Econ 221
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Chapter 6. Combing Sequential and Simultaneous Moves

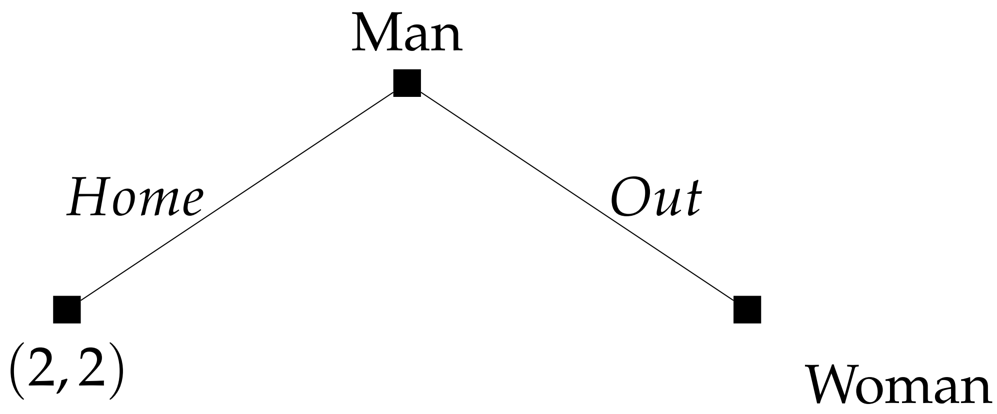
- Multi-stage games with observed actions.
  - Each stage may contain a simultaneous-move game, and actions taken at the stage are observed by all players before they choose their actions in the next stage.
  - Our analysis combines rollback equilibrium with Nash equilibrium.

6.1 Games with both simultaneous and sequential moves

- Battle of the Sexes with a first move by Man.
  - Man first chooses between *Home*, ending the game with payoff of 2 for both Man and Woman, and *Out*, leading to Battle of the Sexes between Man and Woman:

		Woman	
		Boxing	*Opera*
Man	*Boxing*	3, 1	0, 0
	Opera	0, 0	1, 3

– An illustration combining game tree and game table.



Man		*Boxing*	*Opera*
	Boxing	3, 1	0, 0
	Opera	0, 0	1, 3

Battle of the Sexes with a first move by Man.

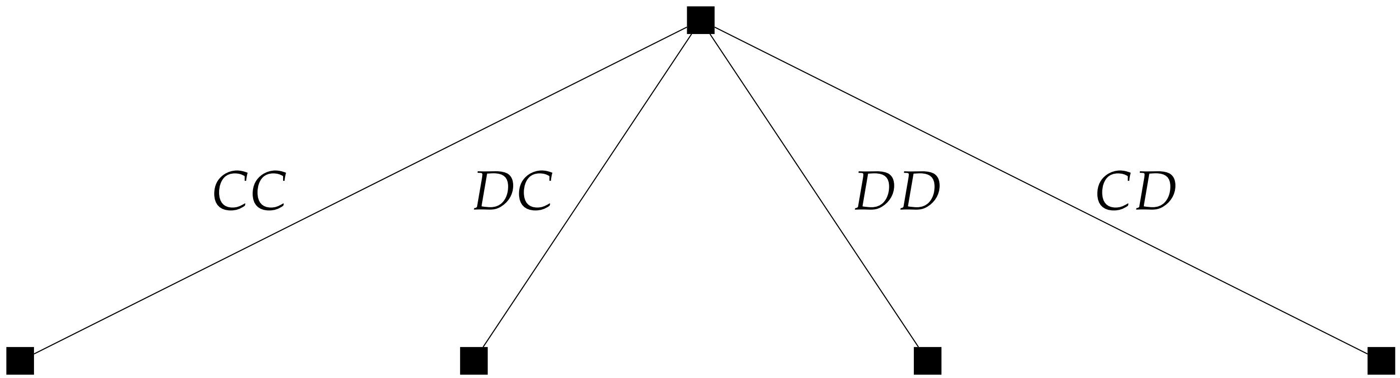
- Twice-played Prisoners' Dilemma.

  – Two players play the following game, observe what has happened, and play it again, with each player's total payoff equal to the sum of the payoffs from the two outcomes.

Player 2

		C	D
Player 1	C	2,2	0,3
	D	3,0	1,1

– An illustration combining game tree and game table.



