Econ 221
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Chapter 11. Collective-Action Games

- We now consider simultaneous-move games played by many identical players.
  - Each player has two strategies.
  - Payoff to each player from each strategy depends on the number of players choosing one versus the other strategy.

11.1 Collective-action games

- N players.
- Each player chooses between P (participating in collective action) and S (non-participation, or shirking).
- Payoffs depend on number n of players choosing P.
  - $P(n)$ denotes payoff to each participant.
  - $S(n)$ denotes payoff to each non-participant.

- Example: public goods provision.
  - P here represents contributing to public goods, and S represents not contributing.
  - Choosing P means paying a private cost, but increases per-player benefit to all.
  - Choosing S means avoiding paying the cost, but enjoys the same benefit as those choosing P.

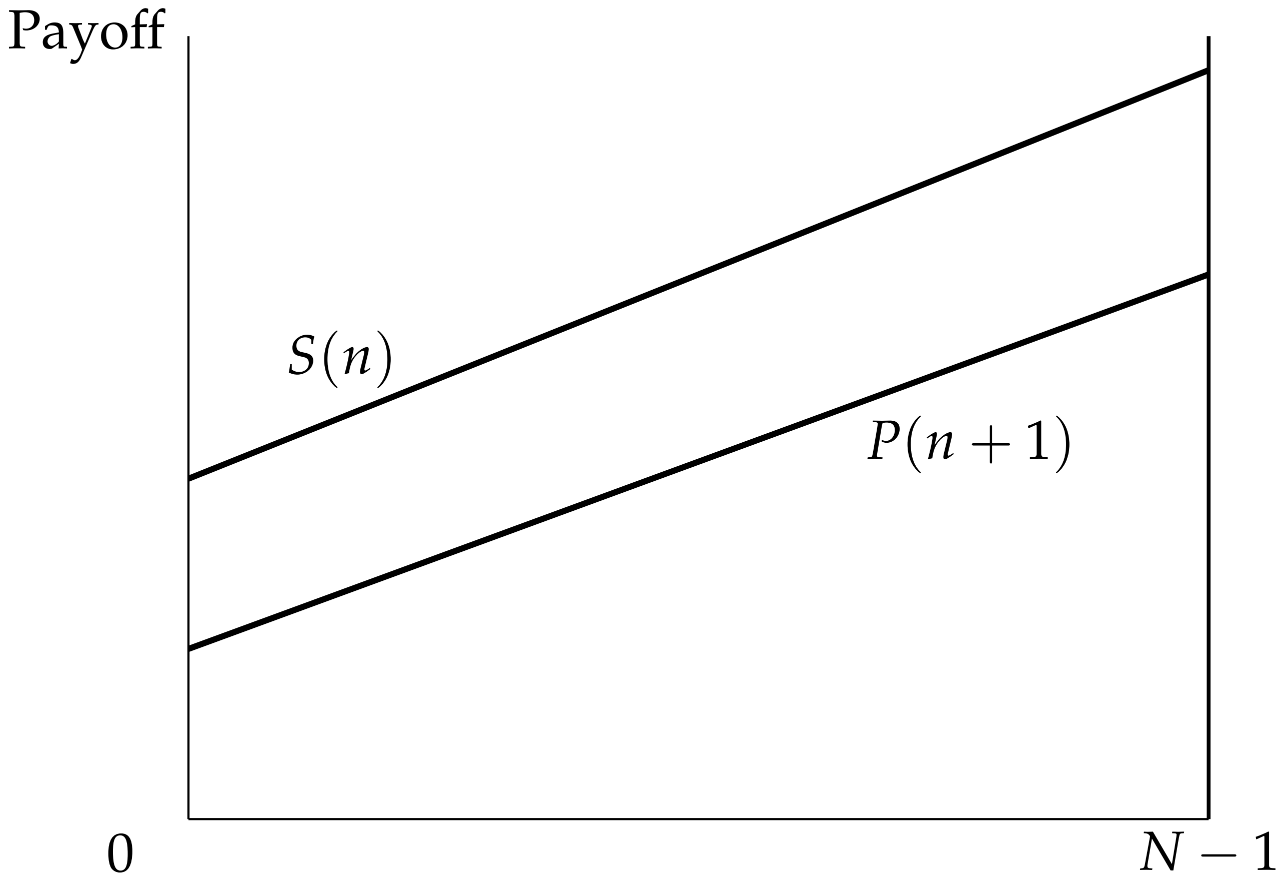
11.2 Collective-action problems

- Nash equilibrium in general collective-action games.
  - Number of participants n between 1 and $N-1$ is a pure-strategy Nash equilibrium of the collection-action game if $P(n) \geq S(n-1)$ and $S(n) \geq P(n+1)$.
  - $n = 0$ is a Nash equilibrium if $S(0) \geq P(1)$.
  - $n = N$ is a Nash equilibrium if $P(N) \geq S(N-1)$.

