Econ 221
Spring, 2023
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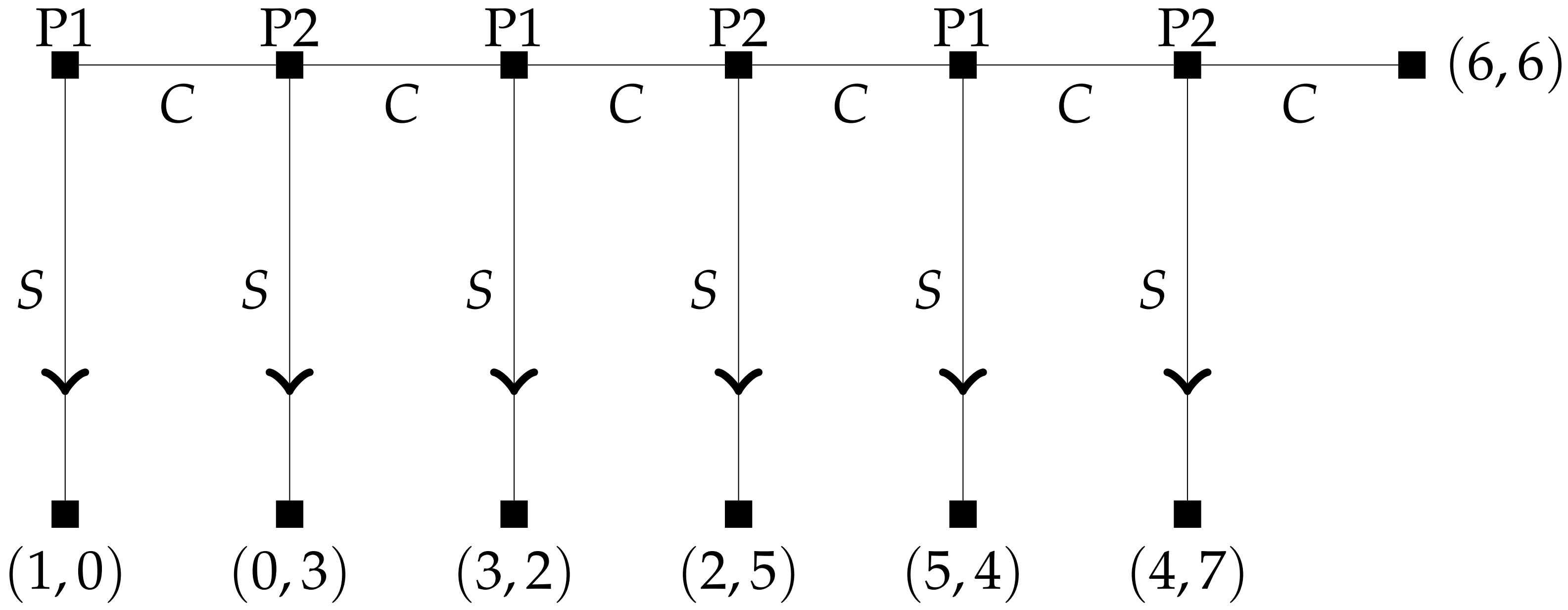
CHAPTER 4. SIMULTANEOUS-MOVE GAMES: DISCRETE STRATEGIES

- Simultaneous instead of sequential moves.
  - Strategy is a single action instead of a complete plan of actions.
  - Common knowledge of rationality is generally no longer sufficient to yield a unique equilibrium predication.



The centipede game again.

4.1 Game table

- A graphical representation of simultaneous-move games.
  - Row player chooses a row, column player a column.
  - Each player has a finite number of strategies.
  - Each cell is marked with the payoff of the row player and the payoff of the column player associated with the row and the column.

- Some famous 2×2 game tables.
  - Pure Coordination: there is no conflict between the two players, but each has to guess which way the other is trying to coordinate.

British

Canadian		*Left*	*Right*
	Left	$1, 1$	$0, 0$
	Right	$0, 0$	$1, 1$

- Matching Pennies: an example of zero-sum (or constant-sum) game, with no common interest at all.

		Child 2	
		Heads	*Tails*
Child 1	*Heads*	$1, -1$	$-1, 1$
	Tails	$-1, 1$	$1, -1$

- Battle of the Sexes: two players share common interest in coordinating, but they each have their own favorite way of coordinating.

		Woman	
		Boxing	*Opera*
Man	*Boxing*	2, 1	0, 0
	Opera	0, 0	1, 2

- Hawk and dove (Game of chicken): two players share common interest in avoiding a bad outcome, and again they have their own favorite way of doing so, but there is also a quite attractive compromise.

