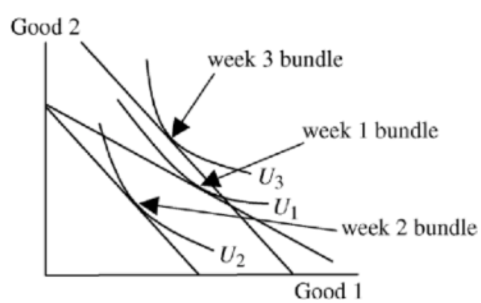
Each week, Bill, Mary, and Jane select the quantity of two goods, x_1 and x_2, that they will consume in order to maximize their respective utilities. They each spend their entire weekly income on these two goods.

a. Suppose you are given the following information about the choices that Bill makes over a three-week period:

	x_1	x_2	P_1	P_2	I
Week 1	10	20	2	1	40
Week 2	7	19	3	1	40
Week 3	8	31	3	1	55

Did Bill's utility increase or decrease between week 1 and week 2? Between week 1 and week 3? Explain using a graph to support your answer.

Bill's utility fell between weeks 1 and 2 because he consumed less of both goods in week 2. Between weeks 1 and 2 the price of good 1 rose and his income remained constant. The budget line pivoted inward and he moved from U_1 to a lower indifference curve, U_2, as shown in the diagram. Between week 1 and week 3 his utility rose. The increase in income more than compensated him for the rise in the price of good 1. Since the price of good 1 rose by \$1, he would need an extra \$10 to afford the same bundle of goods he chose in week 1. This can be found by multiplying week 1 quantities times week 2 prices. However, his income went up by \$15, so his budget line shifted out beyond his week 1 bundle. Therefore, his original bundle lies within his new budget set as shown in the diagram, and his new week 3 bundle is on the higher indifference curve U_3.

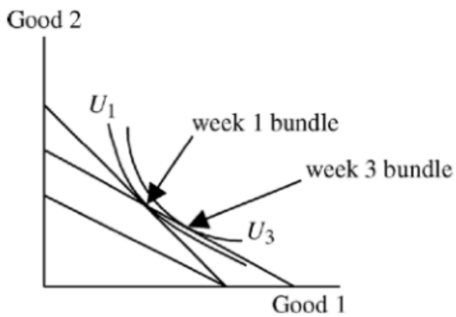


b. Now consider the following information about the choices that Mary makes:

	x_1	x_2	P_1	P_2	I
Week 1	10	20	2	1	40
Week 2	6	14	2	2	40
Week 3	20	10	2	2	60

Did Mary's utility increase or decrease between week 1 and week 3? Does Mary consider both goods to be normal goods? Explain.

Mary's utility went up. To afford the week 1 bundle at the new prices, she would need an extra \$20, which is exactly what happened to her income. However, since she could have chosen the original bundle at the new prices and income but did not, she must have found a bundle that left her slightly better off. In the graph to the right, the week 1 bundle is at the point where the week 1 budget line is tangent to indifference curve U_1, which is also the intersection of the week 1 and week 3 budget lines. The week 3 bundle is somewhere on the week 3 budget line that lies above the week 1 indifference curve. This bundle will be on a higher indifference curve, U_3 in the graph, and hence Mary's utility increased. A good is normal if more is chosen when income increases. Good 1 is normal because Mary consumed more of it when her income increased (and prices remained constant) between weeks 2 and 3. Good 2 is not normal, however, because when Mary's income increased from week 2 to week 3 (holding prices the same), she consumed less of good 2. Thus good 2 is an inferior good for Mary.

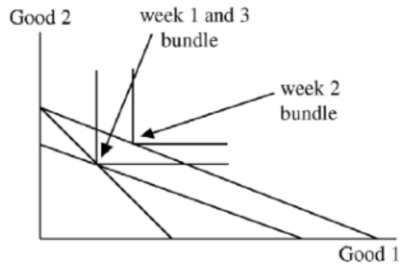


c. Finally, examine the following information about Jane's choices:

	x_1	x_2	P_1	P_2	I
Week 1	**12**	**24**	**2**	**1**	**48**
Week 2	**16**	**32**	**1**	**1**	**48**
Week 3	**12**	**24**	**1**	**1**	**36**

Draw a budget line-indifference curve graph that illustrates Jane's three chosen bundles. What can you say about Jane's preferences in this case? Identify the income and substitution effects that result from a change in the price of good x_1.

