Econ 221
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CHAPTER 5. SIMULTANEOUS-MOVE GAMES: CONTINUOUS STRATEGIES

- Continuous instead of discrete strategies.
  - A strategy is a real number instead of a discrete choice.
  - Rationale is analytical clarity instead of realism.

5.1 Pure strategies that are continuous variables

- Restaurant Pricing: setup of the game.
  - Great Wall and Colosseum choose prices simultaneously to maximize their own revenue: P_w and P_c are non-negative real numbers.
  - Total number of customers $Q = 120 - P_w - P_c$, which is divided between $Q_w = 60 - 5P_w + 4P_c$ for Great Wall, and $Q_c = 60 - 5P_c + 4P_w$ for Colosseum.

- Remarks about the setup.
  - The two restaurants are imperfect substitutes in demand: if Great Wall increases P_w by \$1, it loses 5 customers while Colosseum gains only 4 customers.
  - P_w and P_c can be any real numbers, not just integers.

- Best response function.
  - Great Wall's best response to Colosseum's strategy P_c is P_w that maximizes $P_w(60 - 5P_w + 4P_c)$.
  - The solution is $P_w = 6 + 0.4P_c$.
  - As we vary P_c, the above formula represents Great Wall's best response function.

- Nash equilibrium.
  - Colosseum's best response function is symmetrically given: $P_c = 60 + 0.4P_w$.
  - By definition, Nash equilibrium is an intersection of the two best response functions: we can find it by solving two equations for two unknowns.
  - By substitution, we get Nash equilibrium $P_w = P_c = 10$.