		Animal 2	
		Hawk	*Dove*
Animal 1	*Hawk*	$0, 0$	$3, 1$
	Dove	$1, 3$	$2, 2$

- Prisoners' Dilemma: two players have no interests in coordination, and unlike in Matching Pennies, each player has an obvious way to play that involves no guessing, leading to a collectively bad outcome.

Prisoner 2

		Confess	*Not confess*
Prisoner 1	*Confess*	$1, 1$	$3, 0$
	Not confess	$0, 3$	$2, 2$

4.3 Dominance

- For a given player, if one strategy gives a higher payoff than another strategy no matter what the opponent chooses, we say the first strategy dominates the second strategy, or the second strategy is dominated by the first strategy.
- Among above five 2×2 examples, dominance relationship exists only in the Prisoners' Dilemma.

- A player has a dominant strategy if it dominates all other strategies of this player.
    - Rationality requires a player to play dominant strategy if the player has one.
- If each player has a dominant strategy in a game, then the game is dominance solvable.
- Prisoners' Dilemma is dominance solvable.
    - Both players would be better off if they simultaneously switch to the dominated strategy.

- In a two-player game, if only one player has a dominant strategy, the game remains dominance solvable.
  - Knowledge of rationality requires the player without a dominant strategy to choose a best response to the dominant strategy of the other player.
- A mix of Game of Chicken and Prisoners' Dilemma.

Driver 2

		Straight	*Swerve*
Driver 1	*Straight*	0, 1	3, 0
	Swerve	1, 3	2, 2

- A two-player game where neither player has a dominant strategy may still be dominance solvable through iterated elimination of dominated strategies.
  - This requires at least one player to have at least one dominated strategy.
  - Rationality requires the player not to play it.
  - Knowledge of rationality then requires the other player to eliminate any strategy that becomes dominated.
  - And so on, until one strategy for each player is left.

- An extended Battle of the Sexes: with a third choice added, there is still no dominant strategy for either player, but the game is solvable through 4 rounds of iterated elimination of dominated strategies.

		Woman		
		Boxing	*Opera*	*Home*
Man	*Boxing*	2, 1	0, 0	2,.5
	Opera	0, 0	1, 2	0,1
	Home	1, 0	2, .5	1,1

- Round 1: *Opera* is eliminated for Man.

Woman

		Boxing	*Opera*	*Home*
Man	*Boxing*	2, 1	0, 0	2,.5
	Home	1, 0	2, .5	1,1

- Round 2: *Opera* is eliminated for Woman.

Woman

		Boxing	*Home*
Man	*Boxing*	2, 1	2,.5
	Home	1, 0	1,1

- Round 3: *Home* is eliminated for Man.

		Woman	
		Boxing	*Home*
Man	*Boxing*	2,1	2, .5

- Round 4: *Home* is eliminated for Woman.