- Nash equilibrium depends on the comparison between two functions, $P(n+1)$ and $S(n)$ for $0 \leq n \leq N-1$.
  - Note that $P(0)$ and $S(N)$ have no meaning.
  - Recall that $P(n+1)$ and $S(n)$ are known functions in a given collective action game.
  - Below we will go through different classes of collective action games, depending on comparison of $P(n+1)$ and $S(n)$.

- Multi-player Prisoners' Dilemma.
  - Suppose $S(n) > P(n+1)$ for all $n = 0, \ldots, N-1$, but $S(0) < P(N)$.
  - Then, $n = 0$ is the only Nash equilibrium.
  - For $N = 2$, we have the original Prisoners' Dilemma, with S corresponding to *Confess*, and P to *Don't*.

	S	P
S	$*S(0), S(0)*$	$*S(1), P(1)$
P	$P(1), S(1)*$	$P(2), P(2)$



Multi-player Prisoners' Dilemma.

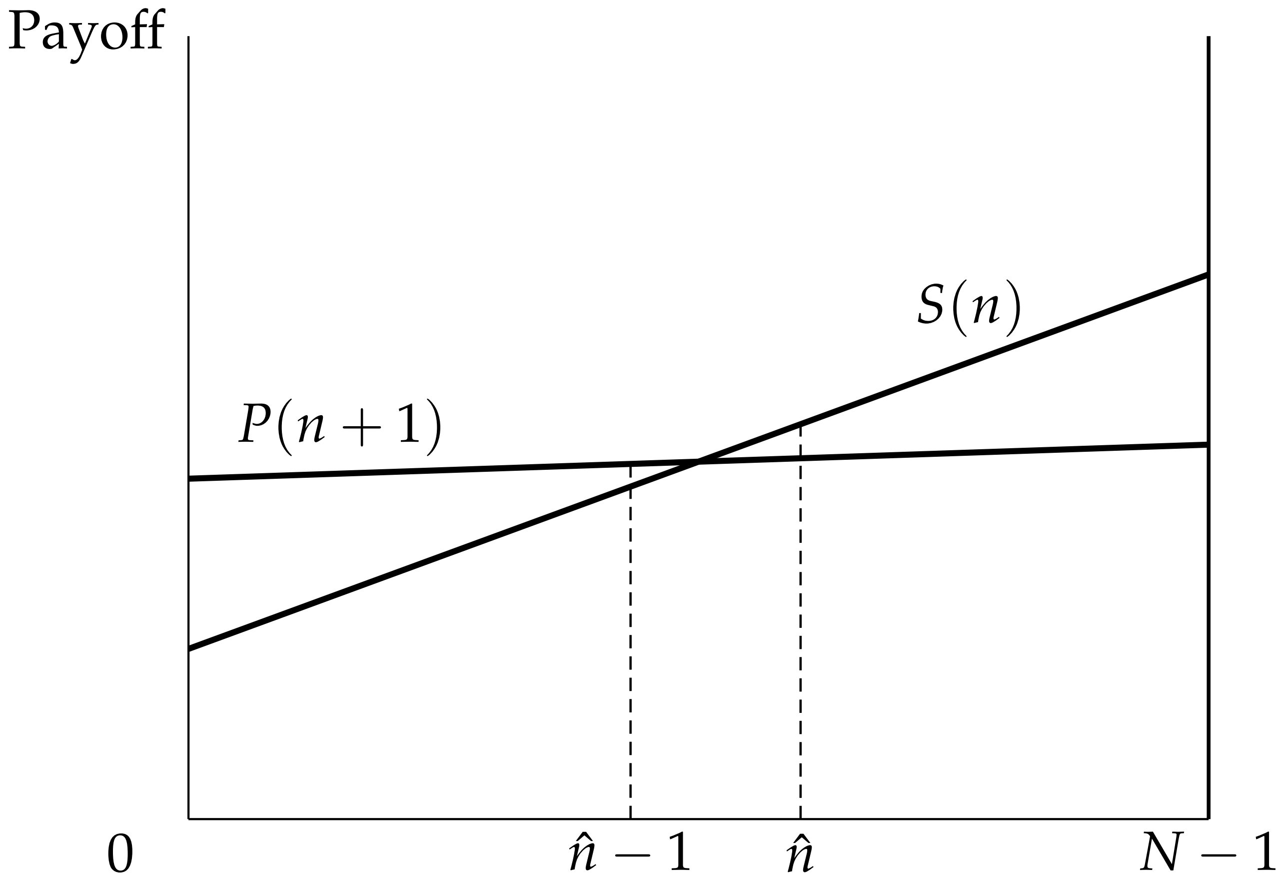
- Multi-player Game of Chicken.

