

Econ 221
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CHAPTER 1. BASIC IDEAS AND EXAMPLES

1.1 What is a game of strategy

- Not all games are mainly about strategies.
- Not all strategizing happens in a game.
- Game of strategy: interactive decision making.
- Strategic thinking in a game of strategy.

1.2 Some examples and stories of games of strategy

- Economics (auctions), political science (voting), international relations (bargaining), biology (evolution).
- Sun Bin's horserace strategy.
- King Solomon's wisdom.
- Are animals capable of strategic thinking?

CHAPTER 2. HOW TO THINK ABOUT GAMES OF STRATEGY

2.2 Classifying games

- Sequential or simultaneous moves.
- Zero-sum or win-win, or somewhere in between.
- One-time encounter or repeated interactions.
- Perfect or imperfect information, and further in the latter case, symmetric or asymmetric information.

2.3 Some terminologies and background assumptions

- Strategies
 - A strategy for a player in a given game is a complete plan of actions.
 - Same concept as any individual decision problem.
 - The collection of feasible strategies for a player may be too complex to fully describe.
 - An outcome of the game is determined once we specify a strategy for each player.

- Payoffs

- To each outcome, a player attaches a number called payoff, with a higher payoff preferred to a lower one.
- Maximizing one's payoff is the objective of the player in the game.
- Expected payoff: players rank uncertain outcomes by computing mathematical expectation of their payoffs. (For example, the expected payoff from 20% probability of a payoff of 10 and 80% probability of a payoff of 20 is $.2 \times 10 + .8 \times 20 = 18$.)

- Rationality
 - Rational behavior: choose a strategy to maximize one's payoff given a belief about strategies other players choose.
 - Rationality is thus best responding to what one believes how others play.
 - Most of our analysis assumes not only rationality of each player but also common knowledge of each player's rationality among all players.

- Equilibrium
 - Equilibrium is Game Theory's answer to: what strategy will each player use in a given game?
 - We have an equilibrium if each player uses a strategy that best responds to strategies of other players.
 - Equivalently, an equilibrium is reached when no single player wishes to change strategy.
 - Two features of equilibrium: non-cooperative, correct beliefs.

CHAPTER 3. GAMES WITH SEQUENTIAL MOVES

3.1 Game trees

- A game tree is a graphical representation of a sequential-move game.
 - Each decision node is marked with the player who makes the move, and the branches leading from the node, each representing a possible move by the player.
 - The initial node, or the root of the game tree, is the first decision node.