		Woman
		Boxing
Man	*Boxing*	2,1

4.4 Stronger and weaker forms of dominance

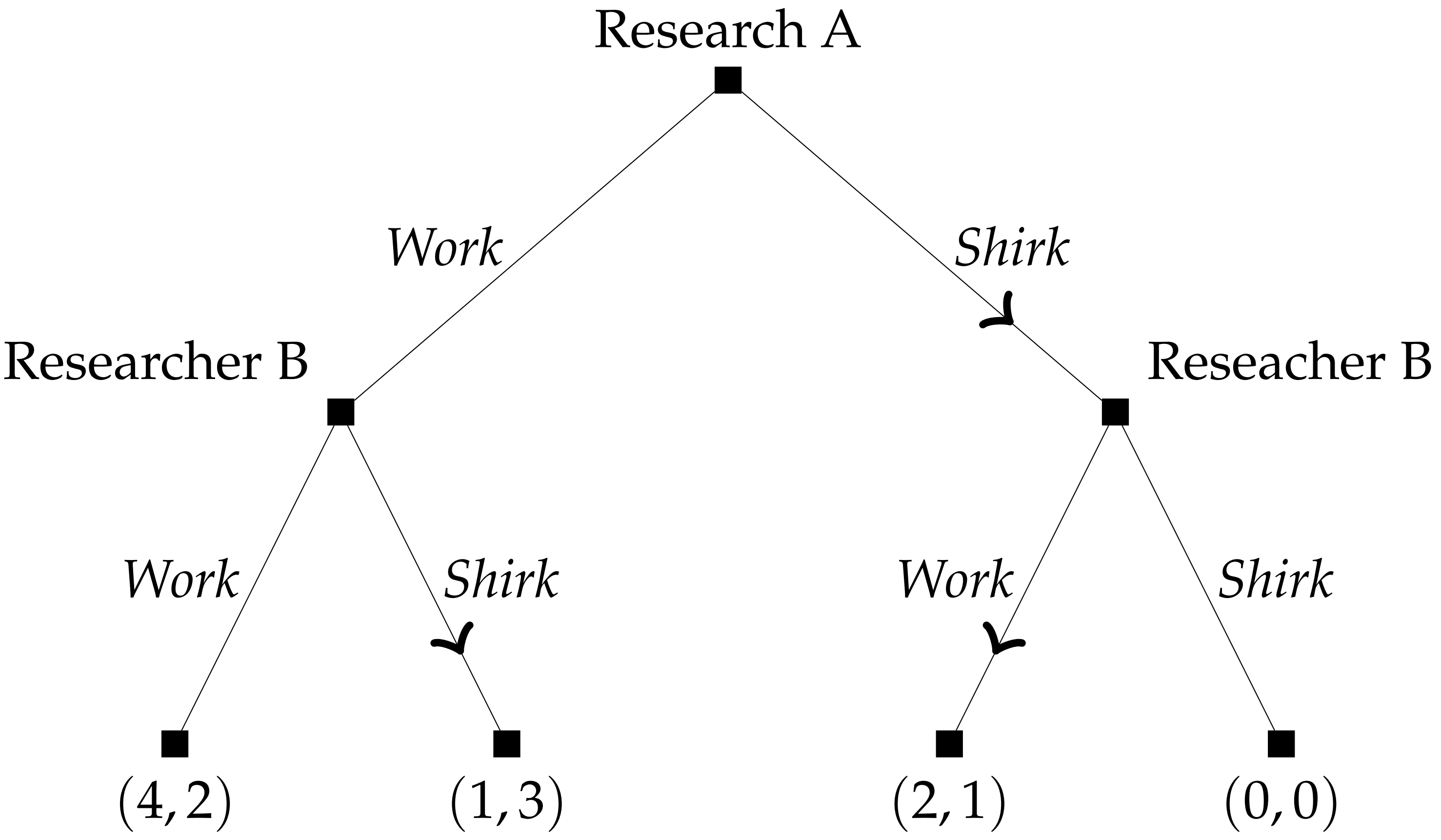
- For a given player, one strategy superdominates another strategy if the lowest payoff from playing the former is higher than the highest payoff from playing the latter.
  - Superdominance implies strict dominance, but reverse is generally not true.
  - Superdominance implies order irrelevance, that is, the superdominant strategy will be played whether the player moves first, second, or simultaneously.

- A game of two researchers.

		Researcher B	
		Work	*Shirk*
Researcher A	*Work*	$4, 2$	$x, 3$
	Shirk	$2, 1$	$0, 0$

  - When two researchers A and B both choose *Shirk*, their joint project remains incomplete and each gets 0. If at least one of them chooses *Work* the project is completed, and the payoffs depend on how much they value the completed project, and how much effort costs.

- If $x = 3$, *Work* superdominates, and, a fortiori, strictly dominates, *Shirk* for Researcher A. The outcome is A choosing *Work* and B choosing *Shirk*, regardless of whether A moves first, second, or simultaneously with B.
- If $x = 1$, for Researcher A *Work* strictly dominates, but does not superdominate, *Shirk*. The outcome is A choosing *Work* and B choosing *Shirk*, if either A moves second or simultaneously with B, but is A choosing *Shirk* and B choosing *Work* if A moves first.



When $x = 1$, Researcher A plays *Shirk* when he moves first.

- Weak dominance: for a given player, if one strategy gives a payoff at least as high as another strategy regardless of what the opponent chooses, and a strictly higher payoff against at least one strategy of the opponent, the first strategy weakly dominates the second strategy.

- Simultaneous-move version of Entry Deterrence.

		Incumbent	
		Accommodate	*Fight*
Startup	*Stay out*	0, 2	0,2
	Enter	1, 1	-1,-1

- We can define a weakly dominant strategy, and method of iterated elimination of weakly dominated strategies.

- Rationality now no long requires a player to eliminate any weakly dominated strategy, but is consistent with it.

- Most games are not dominance-solvable through iterated elimination of strictly or weakly dominated strategies.
  - Common knowledge of rationality is insufficient for a unique equilibrium prediction, but the method remains useful in practice.

4.6 More players

- Games tables can be used to represent simultaneous-move games with three players.
    - Stag Hunt with three hunters.

