Econ 221
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Chapter 9. Uncertainty and Information

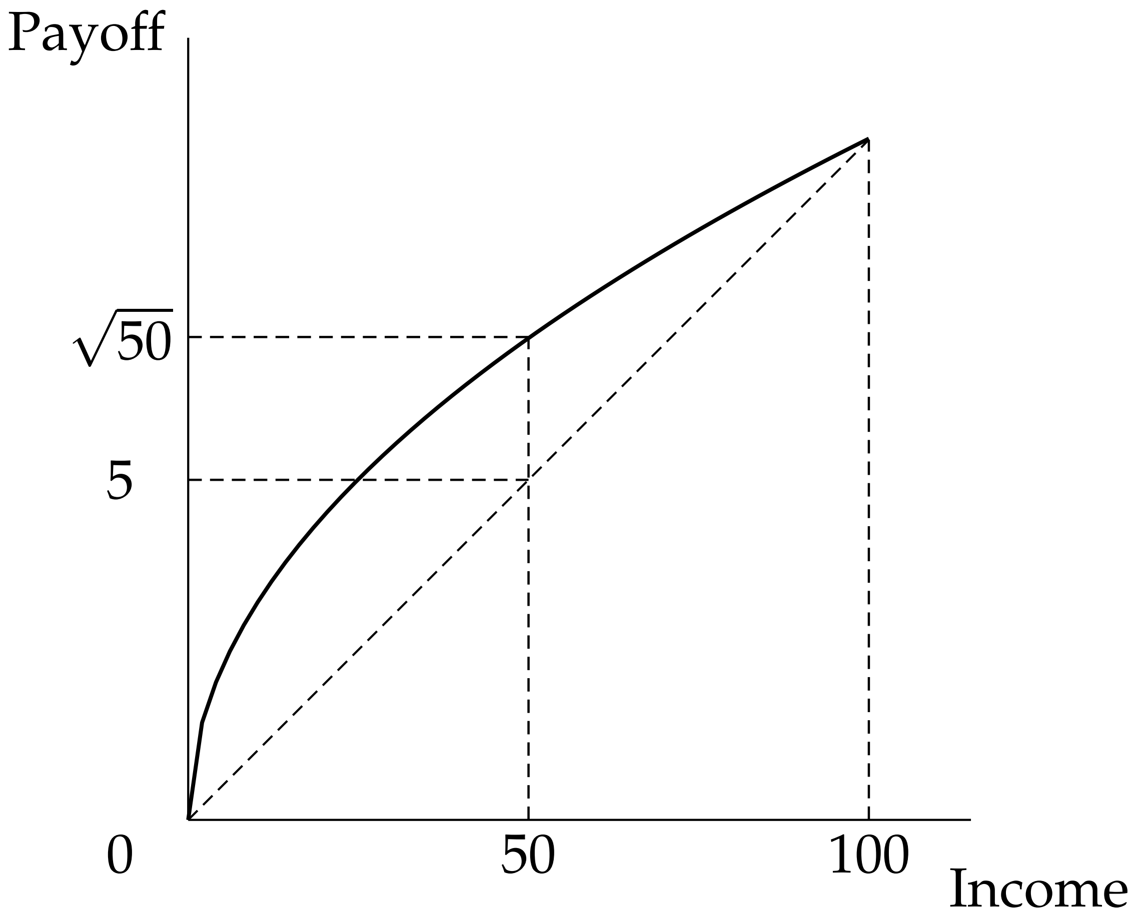
- Games with incomplete information about players.
  - Incomplete information about players' preferences can be symmetric or asymmetric.
  - To analyze games with asymmetric information, we will introduce Bayesian Nash equilibrium and perfect Bayesian equilibrium.

9.1 Uncertainty and risk

- An Okanagan Valley winery's annual income in dollars can vary substantially depending on weather and other factors, so owner faces income uncertainty or risk ahead of season.
  - Expected income, or mean income, is probability-weighted average of prospective incomes.
  - Example: expected income is 50 if income can be either 0 or 100 with equal probabilities, or equally likely to be any number between 0 and 100.

- Owner is risk-neutral if owner makes decisions based only on expected income.
  - Owner is a risk-neutral player in a game if prospective incomes enter payoff only through expected income.
  - Example: an investment changing prospective income from 0 and 100 with equal probabilities to 0 and 400 with equal probabilities is worth 150.
  - A risk-neutral player treats risky income as same as the mean, and does not care about the variance.