  - Suppose there is some $\hat{n}$ between 1 and $N-1$ such that $P(n+1) > S(n)$ for all $n \leq \hat{n}-1$ and $S(n) > P(n+1)$ for all $n \geq \hat{n}$.

  - Then, $n = \hat{n}$ is the only Nash equilibrium.

  - For $N = 2$, we have Game of Chicken, with $\hat{n} = 1$, P corresponding to *Straight* and S to *Swerve*.

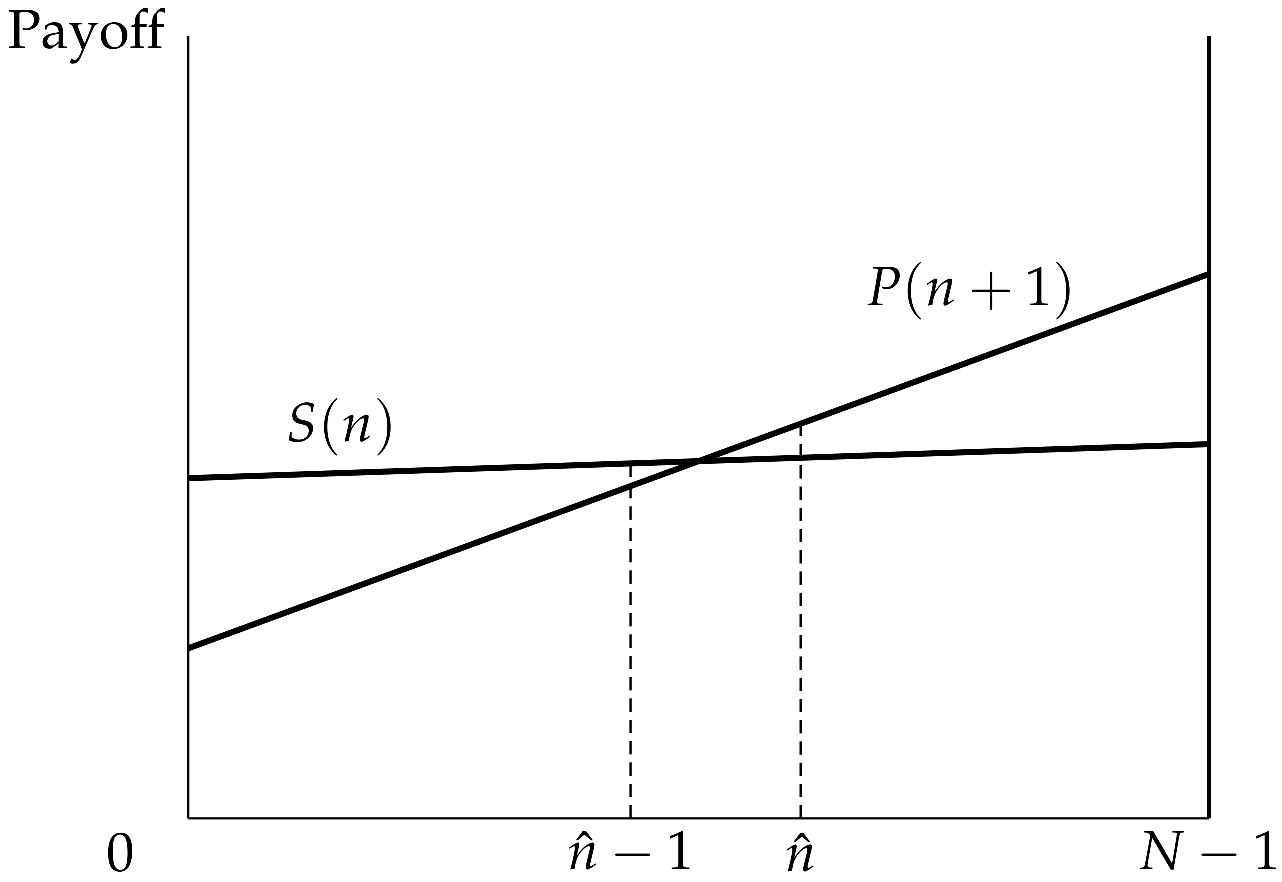
	S	P
S	$S(0), S(0)$	$*S(1), P(1)*$
P	$*P(1), S(1)*$	$P(2), P(2)$



Multi-player Game of Chicken.