	C	D
C	4,4	2,5
D	5,2	3,3

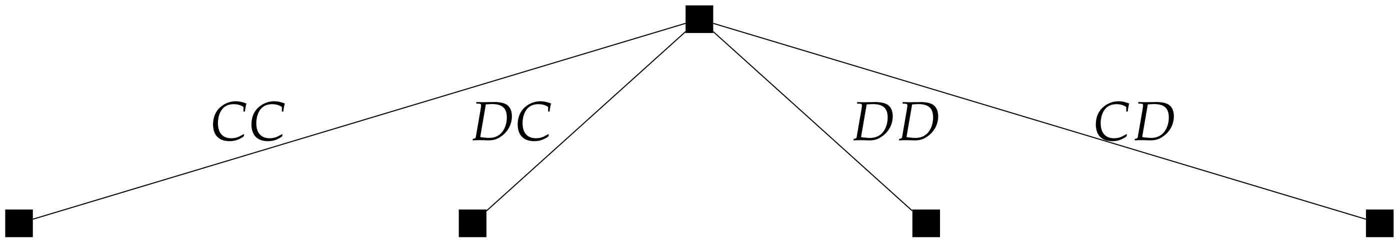
	C	D
C	5,2	3,3
D	6,0	4,1

	C	D
C	3,3	1,4
D	4,1	2,2

	C	D
C	2,5	0,6
D	3,3	1,4

Twice-played Prisoners' Dilemma.

- Solving twice-played Prisoners' Dilemma by combining roll-back method with Nash equilibrium.
  - For each of the four second-stage games, find the Nash equilibrium.
  - In the first stage game, find the Nash equilibrium using the payoffs from the rollback.



	C	D
C	4,4	2,5*
D	*5,2	*3,3*

	C	D
C	5,2	3,3*
D	*6,0	*4,1*

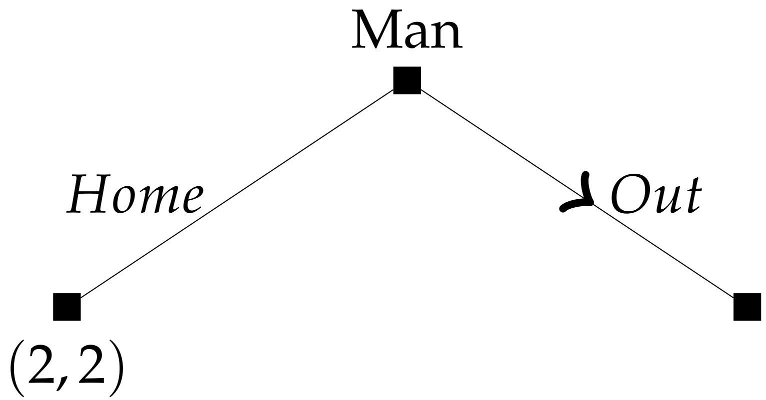
	C	D
C	3,3	1,4*
D	*4,1	*2,2*

	C	D
C	2,5	0,6*
D	*3,3	*1,4*

	C	D
C	3,3	1,4*
D	*4,1	*2,2*

Rollback in twice-repeated Prisoners' Dilemma.

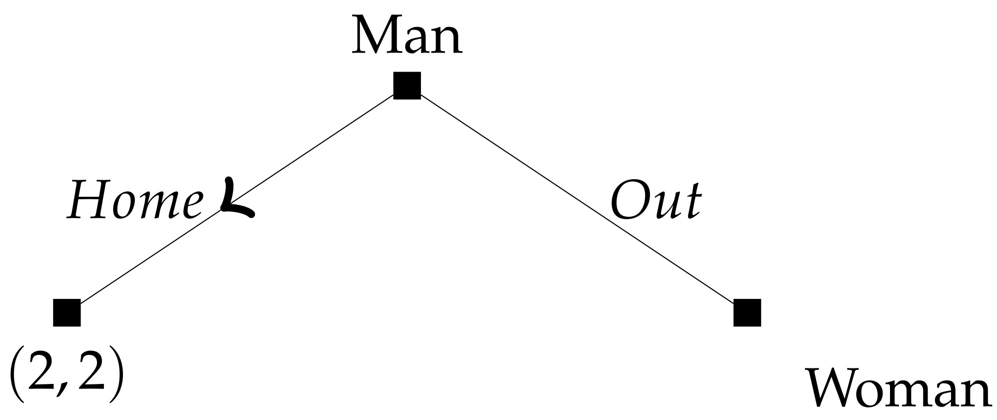
- Solving the Battle of the Sexes with a first move by Man by combining rollback method with Nash equilibrium.
  - For the single second stage game, find the two Nash equilibria.
  - For each of the two Nash equilibria in the second stage, mark the best first move by Man.