- Owner is risk-averse if expected payoff from risky income is lower than payoff from expected income.
  - A risk-averse player uses non-linear scale called payoff function to convert income to payoff.
  - Expected payoff from risky income is lower than payoff from expected income because the payoff function is concave — it's flatter at higher incomes.
  - Example: if payoff function is square root function, then expected payoff from prospective incomes of 0 and 100 with equal probabilities is 5, which is lower than $\sqrt{50}$.



Concave payoff function.

- Insurance contract.
  - A mutually beneficial deal exists between a risk-averse agent and a risk-neutral insurance company.
  - Any deal should eliminate risk for risk-averse agent.
  - In the example, winery will receive some payment x from insurance company when realized income is 0 and make a payment equal to $100 - x$ to insurance company when realized income is 100.
  - x has to be between 25 and 50.

9.2 Asymmetric information: basic ideas

- In many games, some players have more information than others about underlying strategic situation.
  - Examples: adverse selection in insurance, arms race.
- Bayesian Nash equilibrium is just Nash equilibrium under asymmetric information.
  - In simultaneous-move games, informed players will choose their strategies based on their superior information, but this is anticipated by uninformed players, and so on.

- Informed players may communicate, manipulate, or signal their information, while uninformed players may screen in order to reduce their information disadvantage.
  - Perfect Bayesian equilibrium extends rollback method and Bayesian Nash equilibrium to sequential-move games under asymmetric information.
  - Bayes' rule is used by uninformed players in making inference about the information of informed players by observing latter's actions.

9.4 Adverse selection

- Okanagan Valley winery with square root payoff function and risk-neutral insurance company.
  - Consider insurance contract of receiving x when income is 0 and paying $100 - x$ when income is 100.
  - If risky income is 100 with probability 0.5 and 0 with probability 0.5, owner will accept contract if $x \geq 25$.
  - If risky income is 100 with probability 0.9 and 0 with probability 0.1, owner will accept if $x \geq 81$.

- Asymmetric information: insurance company does not know which type of risky income the winery faces, but of course winery owner knows the risk.
  - If insurance company believes that each type is equally likely, then it will offer an insurance contract only if x is smaller than 70.

- Adverse selection: risky income of 100 with probability 0.5 and 0 with probability 0.5 is bad risk type, because expected income is 50 compared to expected income of 90 for the good risk type, but any x that attracts the good risk type will also attract the bad risk type.
    - Adverse selection closes market for the good risk type: insurance company could offer x between 25 and 50 and deal with bad risk type only.

- Fix any x and consider a simultaneous-move game in which Firm chooses whether or not to offer an insurance contract with x and Winery chooses whether to accept or reject it, with good risk type and bad risk type equally likely.

Bad risk (0.5)

		Accept	*Reject*
Firm	*Offer*	$50 - x, \sqrt{x}$	$0, 5$
	Don't	$0, 5$	$0, 5$

Good risk (0.5)

		Accept	*Reject*
Firm	*Offer*	$90 - x, \sqrt{x}$	0,9
	Don't	$0, 9$	0,9

- Firms' strategy is *Offer* or *Don't*; Winery's strategy specifies a choice between *Accept* and *Reject* for each risk type.
  - Bayesian games: in a simultaneous-move game with asymmetric information, private information of informed player is represented by having multiple types.
  - Strategies of uninformed players are defined in the same way as before, while a strategy of an informed players must specify what each type will choose.

- There is no x such that it is a Bayesian Nash equilibrium in the game with x for Firm to choose *Offer* and for both risk types of Winery to choose *Accept*.
- For x between 25 and 50, it is a Bayesian Nash equilibrium in the game with x for Firm to choose *Offer*, for bad risk type to choose *Accept* and for good risk type to choose *Reject*.

- Bayesian Nash equilibrium.
  - Each player's equilibrium strategy is a best response to equilibrium strategies of opponents.
  - For an informed player, this means each type is best responding.

- Adverse selection is a form of market failure.
  - It occurs in other markets (used cars, laid off workers).
  - Market responses include signaling by informed player (warranty, education) and screening by uninformed player (incomplete insurance, probationary employment).
  - Signaling and screening differ in timing of moves, but share same goal of separating different types of informed player.
  - We discuss signaling here, and leave screening to later.

