Econ 221
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Chapter 15. Auctions, Bidding Strategy, and Auction Design

- Auction is a commonly used way of allocating indivisible goods among interested buyers.
  - Letter from John Lennon to Eric Clapton, used bikes, and spectrum.
  - Online platforms (Amazon, eBay) have greatly increased popularity of auctions in the modern digital economy.

15.1 What are auctions?

- Open outcry versus sealed bid.
  - Best known open outcry: English, Dutch auctions.
- First-price versus second-price.
  - In sealed bid auctions, highest bidder wins but price depends on rule.

- Private values.
  - Bidder i's value for the object is denoted as v_i, and is independent of the value of the object to other bidders.
  - Bidder i's payoff from winning the auction is $v_i - p$ if he pays p.
  - Bidder i's type is v_i, only known to i.

- Uniform distribution as a model of types in auctions.
  - Bidders other than bidder i know only v_i is drawn from a uniform distribution over interval $[0, 1]$.
  - Two properties of uniform distribution: the probability that v_i lies on any subinterval from $[0, 1]$ is given by the length of the interval; and the average is given by the mid point of the subinterval.

- How to sell your used bike?
  - You have no use for it — your reservation value is 0.
  - There are two potential buyers.
  - Each buyer's valuation for your bike is drawn from the uniform distribution on the interval $[0, 1]$.

- Select one of the buyers randomly, and then make a take-it-or-leave-it offer.
  - We can model the game after selection as ultimatum game, where you are the proposer and the selected buyer is responder.
  - In the unique subgame perfect equilibrium, the price you choose is $\frac{1}{2}$.
  - Your expected revenue is $\frac{1}{4}$.

- You can do better by making two take-it-or-leave-it offers.
  - This can be modeled as a sequential-move game, where you first choose one buyer randomly to make a take-it-or-leave-it offer, and if the offer is rejected, make a take-it-or-leave-it offer to the other buyer.
  - In the unique subgame perfect equilibrium, first offer is $\frac{5}{8}$, and second offer is $\frac{1}{2}$.
  - Your expected revenue is $\frac{25}{64} > \frac{1}{4}$.

- You can set a buy-it-now price.
  - We can model this as a game that combines sequential and simultaneous moves, where as the first mover you set a price, and the two buyers simultaneously choose whether or not to respond to it, with the bike sold at the set price when there is at least one responder.
  - In the unique subgame perfect equilibrium, the price you set is $\frac{1}{\sqrt{3}}$.
  - Your expected revenue is $\frac{2}{3\sqrt{3}}$, which is between $\frac{1}{4}$ and $\frac{25}{64}$.

- You can invite offers and sell it to the highest offer.
  - This can be modeled as a first price auction.
  - We will show that in the Bayesian game between two bidders, it is an equilibrium for each bidder to offer half of his valuation.
  - Your expected revenue is $\frac{1}{3}$, which is between $\frac{1}{4}$ and $\frac{2}{3\sqrt{3}}$.

- You can sell it on e-Bay.
  - e-Bay has an automatic bidding system, where each buyer is asked to submit their maximum bid, and the system bids in fixed increments for the buyer until the maximum is reached.
  - We will show that in the corresponding Bayesian game, it is a dominant strategy for each buyer to submit their valuation as the maximum bid.
  - Your expected revenue is $\frac{1}{3}$, which is between $\frac{1}{4}$ and $\frac{2}{3\sqrt{3}}$.

- You can sell it on e-Bay with a reserve price.
  - Bidding starts at the reserve price, so if only one buyer submits the maximum bid, bike is sold to the buyer at the reserve price.
  - In the corresponding Bayesian game between the two buyers, it remains a dominant strategy for each buyer to submit their valuation (if above the reserve price) as the maximum bid.
  - The best reserve price is $\frac{1}{2}$, yielding expected revenue of $\frac{5}{12}$, greater than $\frac{25}{64}$.

15.3 Bidding in auctions

- Second price, sealed bid auctions with private values.
  - Each bidder i, $i = 1, \ldots, N$, values an object for sale at v_i; each i knows own valuation v_i, but not any other v_j, $j \neq i$; each i submits a bid b_i independently; bidder i wins the auction if b_i is higher than all other b_j, $j \neq i$, wins with equal probability if b_i is among the highest, and otherwise loses; payoff to each i is $v_i - p$ if i wins, where p is the highest losing bid, and 0 otherwise.

- This is a Bayesian game.
  - To set it up, we will need to specify what each bidder i knows about how each v_j, $j \neq i$, is distributed.
  - Type of each bidder i is own valuation v_i.
  - A bidding strategy of each bidder i specifies the bid b_i depending on v_i.
- Regardless how we specify the Bayesian game, there is a weakly dominant strategy for each bidder i: bidding $b_i = v_i$ weakly dominates all other bids.