Woman

		Boxing	*Opera*
Man	*Boxing*	*3,1*	0,0
	Opera	0,0	1,3

First rollback in Battle of the Sexes with a first move by Man.



		Boxing	Opera
Man	*Boxing*	3,1	0,0
	Opera	0,0	*1,3*

Second rollback in Battle of the Sexes with a first move by Man.

6.2 Changing the order of moves in a game

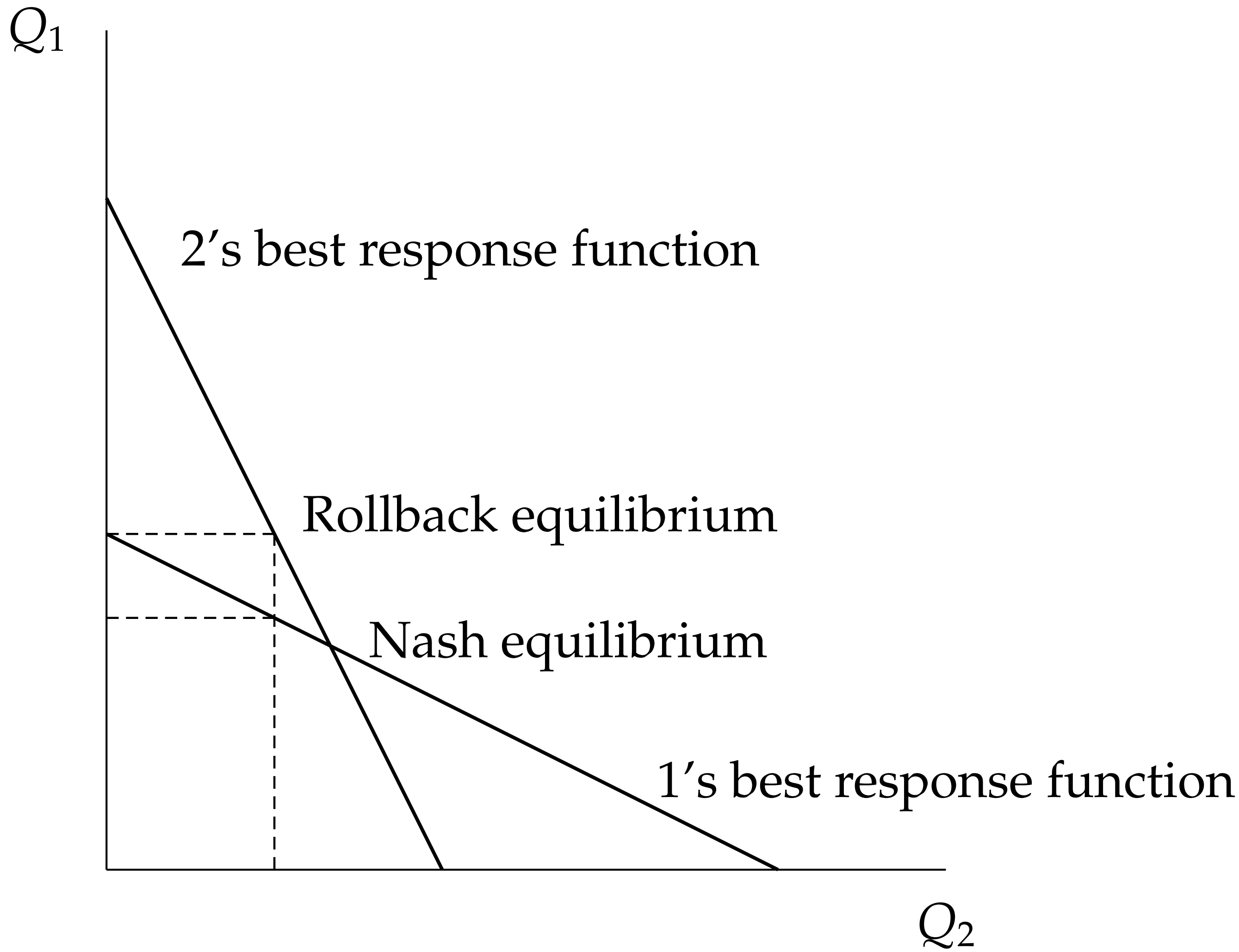
- Stackelberg Duopoly: changing simultaneous moves in Cournot Duopoly to sequential moves.
  - The setup: Firm 1 chooses Q_1, which is observed by Firm 2 before it chooses Q_2; market price is determined by $P = 150 - Q_1 - Q_2$; the marginal production cost is 30 for both firms, with no fixed cost.
  - A sequential-move game with continuous strategies.

