Econ 221
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Midterm Practice Questions

1. (*Tax audit*) A taxpayer (P) chooses between *Cheat* and *Don't*, and an auditor (A) chooses between *Audit* and *Don't*. Choosing *Don't* for P means reporting his entire income for tax at a fixed rate of 0.6 (60% of the income), with a payoff of 0.4 regardless of A chooses *Audit* or *Don't*. Choosing *Cheat* means P reporting zero income, with a payoff of 1 if A chooses *Don't*, and a payoff of 0 if A chooses *Audit* (the entire income is confiscated as penalty for cheating). The cost of choosing *Audit* is 0.5 for A, also a parameter strictly between 0 and 1. For questions (a) and (b), suppose that P and A play a simultaneous-move game, with the following payoff table:

A

		Audit	*Don't*
P	*Cheat*	0, 0.5	1, 0
	Don't	0.4, 0.1	0.4, 0.6

(a) Does there exist a pure-strategy Nash equilibrium? Explain your answer.

(b) Find the mixed-strategy Nash equilibrium.

For the remaining questions, suppose that P can engage in costly tax avoidance by making it more difficult for A to audit his income. Specifically, consider a two-stage game where in the first stage P chooses the amount of cost c he incurs in order to add c to A's cost of choosing *Audit*, and for each c he chooses, in the second stage he plays a simultaneous-move game with A, given by

A

	Audit	Don't
Cheat	$-c, 0.5 - c$	$1 - c, 0$
Don't	$0.4 - c, 0.1 - c$	$0.4 - c, 0.6$

P (row player), A (column player)

The above is a subgame of the two-stage game after P's choice of c. The subgame associated with $c = 0$ is the game analyzed in questions (a) and (b) above.

(c) Is the subgame after P chooses any $c > 0.5$ solvable by iterative elimination of strictly dominated strategies? What is the payoff of P in the Nash equilibrium of this subgame?

(d) In the subgame after P chooses $c = 0.5$, does A have a weakly dominant strategy? What is the payoff of P in the Nash equilibrium of this subgame?

(e) In the subgame after P chooses any $c < 0.5$, what is the mixed-strategy Nash equilibrium? What is the payoff of P in the mixed strategy Nash equilibrium of this subgame?

(f) Using your answers to (c), (d) and (e) to find the unique subgame perfect Nash equilibrium of the two-stage game.

2. (*Inattentive parenting*) Consider a game between a child (C) and her father (F), where C chooses first between *Behave* and *Misbehave*, followed by F's choice between *Discipline* and *Acquiesce*. C's benefit from getting away with misbehaving is 1, with her cost of behaving denoted as 0.5. F's cost of punishing C is 1, with his benefit of disciplining a misbehaving child equal to 2. Below is the payoff table:

$$\begin{array}{c} \\ \text{C} \end{array} \begin{array}{c} \\ \end{array}$$

		F	
		Discipline	*Acquiesce*
	Behave	$-0.5, 0$	$0.5, 1$
C	*Misbehave*	$0, 2$	$1, 1$

For questions (a)-(c), F is inattentive — he does not observe whether C has chosen *Behave* or *Misbehave* when he chooses between *Discipline* and *Acquiesce*.

(a) Does F have a dominant strategy? Is the game solvable by iterated elimination of strictly dominated strategies? Explain your answers.

(b) Is F's strategy of *Acquiesce* rationalizable? Explain your answer.

(c) Find all pure-strategy Nash equilibria of this game. Does there exist a mixed-strategy Nash equilibrium? Explain your answers.

For questions (d) and (e), F is attentive — he observes C's choice before his own choice.

(d) List all pure strategies of C and F, construct the strategic form using a game table, and find all pure-strategy Nash equilibria of this game.

(e) Find the rollback equilibrium. Is there a Nash equilibrium you have found in (d) that is not subgame perfect? Explain your answers.