- Bidding one's own valuation is a weakly dominant strategy.
  - Fix any bidder i, and fix any valuation v_i.
  - Denote as b the highest outstanding bid; this is the price i pays if i wins the auction.
  - Bidding $b_i > v_i$ is weakly dominated by $b_i = v_i$: they give the same payoff when $b < v_i$, when $b = v_i$, and when $b > b_i$, but $b_i > v_i$ is strictly worse than $b_i = v_i$ when $v_i < b \leq b_i$.
  - Bidding $b_i < v_i$ is weakly dominated by $b_i = v_i$.

- First price, sealed bid auctions with private values.
  - Each bidder i, $i = 1, \ldots, N$, values an object for sale at v_i; each i knows own valuation v_i, but not any other v_j, $j \neq i$; each i submits a bid b_i independently; bidder i wins the auction if b_i is higher than all other b_j, $j \neq i$, wins with equal probability if b_i is among the highest, and otherwise loses; payoff to each i is $v_i - b_i$ if i wins.

- There is no weakly dominant bidding strategy.
  - Bidding one's own valuation is weakly dominated by bidding below it; so is bidding above it.
- Equilibrium bidding strategy involves shading the bid, i.e., bidding below one's own valuation.
  - To analyze how much one should shade the bid, we need to specify the Bayesian game in greater detail.

- A Bayesian game.
  - Suppose that $N = 2$.
  - Each bidder i, $i = 1, 2$, privately and independently draws valuation v_i from same uniform distribution over interval $[0, 1]$.

- A Bayesian Nash equilibrium: each bidder i uses bidding strategy $b_i = \frac{1}{2}v_i$.
  - Fix any bidder i, and fix any valuation v_i.
  - Any bid b_i wins when $b_i > b_j = \frac{1}{2}v_j$, i.e. when $v_j < 2b_i$, so b_i wins with probability $2b_i$, with expected payoff $2b_i(v_i - b_i)$.
  - The expected payoff is maximized by setting $b_i = \frac{1}{2}v_i$.

15.4 Auction design

- Revenue equivalence.
  - First price and second price auctions bring the same revenue to the seller.
  - In both auctions, a bidder with a given valuation v wins with probability v, and make an expected payment of $\frac{1}{2}v$ to the seller.
  - The seller's expected revenue is 2 times the integral of $v \cdot \frac{1}{2}v$ from 0 to 1, which is $\frac{1}{3}$.

- Reserve price.
  - Reserve price is minimum bid accepted in an auction.
  - Reserve price can be incorporated into both first price and second price auctions.
  - e-Bay allows seller to set their own reserve prices.
  - A positive reserve price means sometimes no sale, but it can increase seller's revenue, just as the best take-it-or-leave-it offer balances the probability of sale with terms of sale.

- Optimal reserve price.
  - Revenue equivalence holds with reserve price, so we look at second price auction, where bidding one's own valuation is weakly dominant.
  - With any reserve price r, a bidder with valuation v wins with probability v, and makes an expected payment of $r \cdot r + (v - r) \cdot \frac{1}{2}(r + v)$.
  - Optimal r maximizes 2 times the integral of the expected payment from r to 1, which gives $\frac{1}{2}$, with maximized revenue of $\frac{5}{12}$.

15.2 Winner's curse

- Common values.
  - Each bidder has partial information about his valuation for the object in an auction, and his actual valuation depends on other bidders' information as well as his own.
  - Common values is important when there is an objective component to all bidders' valuation of the object, or the object has resale values.

- Second price, sealed bid auctions with common values.
  - Each bidder i, $i = 1, 2$, receives private estimate s_i of the value of the object for sale.
  - Each i believes that s_j, $j \neq i$, is uniform between 0 and 1.
  - Each i's valuation is $v_i = s_i + s_j$.

- Winner's curse.
  - Suppose each bidder i bids expected valuation given own signal: $b(s_i) = s_i + \frac{1}{2}$.
  - Fix i and s_i.
  - Probability of winning is s_i.
  - Expected valuation conditional on winning is $s_i + \frac{1}{2}s_i$.
  - Expected price paid conditional on winning is $\frac{1}{2}s_i + \frac{1}{2}$.
  - Expected payoff conditional on winning is difference, which is negative for $s_i < \frac{1}{2}$.

- A Bayesian Nash equilibrium: $b(s_i) = 2s_i$.
  - Fix i and s_i, and consider any b_i.
  - Probability of winning is $\frac{1}{2}b_i$.
  - Expected valuation conditional on winning is given by $s_i + \frac{1}{4}b_i$.
  - Expected price paid conditional on winning is $\frac{1}{2}b_i$.
  - Expected payoff, which is probability of winning times the difference of expected valuation and expected price conditional on winning, is maximized at $b_i = 2s_i$.