- Find rollback equilibrium.
  - Each last decision node corresponds to some Q_1 by Firm 1 and belongs to Firm 2.
  - Firm 2 chooses Q_2 to maximize $Q_2(150 - Q_1 - Q_2 - 30)$, which gives $Q_2 = 60 - 0.5Q_1$.
  - Rolling back, as the first mover Firm 1 chooses Q_1 to maximize $Q_1(150 - Q_1 - (60 - 0.5Q_1) - 30)$, which gives $Q_1 = 60$.

- Rollback equilibrium in Stackelberg vs Nash equilibrium in Cournot.

  - In rollback equilibrium of Stackelberg Duopoly, we have $Q_1 = 60$ and $Q_2 = 30$, with profit of 1800 to Firm 1 and 900 to Firm 2.

  - In the Nash equilibrium of Cournot Duopoly derived in Lecture 3, we have $Q_1 = Q_2 = 40$, with profit of 1600 to both firms.



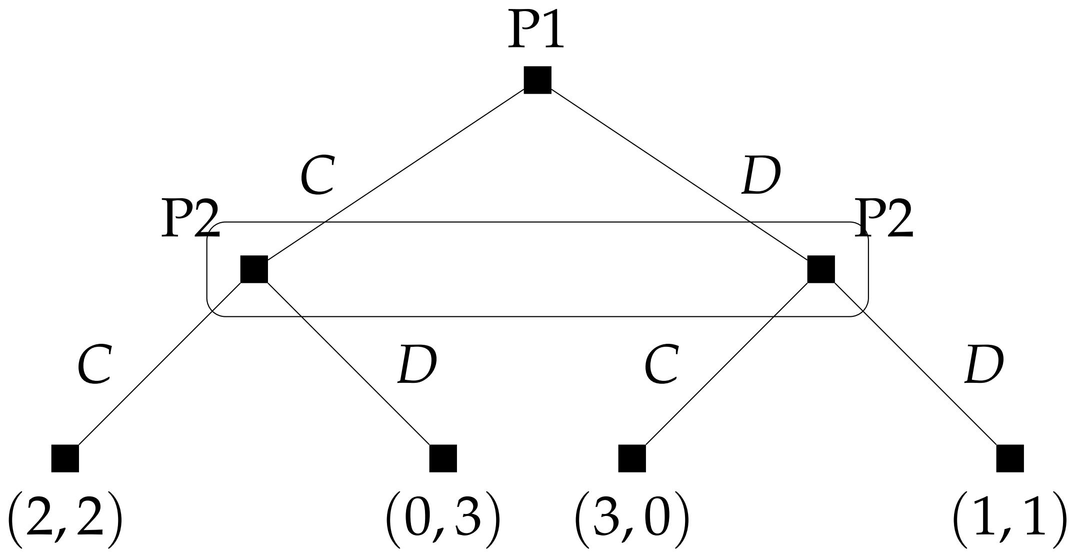
Stackelberg vs. Cournot Duopoly.

- What is the source of the first mover advantage?
  - By choosing Q_1 first, Firm 1 controls what Firm 2 chooses through the latter's best response function.
  - Firm 1 can guarantee the Nash equilibrium profit of 1600 by choosing 40 and making Firm 2 choose 40.
  - By increasing Q_1 from 40, Firm 1 makes Firm 2 produce less than 40.