For the remaining question, suppose that F may be inattentive or attentive, and that when F is inattentive he must choose *Acquiesce*. Whether F is inattentive or attentive is random. More precisely, if C chooses *Behave*, then with probability 0.4 Nature chooses *Inattentive* and the game ends with payoff 0.5 to C and 1 to F; with probability 0.6 Nature chooses *Attentive* and F makes his choice knowing that C has chosen *Behave*, with payoff

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−0.5 to C and 0 to F if he chooses *Discipline* and payoff 0.5 to C and 1 to F if he chooses *Acquiesce*. If C chooses *Misbehave*, then with probability 0.4 Nature chooses *Inattentive* and the game ends with payoff 1 to both C and F; with probability 0.6 Nature chooses *Attentive* and F makes his choice knowing that C has chosen *Misbehave*, with payoff 0 to C and 2 to F if he chooses *Discipline* and payoff 1 to both C and F if he chooses *Acquiesce*.

(f) Find the rollback equilibrium of this game.

3. (*Job Offers*) Two identical firms, A and B, each having a single vacant position to fill and each having received two job applications, must decide which applicant to make an offer to. An applicant receiving a single offer will accept it with probability 1, while an applicant receiving two offers will accept each one with probability 0.5. Applicant 1 is more attractive to both firms than Applicant 2: the value to each firm from filling the position with Applicant 1 is 3, and the value to each firm from filling the position with Applicant 2 is 2. However, note that for each firm, having the offer accepted with probability 0.5 by Applicant 1 is worse than having it accepted by Applicant 2 with probability 1. The payoff table is given by

		B	
		1	*2*
A	*1*	1.5, 1.5	3, 2
	2	2, 3	1, 1

For questions (a)-(c) below, suppose the two firms make their offers simultaneously.

(a) Find all pure-strategy Nash equilibria.

(b) Find the mixed-strategy Nash equilibrium.

For question (c) only, suppose Firm A is more attractive than Firm B to both applicants, so that an applicant receiving both offers will accept A's offer with probability 1.

(c) Argue that there is now a unique pure-strategy Nash equilibrium, and that there is no mixed-strategy Nash equilibrium.

For question (d), suppose Firm A first chooses which applicant to make an offer to, and Firm B observes Firm A's choice before making its own choice.

(d) Find the unique subgame perfect equilibrium. How does the outcome compare with the Nash equilibrium outcomes in (a)? Explain.

For the remaining question, suppose that each firm can attach a signing bonus s to the job offer made to Applicant 1. In other words, in this new game each firm can either make an offer to Applicant 2 (there is no signing bonus), or make an offer to Applicant 1 which also specifies some $s \geq 0$. The strategies in the original game are still available to each firm because making an offer to Applicant 1 is the same as making the offer with $s = 0$. If both firms make their offer to Applicant 1, and if Firm A offers a larger bonus than B, i.e. if $s_A > s_B$, then A's offer is accepted by Applicant 1 with probability 1, resulting in a payoff of $3 - s_A$ to Firm A and 0 to Firm B. If they both make their offer to Applicant 1 and $s_A = s_B = s$, each offer is accepted with probability 0.5, resulting in a payoff of $0.5(3 - s)$ to each firm. Of course if only one firm makes an offer to Applicant 1 with a signing bonus s, the offer is accepted with probability 1 and the firm gets $3 - s$, while the other firm gets 2; and if both firms make an offer to Applicant 2, each gets a payoff of 1.

(e) Do the equilibria in (a) remain equilibria in this new simultaneous-move game? Does there exist a pure-strategy Nash equilibrium? Explain.

4. (*Bank Runs*) Two investors, A and B, having each invested 1 dollar in a project through a bank, must decide simultaneously whether or not to withdraw their investment before completion of the project. If both investors choose *Don't*, the project will be completed by the bank and yield a gross return of 1.2 dollars to each investor (so net return is 20%). If either or both of investors choose *Withdraw*, the project is liquidated by the bank and sold for its the liquidation value of 1.6 (so the total loss is 40%). If only one investor chooses *Withdraw* this investor gets 1 dollar and the other gets $1.6 - 1 = 0.6$ dollars, while if both investors choose *Withdraw* they each get $1.6 \times 0.5 = 0.8$ dollars. The payoff table of this simultaneous-move game is thus given by

B

		Don't	Withdraw
	Don't	1.2, 1.2	0.6, 1
A	*Withdraw*	1, 0.6	0.8, 0.8

(a) Find all pure-strategy Nash equilibria.

(b) Are *Don't* and *Withdraw* both rationalizable? Explain.

(c) Find the mixed-strategy Nash equilibrium.