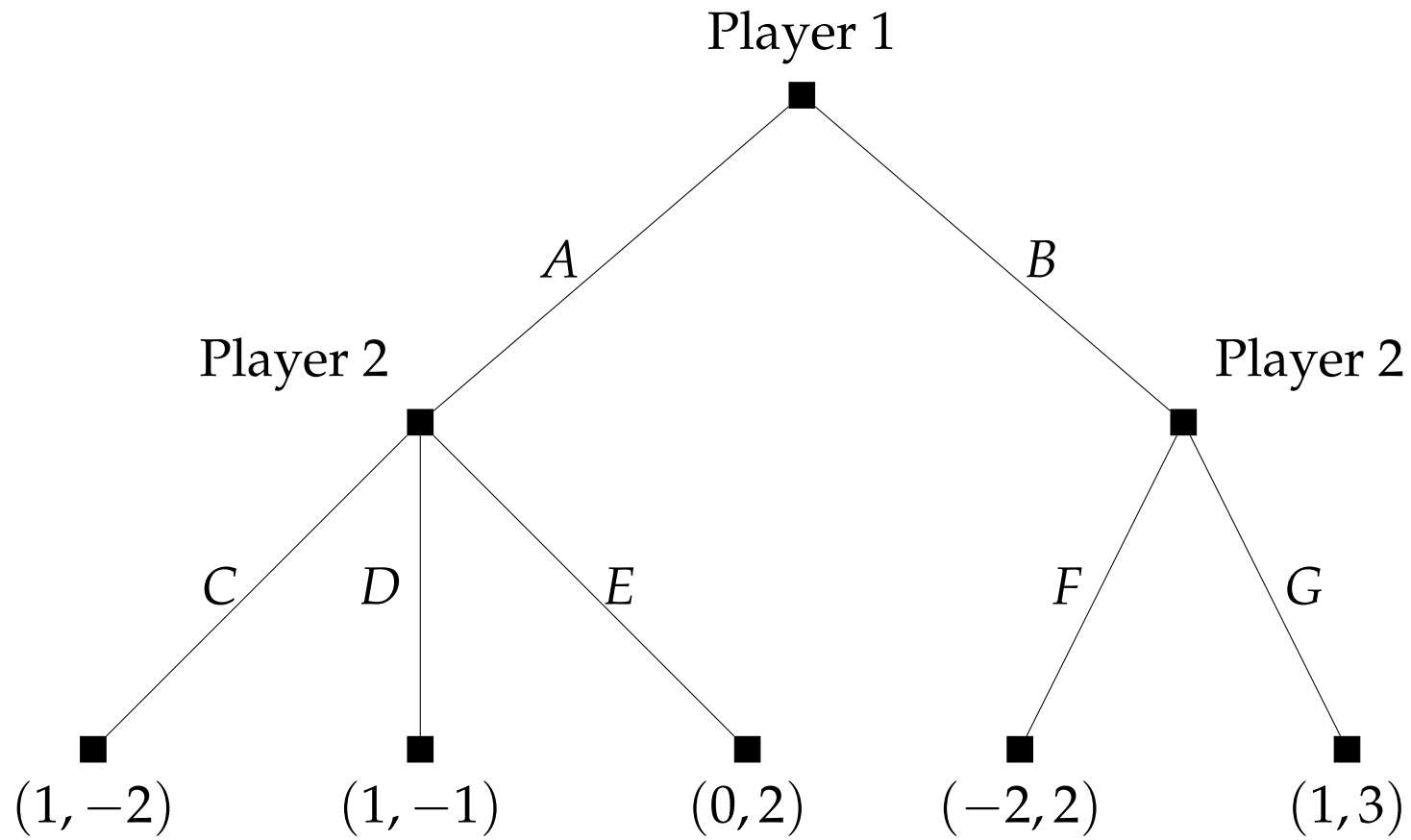


Figure 1. An example with 2 players and 3 decision nodes.

- Game tree continued.
 - Each terminal node is marked with the payoffs to the players, in the order of moves.
 - A decision node marked with “Nature” represents some external uncertainty outside the control of the players in the game, and each branch leading from such a node represents a possible resolution of the uncertainty and is marked with the corresponding probability.