In week 2, the price of good 1 drops, Jane's budget line pivots outward, and she consumes more of both goods. In week 3 the prices remain at the new levels, but Jane's income is reduced. This leads to a parallel leftward shift of her budget line and causes Jane to consume less of both goods. Notice that Jane always consumes the two goods in a fixed 1:2 ratio. This means that Jane views the two goods as perfect complements, and her indifference curves are L-shaped. Intuitively, if the two goods are complements, there is no reason to substitute one for the other during a price change, because they have to be consumed in a set ratio. Thus the substitution effect is zero. When the price ratio changes and utility is kept at the same level (as happens between weeks 1 and 3), Jane chooses the same bundle (12, 24), so the substitution effect is zero.



The income effect can be deduced from the changes between weeks 1 and 2 and also between weeks 2 and 3. Between weeks 2 and 3 the only change is the $12 drop in income. This causes Jane to buy 4 fewer units of good 1 and 8 less units of good 2. Because prices did not change, this is purely an income effect. Between weeks 1 and 2, the price of good 1 decreased by $1 and income remained the same. Since Jane bought 12 units of good 1 in week 1, the drop in price increased her purchasing power by ($1)(12) = $12. As a result of this $12 increase in real income, Jane bought 4 more units of good 1 and 8 more of good 2. We know there is no substitution effect, so these changes are due solely to the income effect, which is the same (but in the opposite direction) as we observed between weeks 1 and 2.

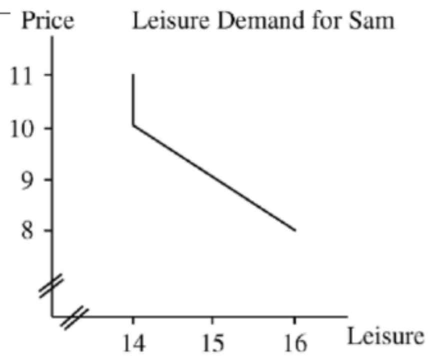
Q2

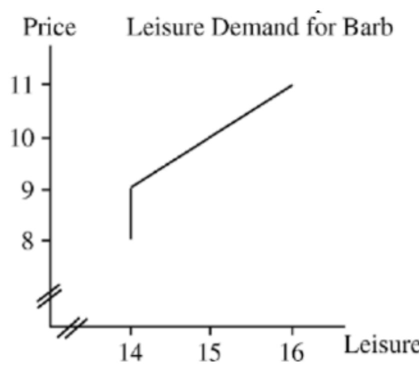
Two individuals, Sam and Barb, derive utility from the hours of leisure (*L*) they consume and from the amount of goods (*G*) they consume. In order to maximize utility, they need to allocate the 24 hours in the day between leisure hours and work hours. Assume that all hours not spent working are leisure hours. The price of a good is equal to $1 and the price of leisure is equal to the hourly wage. We observe the following information about the choices that the two individuals make:

Price of *G*	Price of *L*	Sam *L* (hours)	Barb *L* (hours)	Sam *G* ($)	Barb *G* ($)
1	8	16	14	64	80
1	9	15	14	81	90
1	10	14	15	100	90
1	11	14	16	110	88

Graphically illustrate Sam's leisure demand curve and Barb's leisure demand curve. Place price on the vertical axis and leisure on the horizontal axis. Given that they both maximize utility, how can you explain the difference in their leisure demand curves?

It is important to remember that less leisure implies more hours spent working. Sam's leisure demand curve is downward sloping. As the price of leisure (the wage) rises, he chooses to consume less leisure and thus spend more time working at a higher wage to buy more goods. Barb's leisure demand curve is upward sloping. As the price of leisure rises, she chooses to consume more leisure (and work less) since her working hours are generating more income per hour. See the leisure demand curves below.





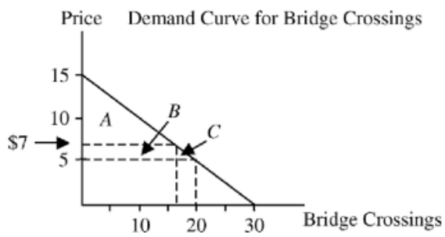
This difference in demand can be explained by examining the income and substitution effects for the two individuals. The substitution effect measures the effect of a change in the price of leisure, keeping utility constant (the budget line rotates along the current indifference curve). Since the substitution effect is always negative, a rise in the price of leisure will cause both individuals to consume less leisure. The income effect measures the effect of the change in purchasing power brought about by the change in the price of leisure. Here, when the price of leisure (the wage) rises, there is an increase in purchasing power (the new budget line shifts outward). Assuming both individuals consider leisure to be a normal good, the increase in purchasing power will increase demand for leisure. For Sam, the reduction in leisure demand caused by the substitution effect outweighs the increase in demand for leisure caused by the income effect, so his leisure demand curve slopes downward. For Barb, her income effect is larger than her substitution effect, so her leisure demand curve slopes upward.