   For the remaining questions, suppose that A and B respectively choose the amount of withdrawal w_A and w_B between 0 and 1. If $w_A = w_B = 0$, then the project will be completed and each investor gets 1.2; otherwise, the project is liquidated and sold for its liquidation value 1.6, with A getting w_A and B getting w_B if $w_A + w_B \leq 1.6$, and A getting the fraction $w_A/(w_A + w_B)$ of 1.6 and B getting the fraction $w_B/(w_A + w_B)$ of 1.6

if $w_A + w_B > 1.6$. For question (d), suppose that A and B make their choices w_A and w_B simultaneously. For question (e), suppose that A chooses w_A first and B then chooses w_B after observing A's choice.

(d) Argue that $w_A = w_B = 0$ and $w_A = w_B = 1$ are both Nash equilibrium. Are there other Nash equilibria? Explain.

(e) Find the unique subgame perfect equilibrium.

5. (*Escalation and Sanction*) The Kingdom of Qin chooses between *Escalate* and *Don't Escalate* with its small neighboring kingdom, while a coalition G6 of all kingdoms except Qin chooses between *Sanction* and *No Sanction*. The payoffs are given by the table below

G6

		Sanction	No
	Escalate	$-1, -3$	$2, 0$
Qin	Don't	$0, 2$	$0, 0$

In questions (a) and (b), Qin and G6 make their choices simultaneously, and in (c) they choose sequentially, with the second-mover observing the choice of the first-mover.

(a) Find all pure-strategy Nash equilibria.

(b) Is there a mixed strategy Nash equilibrium? If yes, find it; if no, explain why it does not exist.

(c) Find the subgame perfect equilibrium in the sequential-move game where Qin moves first, and the subgame perfect equilibrium in the game where G6 moves first. Compare the two equilibrium outcomes.

In questions (d) and (e), Qin has a third choice of *Partially Escalate* and G6 has a third choice of *Partial Sanction*. The payoff table becomes

G6

	Sanction	Partial	No
Escalate	$-1, -3$	$1, -1$	$2, 0$
Qin Partially	$-2, -3$	$0, -1$	$1, 0$
Don't	$0, 2$	$0, 1$	$0, 0$

In question (d), Qin and G6 simultaneously choose one out of their three options.

(d) Is *Partially Escalate* strictly dominated for Qin? Can *Partial Sanction* ever be a best response for G6? Explain that the game has the same Nash equilibria as the one without the third choices.

In questions (e) and (f), consider the following two-stage game, with the first stage outcome observed before the second stage game is played. In the first stage, Qin chooses between *Partially Escalate* and *Don't Escalate*, and G6 simultaneously chooses between *Partial Sanction* and *No Sanction*. If Qin chooses *Don't Escalate* and G6 chooses *No Sanction* in the first stage, then in the second stage, Qin and G6 play the same simultaneous-move game as in (d), with all three options remaining for each side. If Qin chooses *Partially Escalate* and G6 chooses *Partial Sanction* in the first stage, then in the second stage Qin and G6 play the following simultaneous-move game, with each side having only two options (*Partially Escalate* for Qin means no further escalation and *Escalate* means further escalation, and *Partial Sanction* for G6 means no further sanction and *Sanction* means further sanction):

G6

	Sanction	Partial
Qin Escalate	$-1, -3$	$1, -1$
Partially	$-2, -3$	$0, -1$

If Qin chooses *Partially Escalate* and G6 chooses *No Sanction* in the first stage, then in the second stage Qin and G6 play the following simultaneous-move game, with Qin having only two options but all three options remain for G6:

G6

	Sanction	Partial	No
Qin Escalate	$-1, -3$	$1, -1$	$2, 0$
Partially	$-2, -3$	$0, -1$	$1, 0$

Finally, if Qin chooses *Don't Escalate* and G6 chooses *Partial Sanction* in the first stage, then in the second stage Qin and G6 play the following simultaneous-move game, with Qin still having all three options but G6 has only two remaining:

G6

	Sanction	Partial
Escalate	$-1, -3$	$1, -1$
Qin Partially	$-2, -3$	$0, -1$
Don't	$0, 2$	$0, 1$

(e) How many subgames does this two-stage game have? Find all pure-strategy Nash equilibria of each subgame. What are the possible outcomes of subgame perfect equilibria of the two-stage game?

(f) Show that in any subgame perfect equilibrium, the outcome is Qin ends up choosing *Escalate* and G6 ends up choosing *No Sanction*.