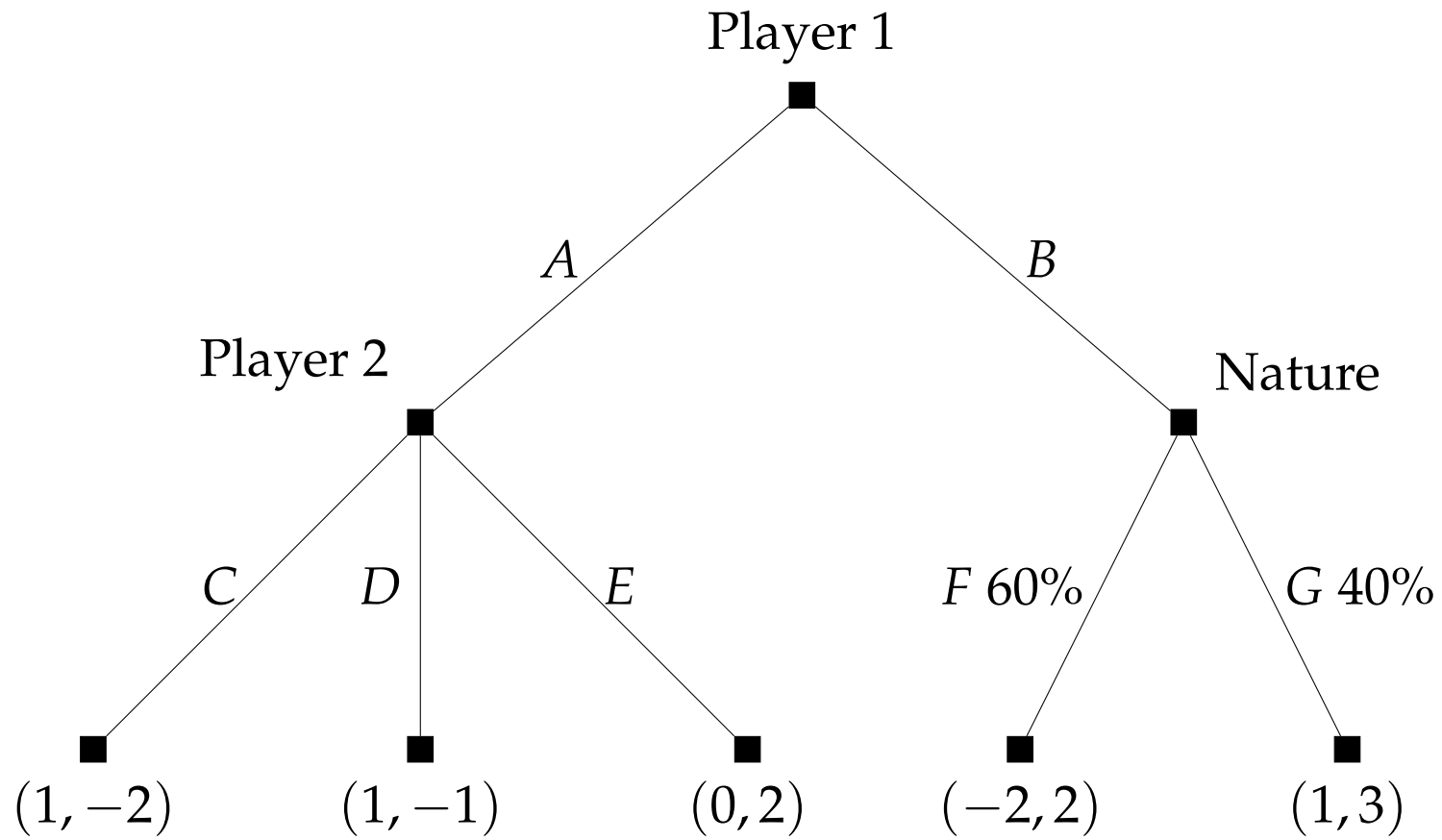


Figure 2. An example with a Nature's move.

- A strategy of a player in a sequential-move game specifies a move for each decision node that belongs to the player.
 - A strategy of a player may specify moves at decision nodes that will not be reached if the player follows the strategy at earlier decision nodes.
 - This is not only required for a strategy to be complete, but also necessary for our analysis.

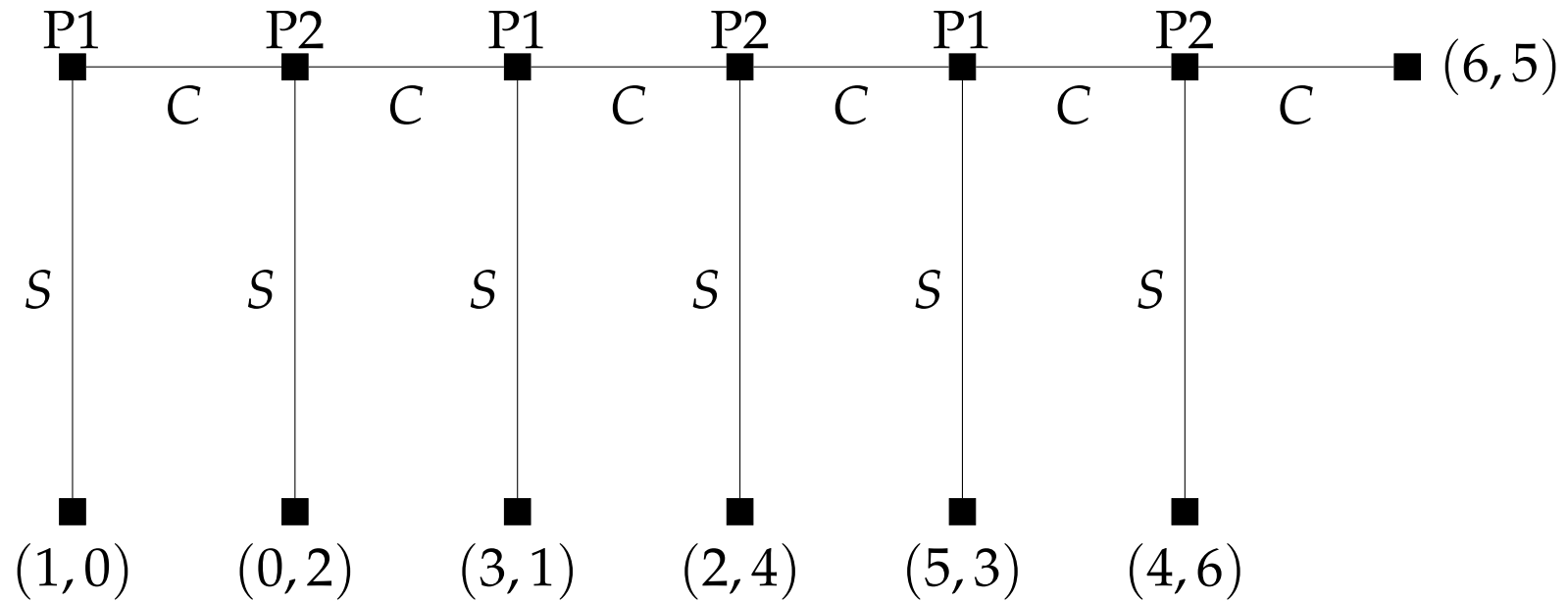


Figure 3. The Centipede Game: Player 1 has 8 strategies, 4 of which lead to $(1, 0)$.