- Commitment: another perspective of first mover advantage.
  - $Q_1 = 60$ is not Firm 1's best response to Firm 2 producing $Q_2 = 30$: if Firm 1 could change quantity after Firm 2 has chosen $Q_2 = 30$, and if this is anticipated by Firm 2, there would be no first mover advantage.
  - $Q_1 = 60$ is Firm 1's best response to Firm 2's equilibrium strategy of producing $Q_2 = 60 - 0.5Q_1$: Firm 1's first mover advantage comes from the ability to commit to not changing quantity after first move.

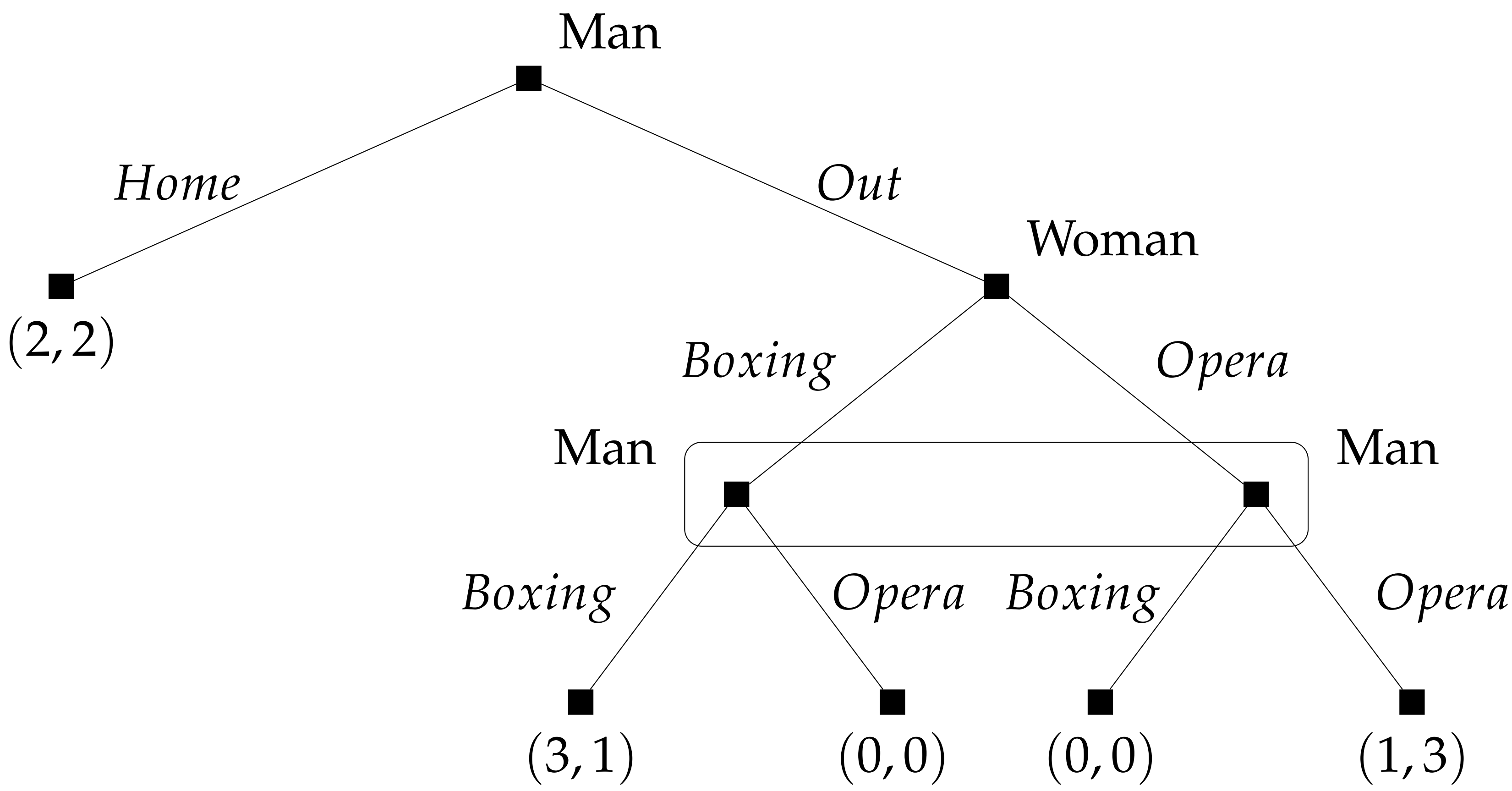
6.3 Alternative Method of Analysis

- Illustrating simultaneous-move games by using trees.