- Multi-player Game of Assurance.
  - Suppose there is some $\hat{n}$ between 1 and $N-1$ such that $S(n) > P(n+1)$ for all $n \leq \hat{n}-1$ and $P(n+1) > S(n)$ for all $n \geq \hat{n}$.
  - Then, $n = 0$ and $n = N$ are the only Nash equilibria.
  - Example is Stag Hunt, for which if $N = 2$ then $\hat{n} = 1$, P corresponds to *Stag* and S to *Hare*.

	S	P
S	$*S(0), S(0)*$	$S(1), P(1)$
P	$P(1), S(1)$	$*P(2), P(2)*$



Multi-player Game of Assurance.

11.3 Spillovers, or externalities

- Define social payoff function $T(n)$ as the sum of payoffs to participants and to non-participants.
  - $T(n) = nP(n) + (N - n)S(n)$.
  - Social optimum is achieved by n^* that maximizes $T(n)$.
- Nash equilibrium typically differs from n^*.
  - For example, in multi-player Prisoners' Dilemma, Nash equilibrium is $n = 0$, but $S(0) < P(N)$ implies that $T(0) < T(N)$.

- Divergence between Nash equilibrium and social optimum can be explained by spillovers, or externalities.
  - Spillover is difference between marginal private gain and marginal social gain.
  - Marginal private gain at n is the change in individual payoff when that individual among $N-n$ non-participants switches to participation: $P(n+1)-S(n)$.
  - Marginal social gain at n is change in social payoff when the number of participants goes up by 1 from n to $n+1$: $T(n+1)-T(n)$.

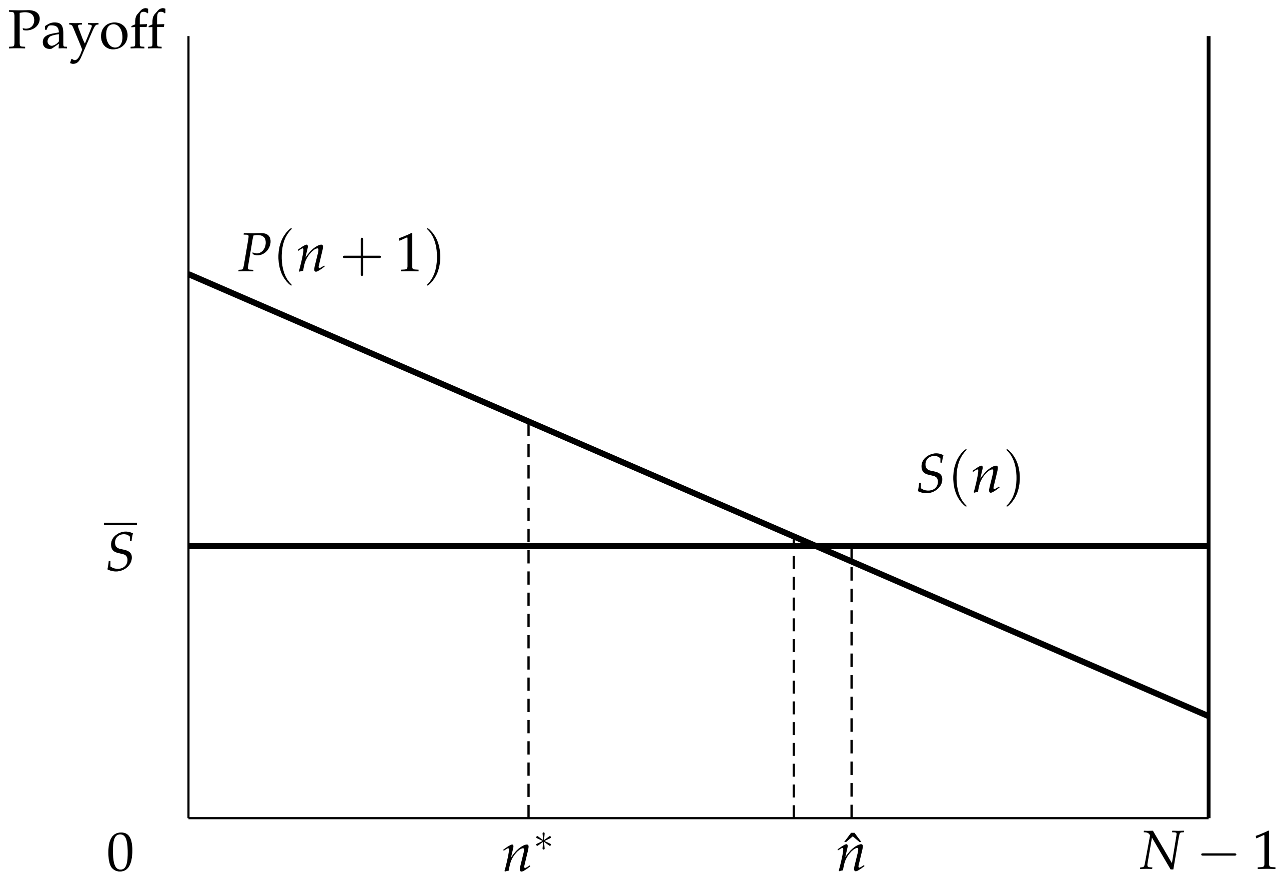
– Using the definition of $T(n)$, we have