		Hunter 2	
		Stag	*Hare*
Hunter 1	*Stag*	2, 2, 2	0,1,0
	Hare	1, 0, 0	1,1,0

Hunter 3 chooses *Stag*

		Hunter 2	
		Stag	*Hare*
Hunter 1	*Stag*	0, 0, 1	0,1,1
	Hare	1, 0, 1	1,1,1

Hunter 3 chooses *Hare*

- With more than 3 players, game table is no longer helpful.
- Number of players does not affect iterated elimination of (strictly or weakly) dominated strategies.
  - Beauty Contest.
  - Application of iterated elimination of weakly dominated strategies.

4.5 Best response analysis

- In two-player games, for each column (row), mark all rows (columns) that give highest payoff for row (column) player.
    - Best response analysis is systematic strategic thinking.
- 2 × 2 examples.

		British	
		Left	*Right*
Canadian	*Left*	0,0	*1,1*
	Right	*1,1*	0,0

Child 2

		Heads	*Tails*
Child 1	*Heads*	*1,-1	-1,1*
	Tails	-1,1*	*1,-1

Woman

		Boxing	*Opera*
Man	*Boxing*	*2,1*	0,0
	Opera	0,0	*1,2*

		Animal 2	
		Hawk	*Dove*
Animal 1	*Hawk*	0,0	*3,1*
	Dove	*1,3*	2,2

		Prisoner 2	
		Confess	*Not confess*
Prisoner 1	*Confess*	*1,1*	*3,0
	Not confess	0,3*	2,2

- Ties in payoffs lead to multiple best responses.

		Incumbent	
		Accommodate	*Fight*
Startup	*Stay out*	0,2*	*0,2*
	Enter	*1,1*	-1,-1

- Best response analysis with more than two players.
  - In Stag Hunt, *Stag* is the best response for any hunter if all others choose *Stag*, and *Hare* is the best response otherwise.

4.2 Nash equilibrium

- We have an equilibrium if each player uses a strategy that best responds to strategies of other players.
  - Equivalently, an equilibrium is reached when no single player wishes to change strategy.
- Two features of equilibrium.
  - Non-cooperative: consider only unilateral deviations.
  - Correct beliefs: each player′ equilibrium strategy is a best response to equilibrium strategies of other players.

- Nash equilibrium and best response analysis.
  - In two-player games, a Nash equilibrium corresponds to a cell whose row and column are both marked.

Woman

		Boxing	*Opera*	*Home*
	Boxing	*3,2*	0,0	*2,1
Man	*Opera*	0,0	1,4*	0,1
	Home	2,0	*2,0	1,1*

- Nash equilibrium and strict dominance.
  - Only strategies that survive iterated elimination of strictly dominated strategies can be a player's candidates for equilibrium strategy.
  - A solution obtained through iterated elimination of strictly dominated strategies is the only Nash equilibrium.

- Nash equilibrium and weak dominance.
  - A solution obtained through iterated elimination of weakly dominated strategies is still a Nash equilibrium, but there may be other (less appealing) Nash equilibria.

Incumbent

		Accommodate	*Fight*
Startup	*Stay out*	0,2*	*0,2*
	Enter	*1,1*	-1,-1

4.7 Multiple Nash equilibria

- Nash equilibrium as social convention.
  - Recall two features: non-cooperative and correct belief.
  - Nash equilibrium rules out non-conventions, but is silent about which convention will be formed.

American Man

		Push	*Pull*
Canadian Woman	*Push*	0,0	*1,1*
	Pull	*1,1*	0,0

- Nash equilibria in assurance games.
  - In Stag Hunt, (*Stag*, *Stag*, *Stag*) is a risky Nash equilibrium, and (*Hare*, *Hare*, *Hare*) a safe one; Nash equilibrium makes no prediction between the two.

Hunter 2

		Stag	*Hare*
Hunter 1	*Stag*	*2,2*,2*	0,1,0
	Hare	1,0,0	*1,1*,0

Hunter 3 chooses *Stag*

Hunter 2

		Stag	*Hare*
Hunter 1	*Stag*	0,0,1	0,1*,1*
	Hare	*1,0,1*	*1,1*,1*

Hunter 3 chooses *Hare*

- In games with some conflicts, Nash equilibrium rules out some resolutions but is silent about which of the remaining ones will emerge.

		Woman	
		Boxing	*Opera*
Man	*Boxing*	*2,1*	0,0
	Opera	0,0	*1,2*

		Animal 2	
		Hawk	*Dove*
Animal 1	*Hawk*	0,0	*3,1*
	Dove	*1,3*	2,2

4.8 No Nash equilibrium in pure strategies

- Matching Pennies does not have a Nash equilibrium.

		Child 2	
		Heads	*Tails*
Child 1	*Heads*	*1,-1	-1,1*
	Tails	-1,1*	*1,-1

- Some zero-sum games have a Nash equilibrium; some non-zero-sum games have no Nash equilibrium.