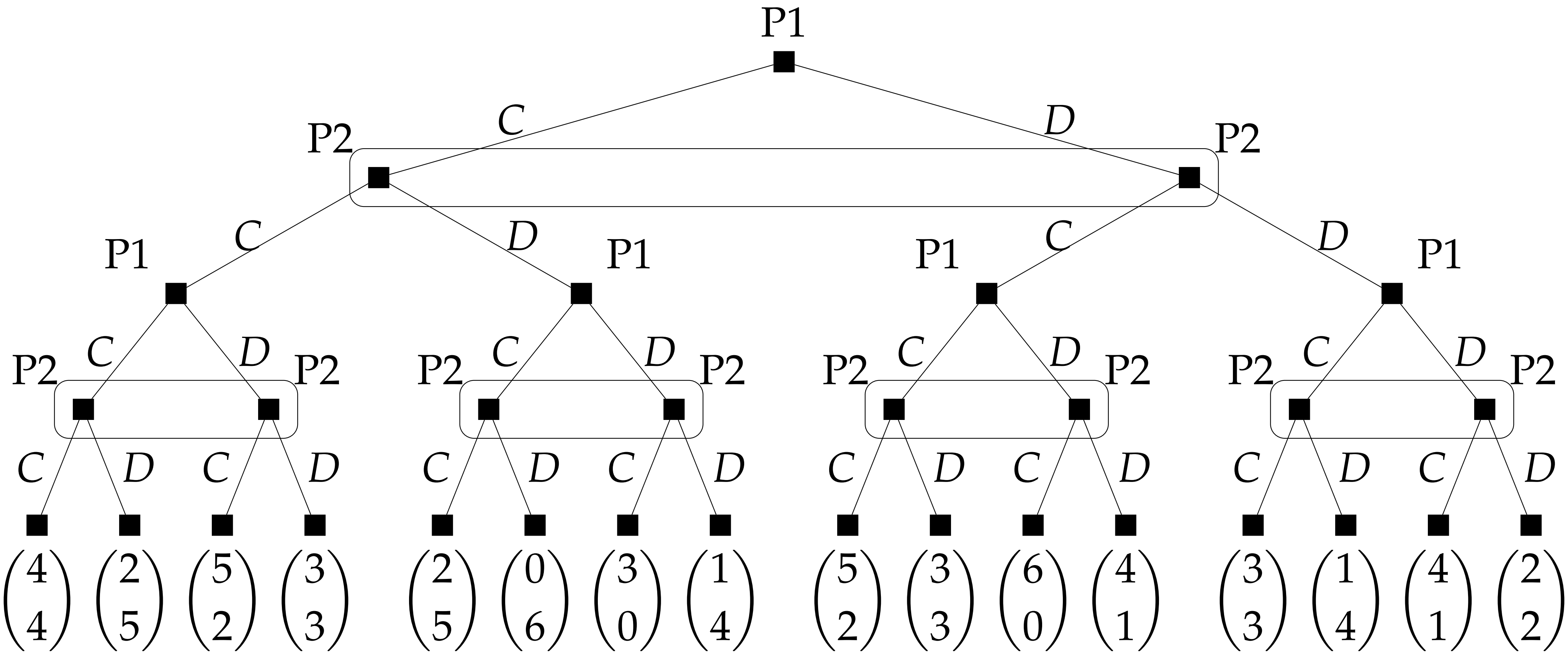


Prisoners' Dilemma in a game tree.

- An information set of a player in a game tree represents the collection of decision nodes that belong to this player who must take the same action for all the nodes because the player cannot distinguish between them.
  - The concept of information set can be applied to any multi-stage game with observed actions.
  - Non-degenerate information sets represent the presence of imperfect information.
  - Sequential-move games have perfect information, with each decision node a degenerate information set.



Battle of the Sexes with first move by Man.