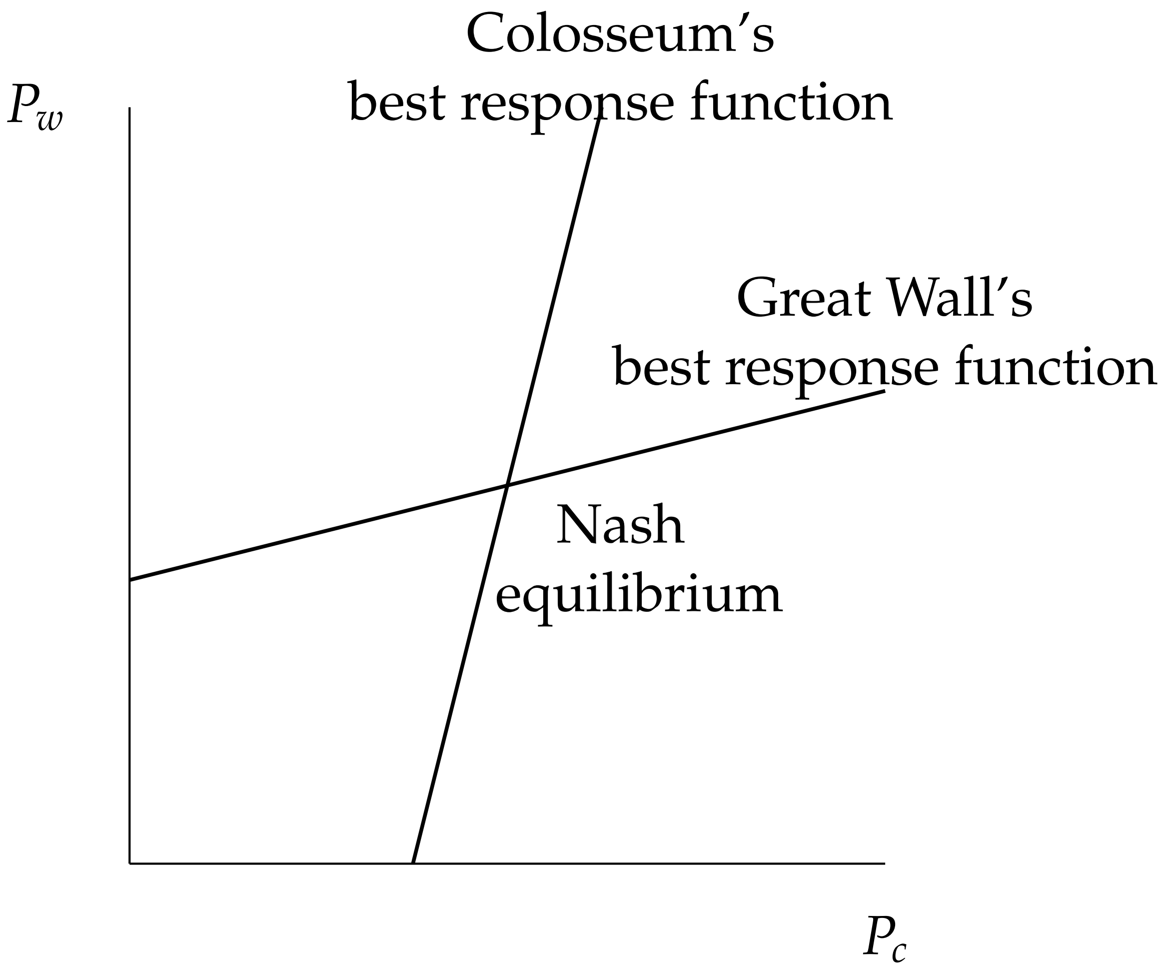

Figure 1. Restaurant Pricing.

- If Great Wall and Colosseum could collude with each other, they would maximize their joint revenue $P(120 - 2P)$ by choosing $P = 30$.
  - Collusive price is higher than Nash equilibrium price.
  - Each restaurant is trying to undercut its opponent, and in Nash equilibrium they both fail.

- Cournot Duopoly: setup of the game.
  - Firm 1 and Firm 2 choose their output simultaneously to maximize their own profit: Q_1 and Q_2 are non-negative real numbers.
  - Market price is $P = 150 - Q_1 - Q_2$.
  - Marginal production cost is 30 for both firms; there is no fixed cost for either firm.

- Remarks about the setup.
  - Outputs by the two firms are perfect substitutes in the same market.
  - Instead of price competition in Restaurant Pricing, we have quantity competition.
  - $P = 150 - Q_1 - Q_2$ is the inverse demand function.

- Best response function.

  - Firm 1's best response to Firm 2's strategy Q_2 is the Q_1 that maximizes $Q_1(150 - Q_1 - Q_2 - 30)$.

  - The solution $Q_1 = 60 - 0.5Q_2$.

  - As we vary Q_2, the above formula represents Firm 1's best response function.

- Nash equilibrium.
  - Firm 2's best response function is symmetrically given: $Q_2 = 60 - 0.5Q_1$.
  - By definition, Nash equilibrium is an intersection of the two best response functions: we can find it by solving two equations for two unknowns.
  - We get the Nash equilibrium $Q_1 = Q_2 = 40$.