3.2 Solving games by using trees

- Entry Deterrence

- A start-up car-sharing company S decides whether or not to enter a market monopolized by an incumbent company I . If S does not enter, nothing happens. If S enters, I has to decide whether to force S out (by cutting the price) or to accommodate S ; in the first case both companies will lose money, while in the second case S will make positive profit and I will make a positive but smaller profit than if S does not enter.

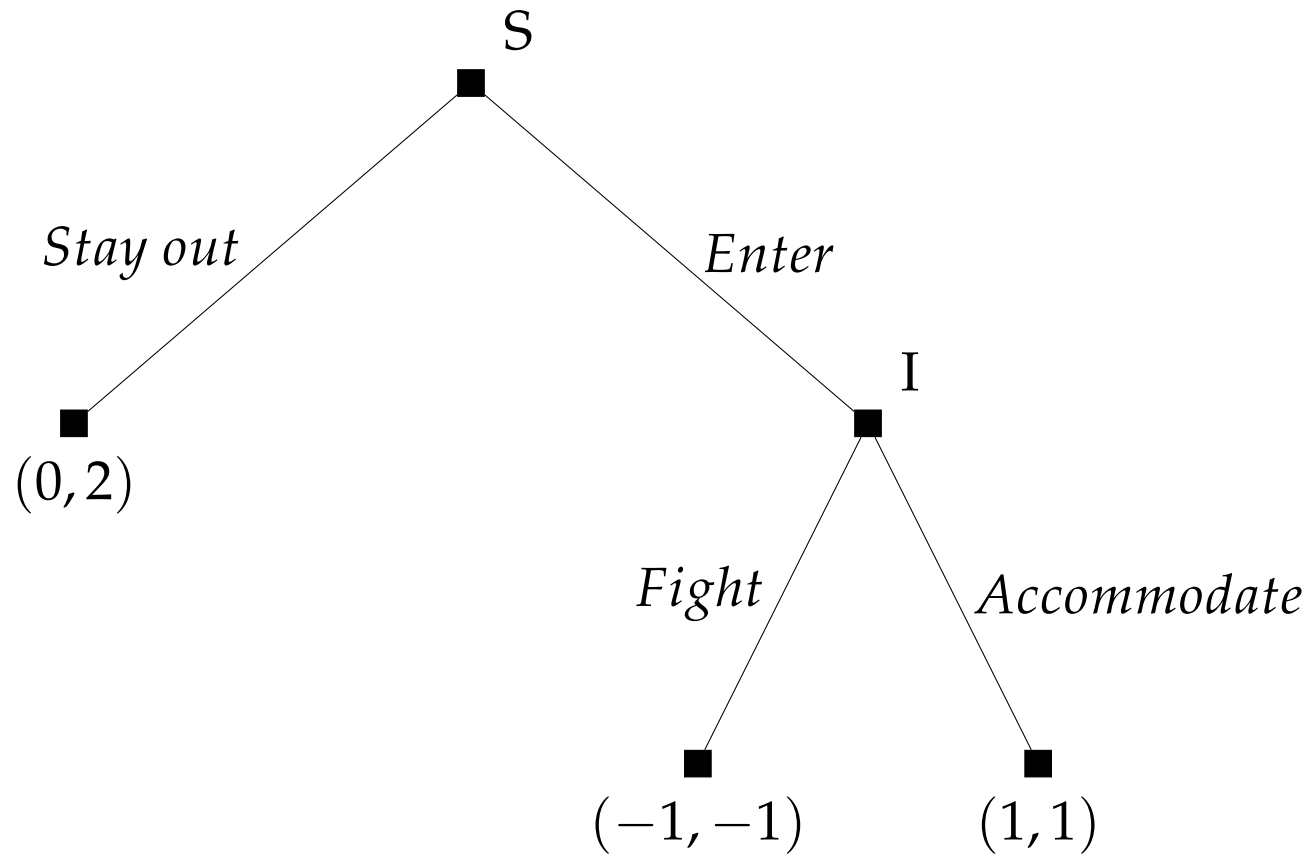


Figure 4. Entry Deterrence.

- Rollback: look ahead and reason back.
 - Rollback is a necessary implication of strategic thinking in a sequential-move game.
 - Rollback equilibrium is game theory's prediction of the strategies that will be used by players in such a game.

- Rollback: the arrowhead form.
 - Mark selected branches with arrows to find the rollback equilibrium and the equilibrium outcome.