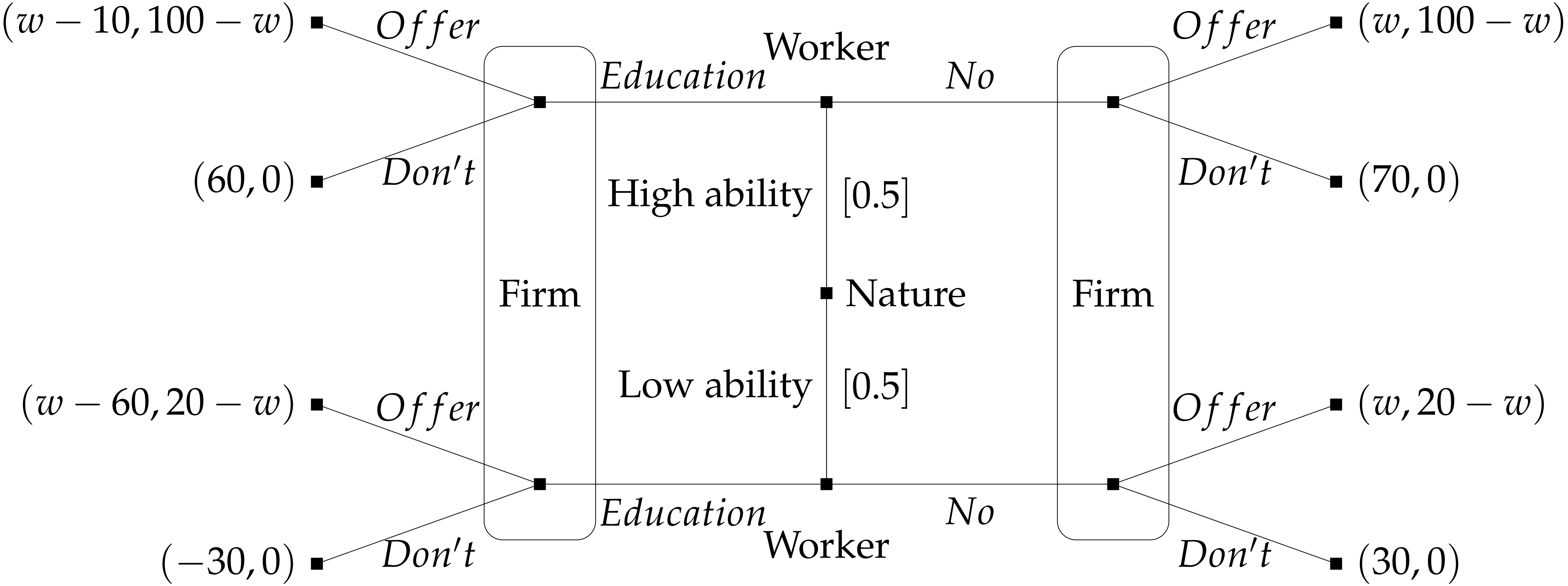
9.5 Signaling in the labor market

- In entry-level job market, workers often have asymmetric information about their innate ability that firms care about.
  - Given the same productivity, firms are willing to pay more for high-ability workers.
  - Education emerges as market response to asymmetric information.

- Asymmetric information.
  - Worker knows whether his actual ability is high or low, but Firm only knows prior probability of high ability is $p = 0.5$.
  - Firm is willing to pay Worker up to $w_H = 100$ if he has high ability, but only $w_L = 20$ if he has low ability, while Worker has outside option equal to $u_H = 70$ for high ability and $u_L = 30$ for low ability.
  - $w_H > u_H$ but $w_L < u_L$: if no asymmetric information, Worker would be hired only if he has high ability.

- Adverse selection.
  - For given wage offer w between w_L and w_H, consider a simultaneous-move game in which Worker chooses between *Accept* and *Reject* and Firm chooses between *Offer* and *Don't*.
  - Since $u_H = 70 > pw_H + (1 - p)w_L = 60$, there is no w such that it is a Bayesian Nash equilibrium for Worker of only high ability to choose *Accept* and for Firm to choose *Offer*.
  - Adverse selection leads to failure of labor market.

- Education as a costly signal.
  - Worker can get education at cost $c_H = 10$ for the high ability type and $c_L = 60$ for low ability, before Firm decides to whether or not make wage offer w between $w_L = 20$ and $w_H = 100$.
  - This is a game with asymmetric information where Worker is informed with two types and Firm is uninformed, and Worker moves first and Firm moves second after observing Worker's move but not Worker's type.



Job market signaling.

- Perfect Bayesian equilibrium.
  - As in any Nash equilibrium, each player's equilibrium strategy is a best response to equilibrium strategies of other players, and as in Bayesian Nash equilibrium, this requires each type of informed player to best respond.
  - There is no subgame in sequential-move games with asymmetric information, but the idea of rollback still applies: uninformed player is required to form belief about type of informed player at each information set and must best respond given the belief.