$$\begin{aligned} T(n+1) - T(n) &= (n+1)P(n+1) + [N-(n+1)]S(n+1) \\ &\quad -nP(n) - [N-n]S(n) \\ &= [P(n+1) - S(n)] + n[P(n+1) - P(n)] \\ &\quad +[N-(n+1)][S(n+1) - S(n)] \end{aligned}$$

– The second and third terms represent spillover.

– Positive spillover in Prisoners' Dilemma: social gain can be positive even though private gain is negative.

- Example: Traffic Congestion as collective-action problem.
  - Story: each commuter chooses between a freeway and a local highway; commuting time by local highway is fixed; commuting time by freeway is faster when few commuters choose it but is slower when many do.
  - Collective-action game: $N \geq 3$ players; P is choosing freeway; S is choosing local highway; $S(n) = \overline{S} > 0$; $P(n+1)$ decreases with n and there exists $\hat{n}$ such that $P(n+1) > \overline{S}$ for all $n \leq \hat{n} - 1$ and $\overline{S} > P(n+1)$ for all $n \geq \hat{n}$.



Traffic Congestion.

- Too much congestion in equilibrium.
  - As we have seen from multi-player Game of Chicken, Nash equilibrium is $\hat{n}$, defined by the largest number n such that private marginal gain is positive.
  - Since $S(n) = \overline{S}$, from the earlier formula we have that $T(n+1) - T(n) = P(n+1) - \overline{S} + n(P(n+1) - P(n))$.
  - Spillover is always negative.
  - As a result, $n^* < \hat{n}$.

- Social optimum can be achieved by privatizing freeway.
  - Owner charges entrance fee t to maximize revenue nt; commuters pay t so long as $P(n) - t \geq \overline{S}$, and thus n satisfies $t = P(n) - \overline{S}$; since $S(n) = \overline{S}$, maximizing nt is same as maximizing $T(n)$, with revenue-maximizing price equal to $P(n^*) - \overline{S}$.
  - Social optimum n^* is achieved so long as commuters face appropriate congestion pricing, but with private ownership, there is no need to rely on any authority to choose the price.

11.5 A Game of Chicken with mixed strategies

- So far, we have only looked at pure-strategy Nash equilibria in collective-action games.
  - Such equilibria require a degree of coordination that is not always realistic, especially in relatively small groups.
  - Coordination is needed whenever a Nash equilibrium involves different actions by identical players.
- Mixed-strategy Nash equilibria do not require coordination.

- Reporting a Crime as a collective-action game.
  - Same setup as before, except there are many witnesses: N witnesses of crime decide whether to report it or not; reporting it costs C individually; each witness receives $B > C$ if at least one of them reports it.
  - Collective-action game: P is reporting the crime; S is not reporting; $P(n) = B - C$ for all $1 \leq n \leq N$; $S(0) = 0$ and $S(n) = B$ for all $1 \leq n \leq N - 1$.

- Pure-strategy Nash equilibrium.
  - This is a multi-player Game of Chicken: $P(n+1)$ cuts $S(n)$ from above, with $\hat{n} = 1$.
  - There is a unique pure-strategy Nash equilibrium, with $n = \hat{n} = 1$: this is the only n satisfying $P(n) \geq S(n-1)$ and $S(n) \geq P(n+1)$.
  - Nash equilibrium is socially optimal: $n^* = 1$ because $T(0) = 0$ and $T(n) = n(B-C) + (N-n)B$ for $n \geq 1$.
  - But who should be the one reporting the crime?

- Mixed-strategy Nash equilibrium.
  - Suppose that each player chooses S with probability q.
  - By principle of indifference, $B - C = (1 - q^{N-1}) \cdot B$.
  - Equilibrium $q = (C/B)^{1/(N-1)}$.
  - Comparative statics: q^N is increasing in N, and so more witnesses, smaller probability crime is reported.