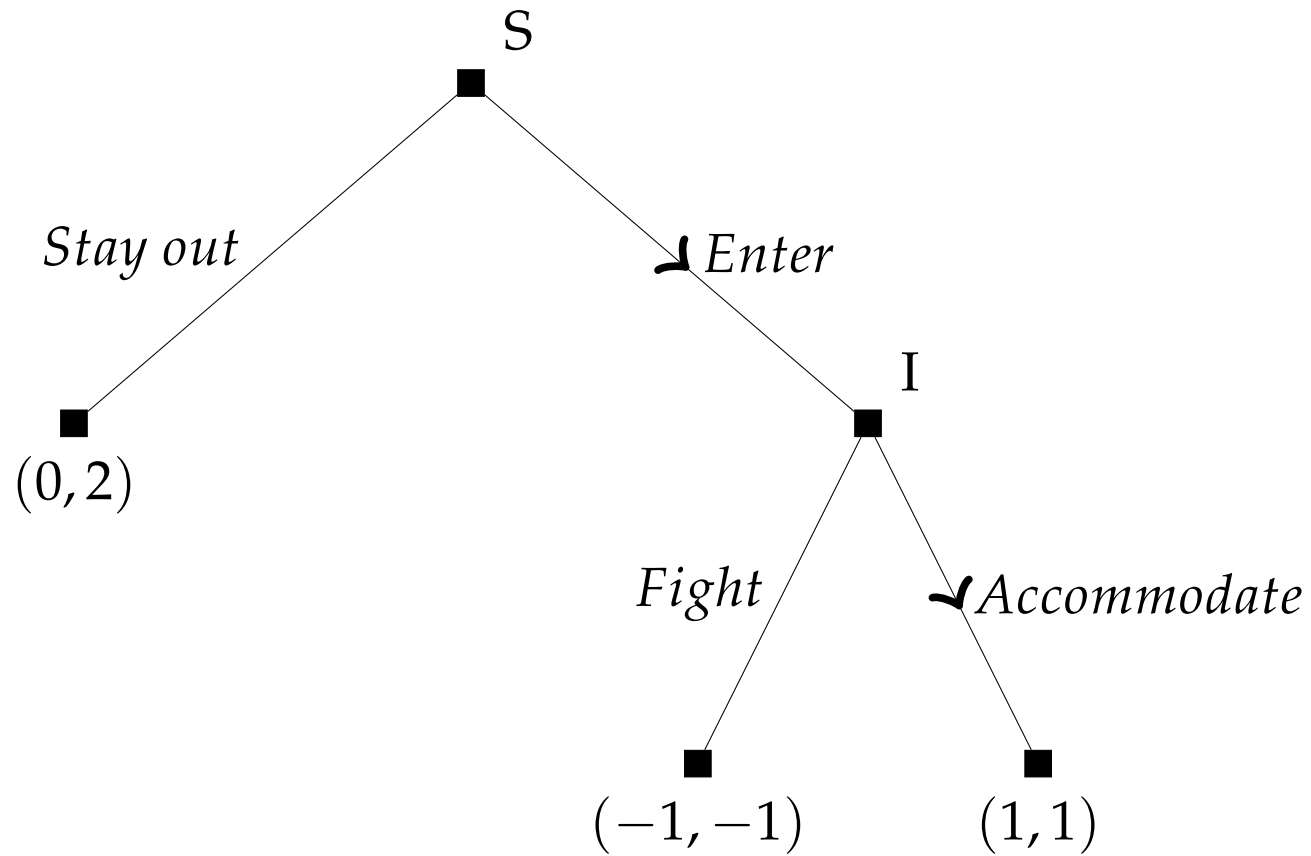


Figure 5. Rollback in Entry Deterrence.