- Bayes' rule.
  - Uninformed player's belief is a probability for each type such that probabilities sum up to 1 over all types.
  - Uninformed player uses given prior belief and strategy of informed player to update at each information set.
  - Updated probability of any type at an information set is ratio of probability of this type reaching it and total probability over all types reaching it.
  - Bayes' rule does not apply if information set is not reached.

- Consider a perfect Bayesian equilibrium in which different types of the informed player make different choices.
  - Equilibrium strategies: high-ability chooses *Education*, and low-ability chooses *No*; Firm chooses *Offer* after seeing *Education* and *Don't* after seeing *No*.
  - Verification: for high-ability type, $w - 10 \geq 70$; for low-ability type, $30 \geq w - 60$; for Firm after *Education* through Bayes' rule, $100 - w \geq 0$; and for Firm after *No* through Bayes' rule, $20 - w \leq 0$.
  - Any w between 80 and 90 works.

- The labor market outcome of the equilibrium is the same as under symmetric information.
  - The equilibrium can occur even though labor market fails without costly education.
  - Condition $c_H = 10 \leq w_H - u_H = 30$ ensures education is not too costly for high ability.
  - Condition $c_L = 60 \geq c_H + u_H - u_L = 40$ means that education is too costly for low ability even though low ability could masquerade as high ability.

9.6 Equilibria in two-player signaling games

- Perfect Bayesian Nash equilibria can take different forms.
  - Equilibrium constructed in the job market signaling game is an example of separating equilibrium.
  - Pooling equilibrium is perfect Bayesian equilibrium where all types of informed players adopt the same strategy.
  - Between separating and pooling equilibria, there may also be perfect Bayesian Nash equilibria with partial separation, such as semi-separating equilibrium.

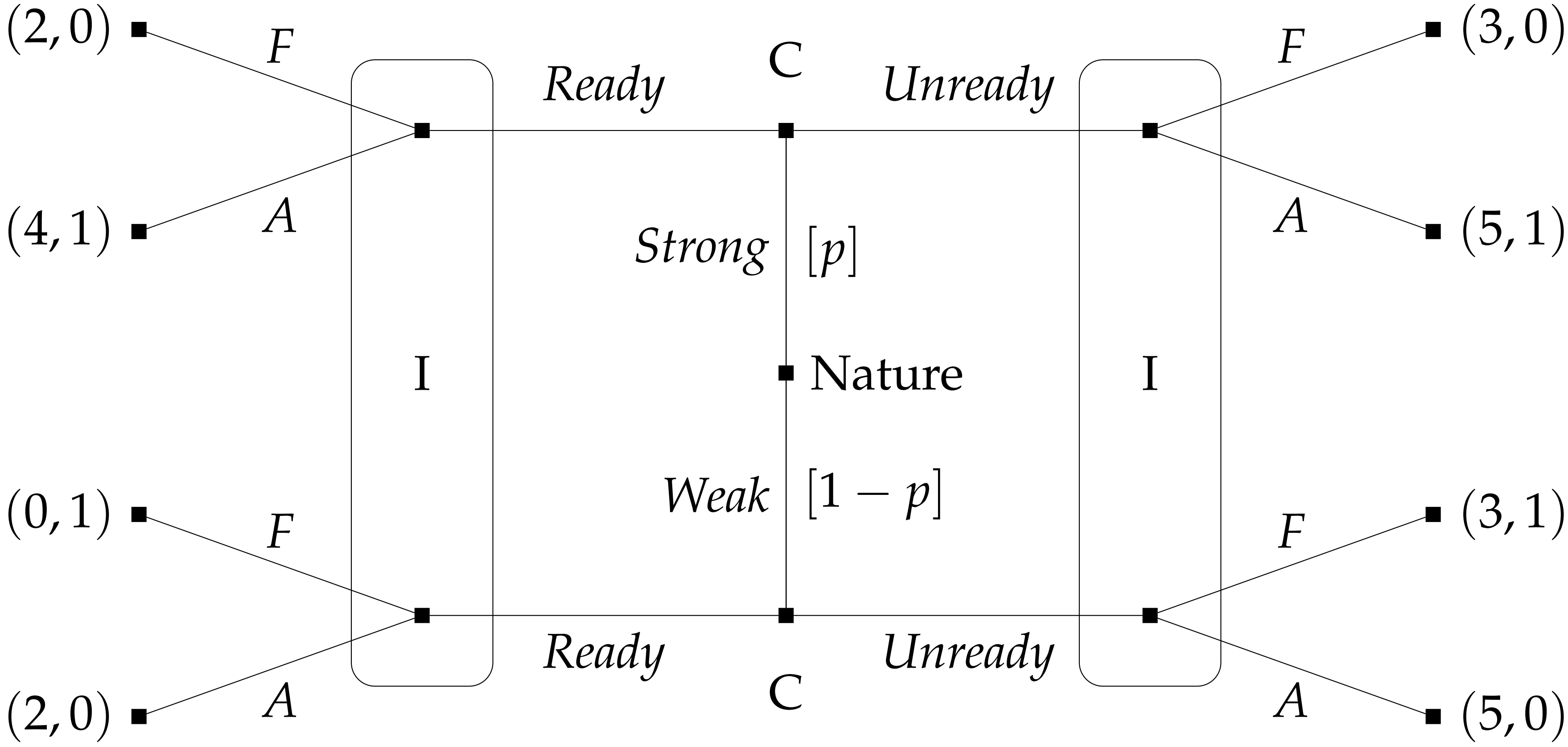
Signaling games with two players, two types, two signals, and two actions.

