Econ 221
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Chapter 10. The Prisoners' Dilemma and Repeated Games

- Prisoners' Dilemma.

P2

P1		*Cooperate*	*Defect*
	Cooperate	2, 2	0, 3*
	Defect	*3, 0	*1, 1*

- Prisoners' Dilemma, general formulation: $h > c > d > l$.

P2

P1		*Cooperate*	*Defect*
	Cooperate	c, c	l, h*
	Defect	*h, l	*d, d*

- Meaning of parameters.
  - $h - c$ is gain from unilateral defection; $c - l$ is loss from being defected on; and $c - d$ is loss from simultaneous defections.

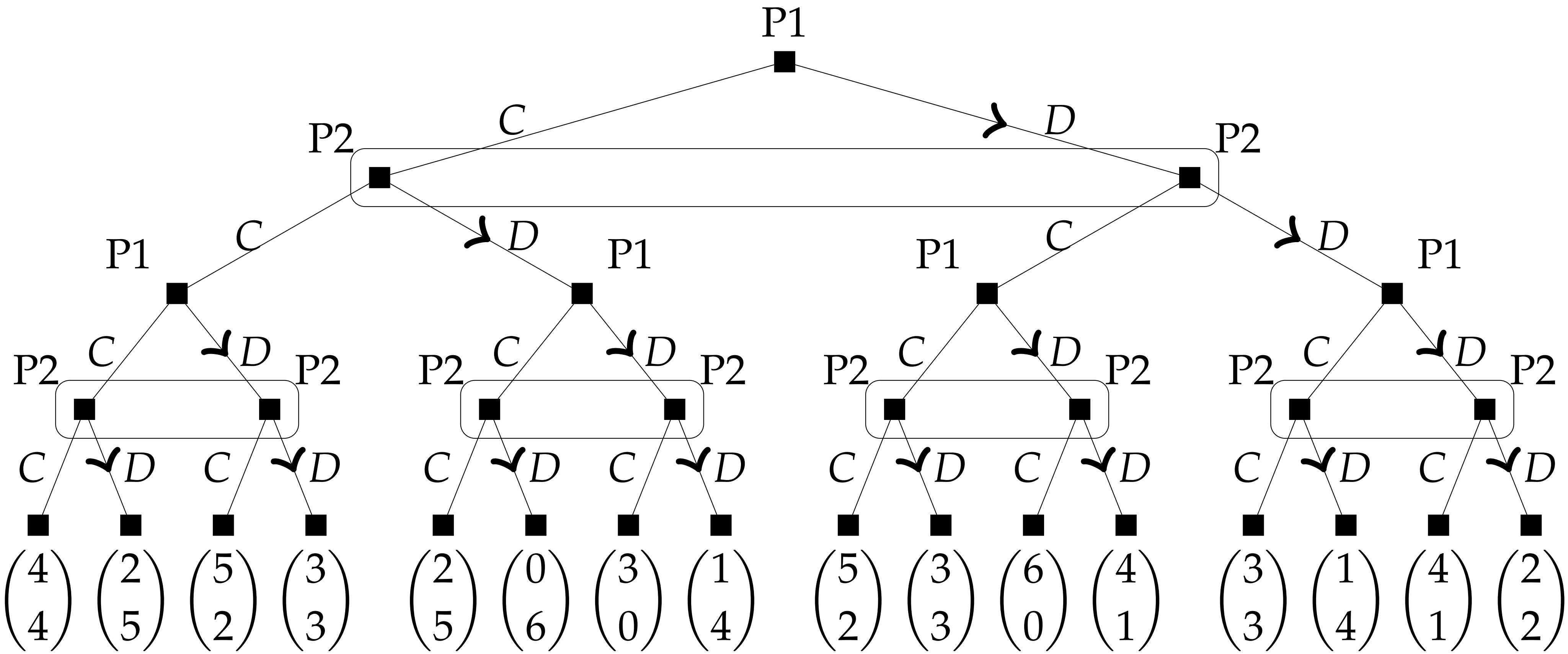
10.1 The basic game

- *Defect* strictly dominates *Cooperate*.
  - (*Defect*, *Defect*) is the unique Nash equilibrium.
- Why is Prisoners' Dilemma an important game?
  - Pursuit of individual interests vs collective welfare.
  - Examples: international trade, environment, pricing.
- Can players sustain mutually beneficial cooperation?

10.2 Solution 1: repetition

- Often same players engage in long-term relationship.
  - Repeated interactions between same players provide hope for sustaining cooperation, if players care about their future payoffs, because players can condition their own future plays on rival's current play.
  - Key is whether promises of future cooperation and threats of future defections are credible.

- Finite repetition.
  - Having ending date of long term relationship destroys credibility of any promise or threat: cooperation cannot be sustained.
  - Recall the example of twice-played Prisoners' Dilemma, with $h = 3$, $c = 2$, $d = 1$, and $l = 0$, and each player maximizes sum of payoffs in the two stage games.
  - Argument does not depend on number of times stage game is played: unique subgame perfect equilibrium is each player choosing *Defect* at each information set.



Subgame perfect equilibrium in a twice-played Prisoners' Dilemma.

- Infinite repetition.
  - Rollback method does not apply here because there are no smallest subgames to start with.
  - Subgame perfect equilibrium analysis still works here, but requires us to model how players discount future payoffs.
  - We will show that when players do not discount future payoffs too heavily, cooperation can be sustained.

- Discounting: common discount factor is β between 0 and 1.
  - Payoff x in next stage is same as βx in current stage.
  - Greater β means less discounting, and greater patience.
  - Each player's payoff from infinitely repeated game is discounted sum of payoffs from stage games.
  - Getting x in every future stage game is same as getting $\beta x + \beta^2 x + \ldots = \beta x / (1 - \beta)$ in current stage.

- Alternative interpretations of discount factor β.
  - β as reciprocal of gross interest rate, and discounted sum of payoffs as present value.
  - β as probability that relationship lasts to the next time period, with 0 payoff if relationship breaks down.

- Trigger strategy: start with C, and continue with it as long as all previous stage game outcomes are (C,C); switch to D permanently otherwise.
  - Claim: pair of trigger strategies forms subgame perfect equilibrium when β is sufficiently high, and sustains cooperation indefinitely.
  - Verification: there are no smallest subgames to start rolling back, and there are many subgames, but there are only two kinds of them.

- In any subgame in which not all previous stage game outcomes are (C,C), both players playing D now and forever is a Nash equilibrium.

- In any subgame where previous stage game outcomes are all (C,C), including beginning of the game, we have a Nash equilibrium if $c + \beta c/(1-\beta) \geq h + \beta d/(1-\beta)$, which is same as $\beta \geq (h-c)/(h-d)$.

- Cooperation is sustained so long as $h - c$ is small, $c - d$ is large, and β is high.

- Tit-for-Tat: start with C, and choose the action in current stage what the rival chose in the previous stage.
  - Unlike the trigger strategy, tit-for-tat allows return to cooperation after unilateral defection.
  - However, the same feature means that tit-for-tag is not a subgame perfect equilibrium strategy for any β.
  - In any subgame after (C, C) in the previous stage, we need $c + \beta c + \beta^2 c + \ldots \geq h + \beta l + \beta^2 h + \beta^3 l + \ldots$, but in any subgame after (C, D) in the previous stage, we require the opposite.