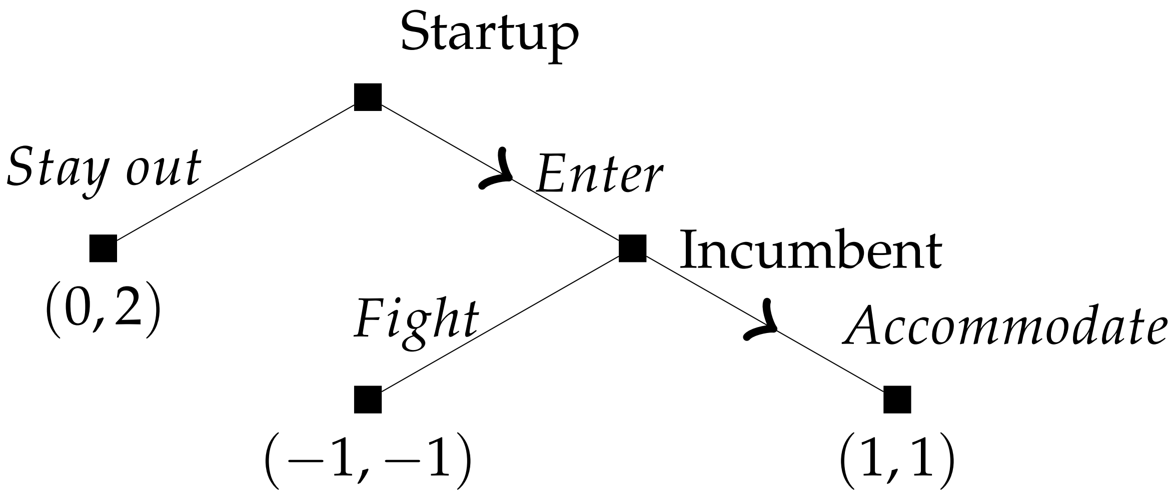


Twice-played Prisoners' Dilemma in a tree.

- Definition of strategy adapted to information set.
  - For all decision nodes contained in an information set, the player must now choose the same action.
  - Any strategy of any player remains a complete plan of actions, and so it needs to specify an action for each information set that belongs to the player.
  - In Battle of the Sexes with a first move by Man, Man has 4 strategies while Woman has 2; in twice-played Prisoners' Dilemma, each player has 32 strategies.

- Nash equilibrium in sequential-move games.
  - Strategic form of a sequential-move game is the game table obtained from the game tree through listing all strategies of the players.
  - The original game tree is known as extensive form.
  - All Nash equilibria of a sequential-move game can then be found from the strategic form.
  - Rollback equilibrium is one Nash equilibrium, but there are other Nash equilibria.

- Example: Entry Deterrence.



Incumbent

		Accommodate	*Fight*
Startup	*Stay out*	0,2*	*0,2*
	Enter	*1,1*	$-1, -1$

- Nash equilibrium and rollback equilibrium.
  - Strategic form is a simultaneous-move game in which players choose their plans once for all, and is a different game from the original game tree.
  - Nash equilibria of strategic form that do not correspond to the rollback equilibrium can involve empty promises or threats, because Nash equilibrium does not check subgames that are never reached.

- Subgame perfect equilibrium extends rollback equilibrium to games with imperfect information.
  - A subgame of an extensive form game is part of the original game that starts with a degenerate information set containing a single decision node and includes all the information sets and terminal nodes following the node.
  - A subgame perfect equilibrium of an extensive form game is a Nash equilibrium such that the equilibrium strategies form a Nash equilibrium in every subgame.

- In simultaneous-move games, subgame perfect equilibrium is the same as Nash equilibrium.
  - There is only one subgame – the game itself.
- In sequential-move games, subgame perfect equilibrium is the same as rollback equilibrium.
  - Every decision node starts a subgame.
- In games that combine sequential moves and simultaneous moves, subgame perfect equilibrium uses the rollback method to refine Nash equilibrium.

- Entry Deterrence meets Cournot Duopoly
  - The setup: Startup chooses between *Out* and *In*; if Startup chooses *Out*, Incumbent chooses a quantity; if Startup chooses *In*, Incumbent chooses quantity Q_i and Startup chooses Q_s simultaneously; market price P is given by $150 - Q_1 - Q_2$; the marginal production cost is 30 for both firms, with no fixed cost.

– Two subgames besides the game itself: after Startup chooses *Out*, and after Startup chooses *In*.

– In the subgame after Startup chooses *Out*, Incumbent will choose $Q_i = 60$, with payoff 0 to Startup and 3600 to Incumbent.