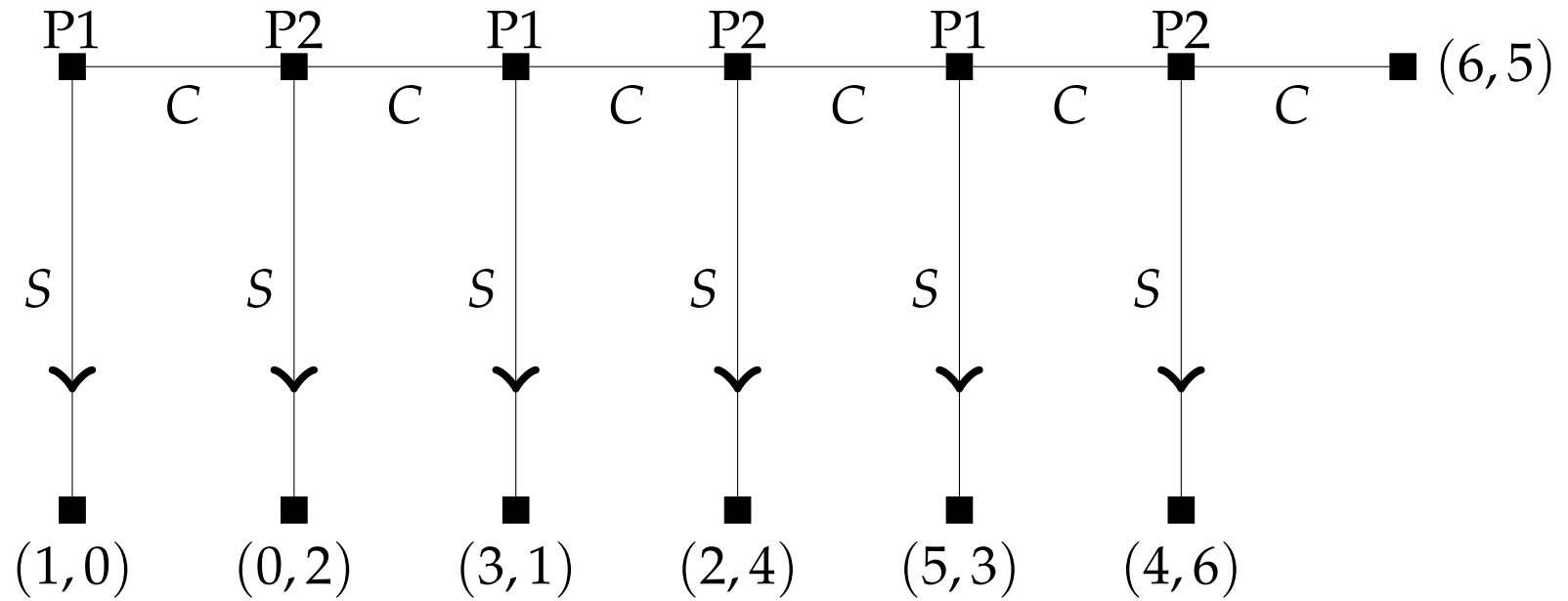


Figure 6. Rollback in the Centipede Game.