- Game tree: Nature moves first and privately informs Sender of his type; Sender chooses a signal; Receiver observes the signal and chooses an action.

- Strategies: Sender has 2 information sets, corresponding to the two types, and so has 4 possible strategies; Receiver has 2 information sets, corresponding to the two signals, and so has 4 possible strategies.

Entry game under incomplete information.

- A challenger (C) contests an incumbent (I). C is strong with probability p and weak with probability $1 - p$; it knows its type, but I does not. C may either ready itself for battle, which costs 1 for strong type and 3 for weak type, or remain unready. I observes the challenger's readiness, and chooses whether to fight (F) or acquiesce (A). Regardless of type, an unready C's payoff is 5 if I chooses A, and 3 if I chooses F. I prefers F (with payoff 1) to A (payoff 0) if C is weak, and prefers A (payoff 1) to F (payoff 0) if C is strong.


(2,0)
F
Ready
C
Unready
F
(3,0)
(4,1)
A
Strong
[p]
A
(5,1)
I
Nature
I
(0,1)
F
Weak
[1 − p]
F
(3,1)
Ready
Unready
C
(2,0)
A
A
(5,0)

- Separating equilibrium.
  - Two Sender types use two different signals.
  - By Bayes' rule, Receiver's belief puts probability 1 on the type after the signal the type is supposed to send.
  - Receiver best responds to each signal given the above beliefs, and each Sender type chooses best signal given Receiver's responses.

- In Entry game under incomplete information, there is only one separating equilibrium.
  - Construct a separating equilibrium where C chooses *Ready* when type is strong and *Unready* when type is weak.
  - Show there is no separating equilibrium where C chooses *Ready* when type is weak and *Unready* when type is strong.

- Pooling equilibrium.
  - Two Sender types use the same signal (pooling signal).
  - By Bayes' rule, Receiver's belief about the type of Sender after seeing the pooling signal is the same as prior, and Receiver best responds given prior belief.
  - Receiver cannot use Bayes' rule to derive belief after seeing the other signal, but is required to specify the belief, and best responds given this belief.
  - Each type chooses best signal given Receiver responses.

- In Entry game under incomplete information, there is only one kind of pooling equilibrium.
  - Construct a pooling equilibrium where both types of C choose *Unready*, and for any $p < 0.5$ we must specify after *Ready* I's updated belief that C is strong is lower than 0.5 so as to respond to it with F.
  - Verify that there is no pooling equilibrium where both types of C choose *Ready*.

- Semi-separating equilibrium: One signal (separating signal) is used by one Sender type only (separating type), and the other signal is used by both types.
  - By Bayes' rule, Receiver's belief puts probability 1 on Sender being separating type after seeing separating signal, and Receiver best responds given this belief.
  - By Bayes' rule, Receiver's belief puts smaller than prior probability on Sender being the separating type after seeing the other signal, and Receiver best responds given this belief.

- In Entry game under incomplete information, there is only one kind of semi-separating equilibrium.
  - Construct a semi-separating equilibrium for $p > 0.5$ where the strong type of C is separating type, *Ready* is separating signal, and I responds to *Unready* by mixing between F and A with equal probabilities.
  - Show that there is no semi-separating equilibrium where weak type of C is separating type, or where *Unready* is separating signal.