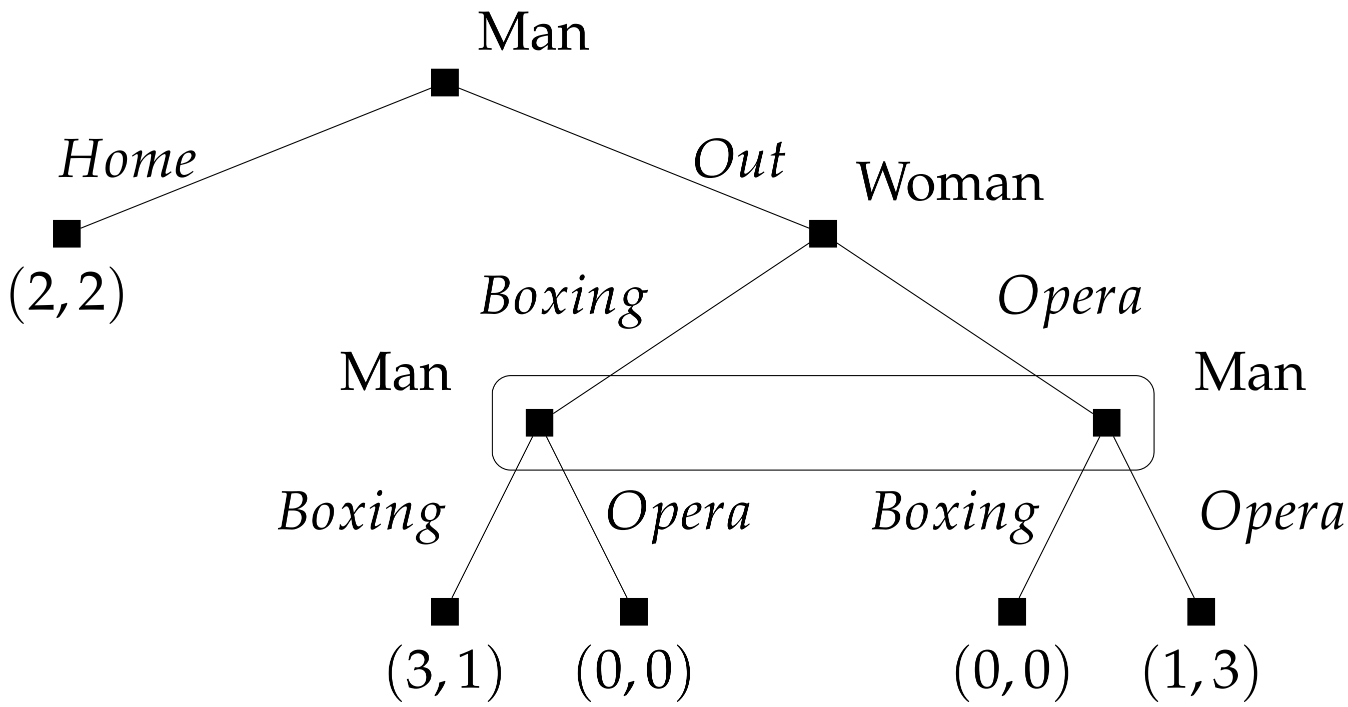
– In the subgame after Startup chooses *In*, in the only Nash equilibrium is $Q_s = Q_i = 40$, with payoff 1600 to both Startup and Incumbent.

– Rolling back, Startup will choose *In* at initial decision node.

- Subgame perfect equilibrium: Startup chooses *In* and $Q_s = 40$; Incumbent chooses $Q_i = 60$ after *Out*, and $Q_i = 40$ after *In*.

- It is a Nash equilibrium for Startup to choose *Out* and $Q_s = 0$, and for Incumbent to choose $Q_i = 60$ after *Out*, and $Q_i = 120$ after *In*, but this is not subgame perfect because $Q_s = 0$ and $Q_i = 120$ is not a Nash equilibrium in the subgame after *In*.

- There is a single subgame in Battle of the Sexes with first move by Man besides the game itself, which has two Nash equilibria.



- Two subgame perfect equilibria: Man chooses *Out* and then *Boxing*, and Woman chooses *Boxing*; Man chooses *Home* but then *Opera*, and Woman chooses *Opera*.

- ((*Home*, *Boxing*), *Opera*) is Nash but not subgame perfect.

		Woman	
		Boxing	*Opera*
Man	*Home, Boxing*	2,2*	*2,2*
	Home, Opera	2,2*	*2,2*
	Out, Boxing	*3,1*	0,0
	Out, Opera	0,0	1,3*