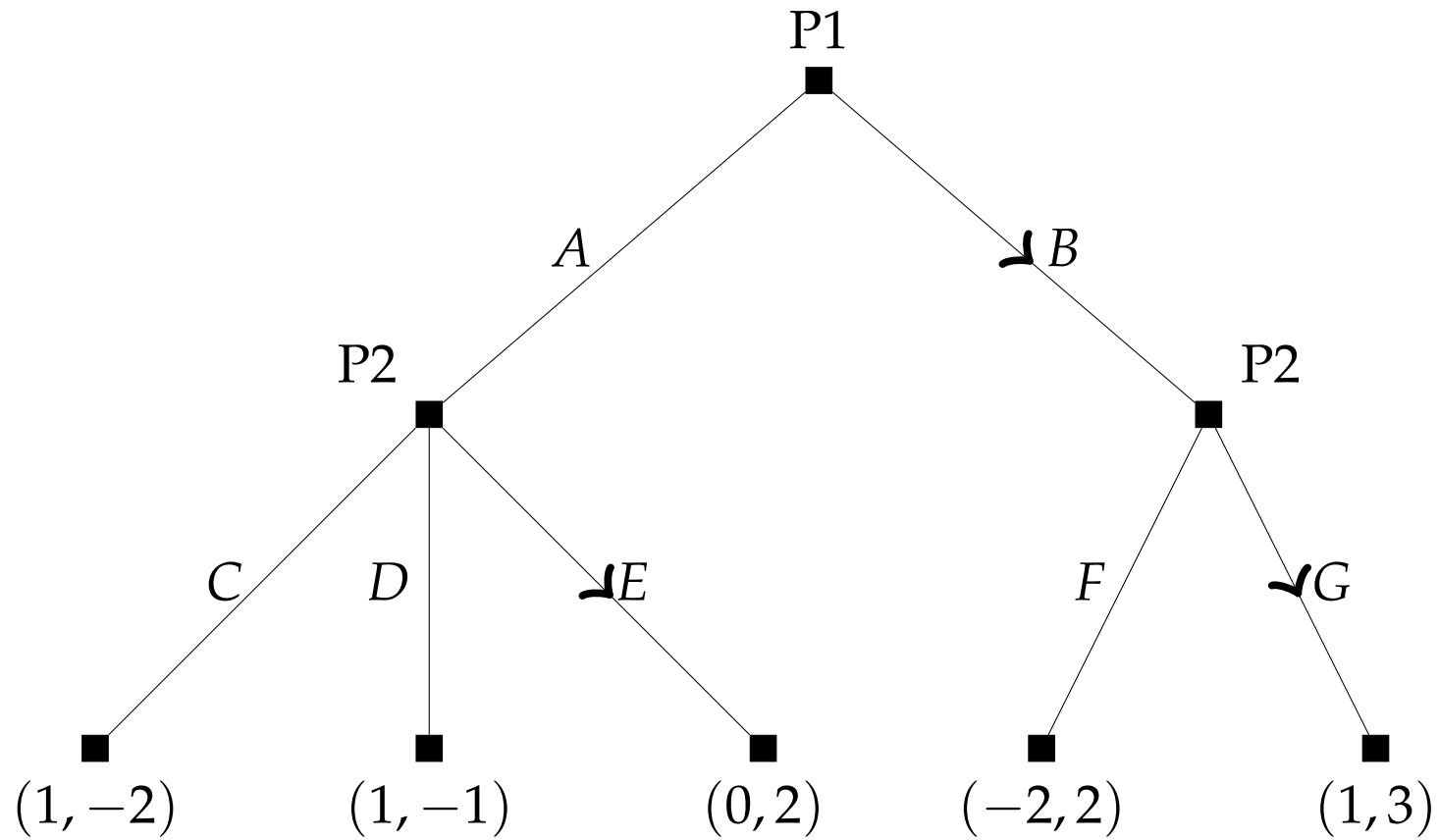


Figure 7. Rollback in the example with 2 last moves.

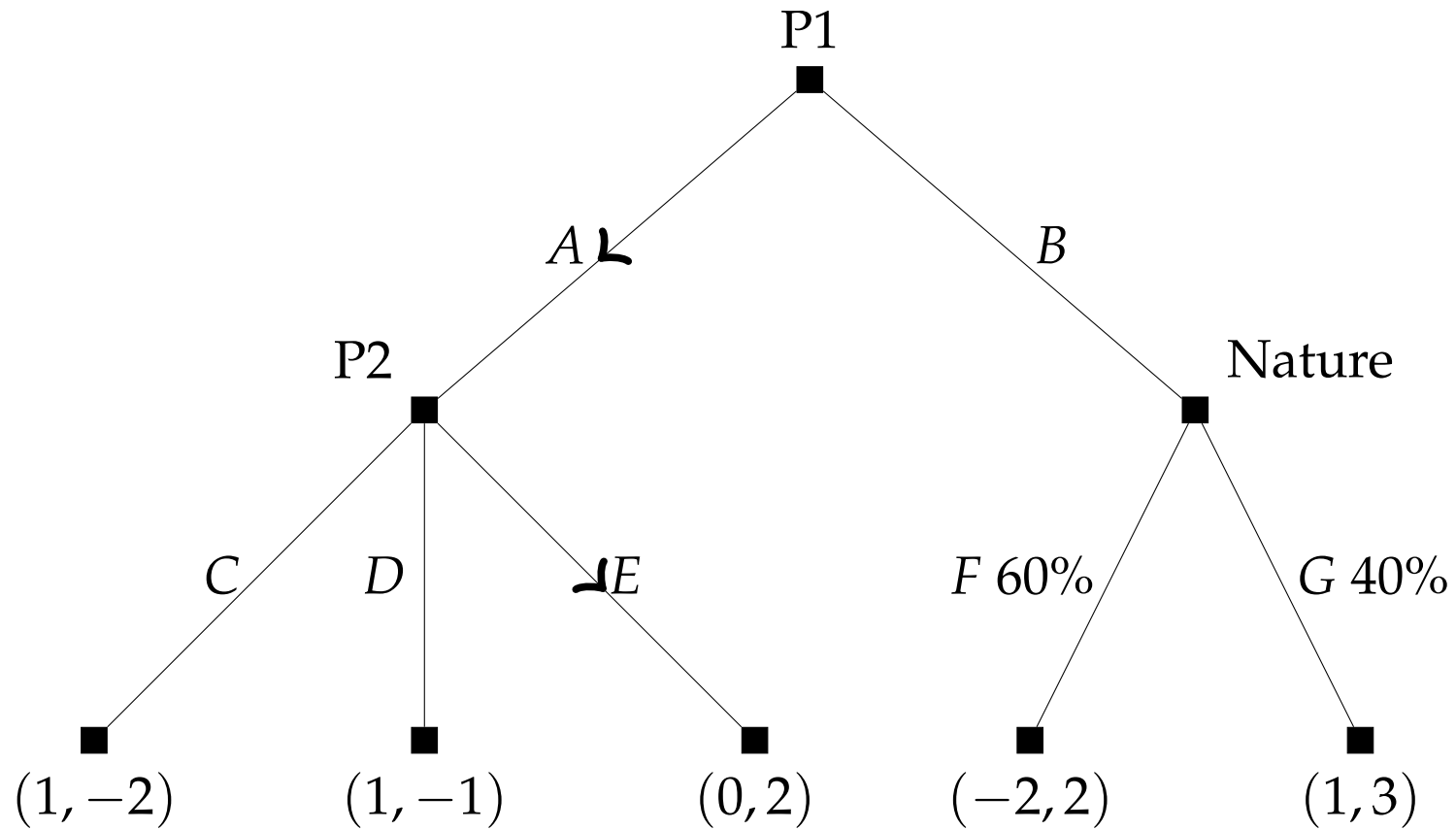


Figure 8. Rollback in the example with Nature's move.