Q3

13. **Suppose you are in charge of a toll bridge that costs essentially nothing to operate. The demand for bridge crossings Q is given by $P = 15 - \frac{1}{2}Q$.**

a. **Draw the demand curve for bridge crossings.**

The demand curve is linear and downward sloping. The vertical intercept is 15 and the horizontal intercept is 30.



b. **How many people would cross the bridge if there were no toll?**

At a price of zero, $0 = 15 - (1/2)Q$, so $Q = 30$. The quantity demanded would be 30.

c. **What is the loss of consumer surplus associated with a bridge toll of \$5?**

If the toll is \$5 then the quantity demanded is 20. The lost consumer surplus is the difference between the consumer surplus when price is zero and the consumer surplus when price is \$5. When the toll is zero, consumer surplus is the entire area under the demand curve, which is (1/2)(30)(15) = 225. When $P = 5$, consumer surplus is area $A + B + C$ in the graph above. The base of this triangle is 20 and the height is 10, so consumer surplus = (1/2)(20)(10) = 100. The loss of consumer surplus is therefore \$225 – 100 = \$125.

d. **The toll-bridge operator is considering an increase in the toll to \$7. At this higher price, how many people would cross the bridge? Would the toll-bridge revenue increase or decrease? What does your answer tell you about the elasticity of demand?**

At a toll of \$7, the quantity demanded would be 16. The initial toll revenue was \$5(20) = \$100. The new toll revenue is \$7(16) = \$112, so revenue increases by \$12. Since the revenue goes up when the toll is increased, demand is inelastic (the 40% increase in price outweighs the 20% decline in quantity demanded).

e. **Find the lost consumer surplus associated with the increase in the price of the toll from \$5 to \$7.**

The lost consumer surplus is area $B + C$ in the graph above. Thus, the loss in consumer surplus is (16) × (7 – 5) + (1/2) × (20 – 16) × (7 – 5) = \$36.

Q4.

a. David's Lagrangian for utility maximization is

$$L = 10\, q_1^{0.25}\, q_2^{0.75} + \lambda(Y - p_{q1}\, q_1 - p_2 q_2).$$

The first order conditions are

$$\frac{dL}{dq1} = 2.5\, q_1^{-0.75}\, q_2^{0.75} - p_1\lambda = 0$$

$$\frac{dL}{dq2} = 7.5\, q_1^{0.25}\, q_2^{-0.25} - p_2\lambda = 0$$

$$\frac{dL}{d\lambda} = Y - p_{q1}\, q_1 - p_2\, q_2 = 0.$$

Dividing the first condition by the second,

$$\frac{2.5q2}{7.5q1} = \frac{p_{q1}}{p_{q2}}.$$

Rewriting,

$$q_2 = 3q_1\left(\frac{p_{q1}}{p_{q2}}\right). \qquad (1)$$

Substituting this into the third condition and solving for G, David's optimal value for gasoline is

$$Y - p_{q1}\, q_1 - p_{q2}\, 3q_1\left(\frac{p_{q1}}{p_{q2}}\right) = 0$$

$$Y - p_{q1}\, q_1 - 3q_1\, p_{q1} = 0$$

$$Y = 4p_{q1}\, q_1$$

$$q_1 = \frac{Y}{4p_{q1}}.$$

Substituting this into equation (1) above and solving for B, David's optimal value of bread is

$$q_2 = 3\left(\frac{Y}{4pq1}\right)\left(\frac{p_{q1}}{p_{q2}}\right)$$

$$q_2 = \frac{3Y}{4p_{q2}}.$$

b. The partial derivative of David's optimal value of gasoline is

$$\frac{dq1}{dp_{q1}} = -\frac{Y}{4(p_{q1}^{\;2})},$$

which is negative, indicating that David will demand less gasoline when the price of gasoline increases.

c. The partial derivative of the effect of a change in the price of gasoline on David's optimal quantity of gasoline is

$$\frac{d^2q1}{dp_{q1}dY} = -\frac{1}{4(p_{q1}^{\;2})},$$

which is negative, indicating that as income increases, David's response to a change in the price of gasoline on his demand for gasoline becomes more negative.