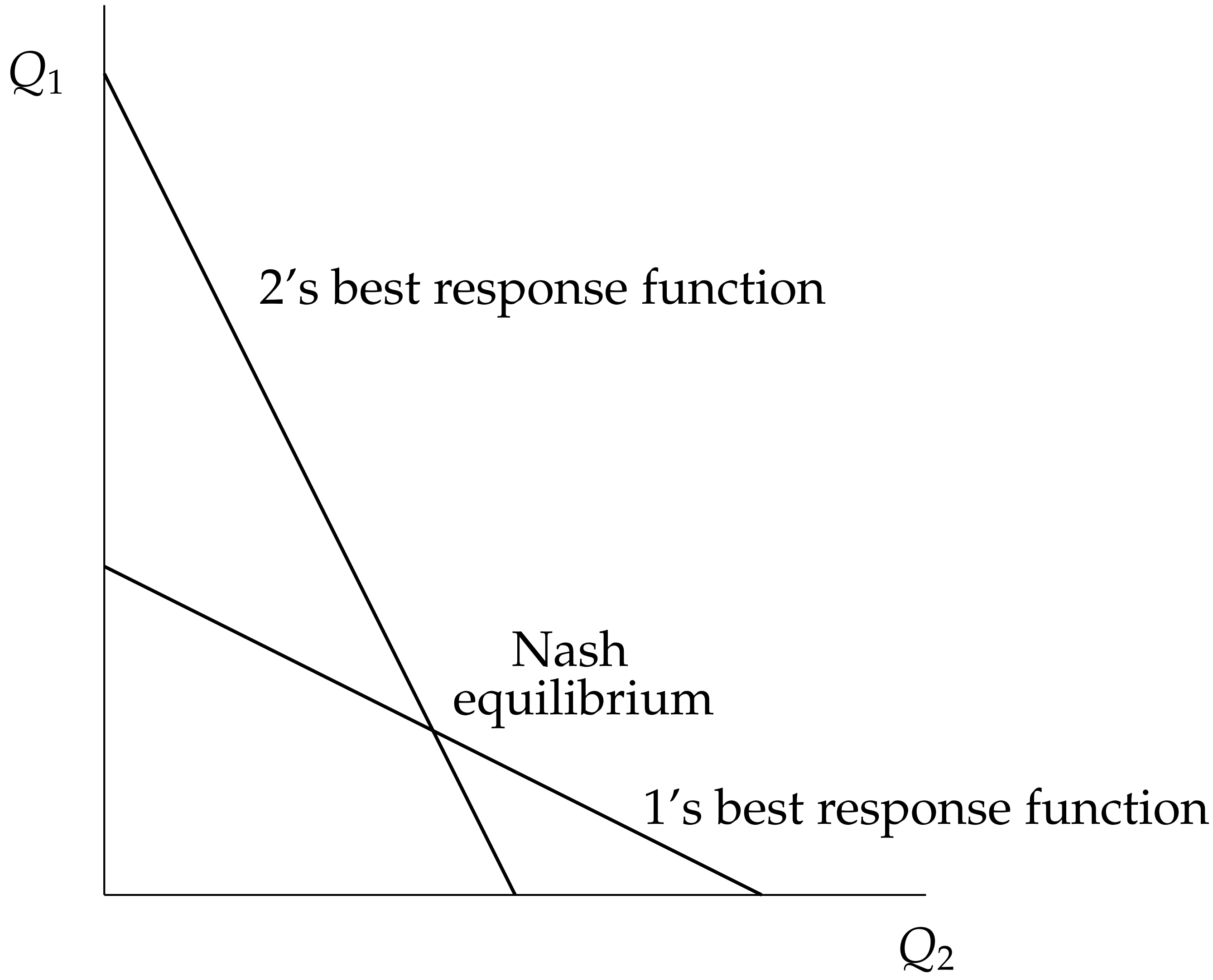

Figure 2. Cournot Duopoly.

- If the two firms could collude with each other, they would maximize their joint profit $Q(150 - Q - 30)$ by choosing a total quantity of $Q = 60$.
  - Collusive quantity is lower than the Nash equilibrium quantity, and price is higher.
  - Each firm cares only about impact on their own profit from increasing its output, and in Nash equilibrium they both produce too much.

5.2 Critical discussion of Nash equilibrium

- Nash equilibrium requires correct belief, which is not based on common knowledge of rationality.
  - In games with multiple Nash equilibria such as Battle of the Sexes, players may not be able to coordinate on a given Nash equilibrium.
  - In games with no Nash equilibrium in pure strategies such as Matching Pennies, Nash equilibrium fails to make any prediction.

- In Battle of the Sexes, miscoordination between the two — one paying *Boxing* and the other playing *Opera* — might occur because both *Boxing* and *Opera* are rationalizable for each player.

		Woman	
		Boxing	*Opera*
Man	*Boxing*	$2, 1$	$0, 0$
	Opera	$0, 0$	$1, 2$

- In Matching Pennies, every one of the four outcomes may occur, because both *Heads* and *Tails* are rationalizable for each player.

		Child 2	
		Heads	*Tails*
Child 1	*Heads*	$1, -1$	$-1, 1$
	Tails	$-1, 1$	$1, -1$

5.3 Rationalizability

- Through iterated elimination of strategies that are never best responses, we find rationalizable strategies of each player.

- Battle of the Sexes with a never-best-response third choice.

		Woman		
		Boxing	*Opera*	*Home*
	Boxing	2,1	0,0	1,.5
Man	*Opera*	0,0	1,2	0,.5
	Home	.5,0	.5,1	.5,.5

- Iterated elimination of never best responses versus iterated elimination of strictly dominated strategies.
  - If a strategy becomes strictly dominated, then it is never a best response and is eliminated in process of finding rationalizable strategies.
  - But some strategies are never best responses and are eliminated even though they are not strictly dominated.

- Rationalizability and Nash equilibrium.
  - A Nash equilibrium strategy of any player is always rationalizable, through strategies of other players in the same equilibrium.
  - Unlike Nash equilibrium, rationalizability requires only common knowledge of rationality, and makes different predictions (Battle of the Sexes, Matching Pennies).
  - If rationalizability leads to the unique Nash equilibrium, then the prediction is more appealing.