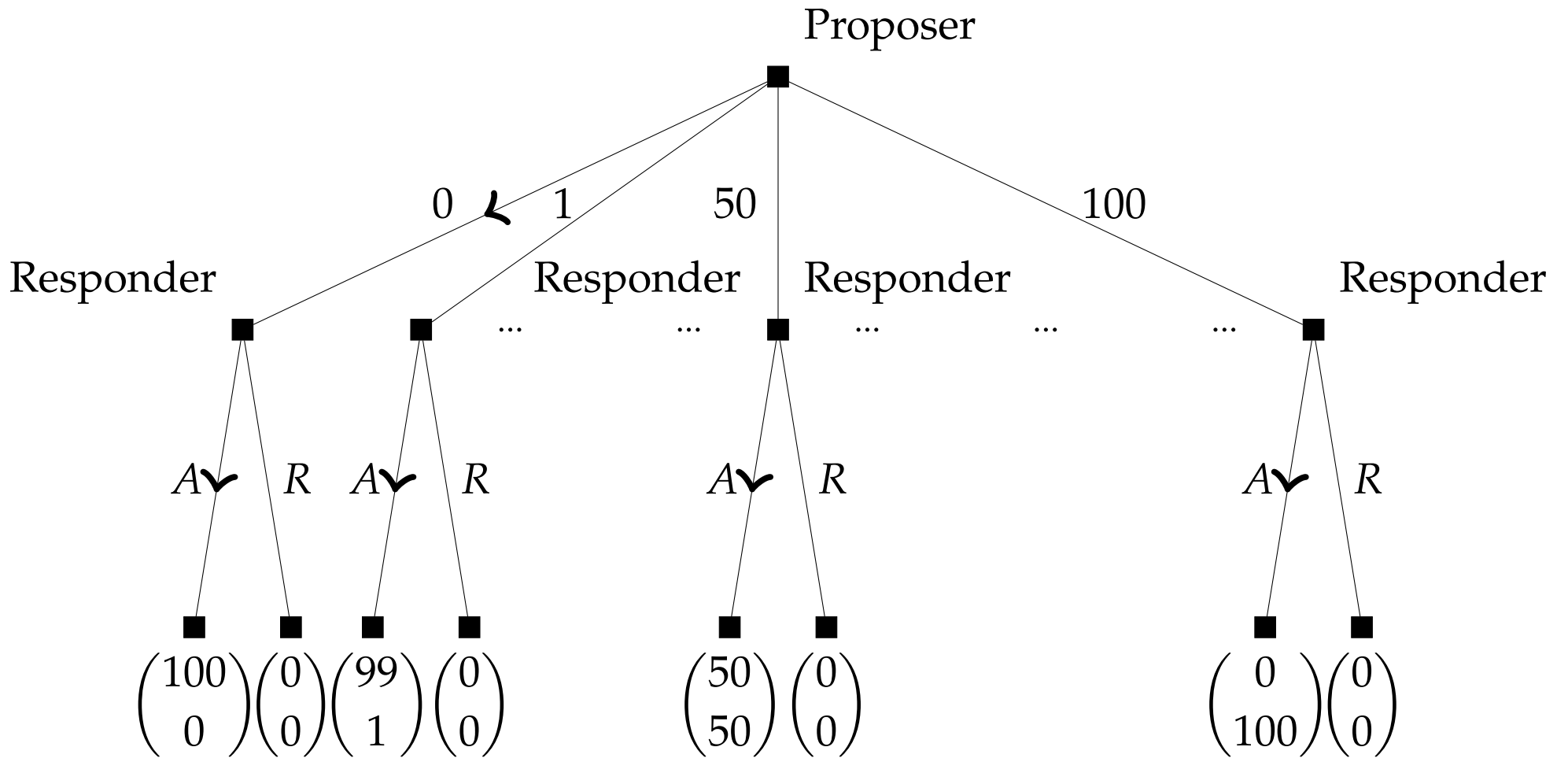


Figure 9. The Ultimatum Game: first rollback.

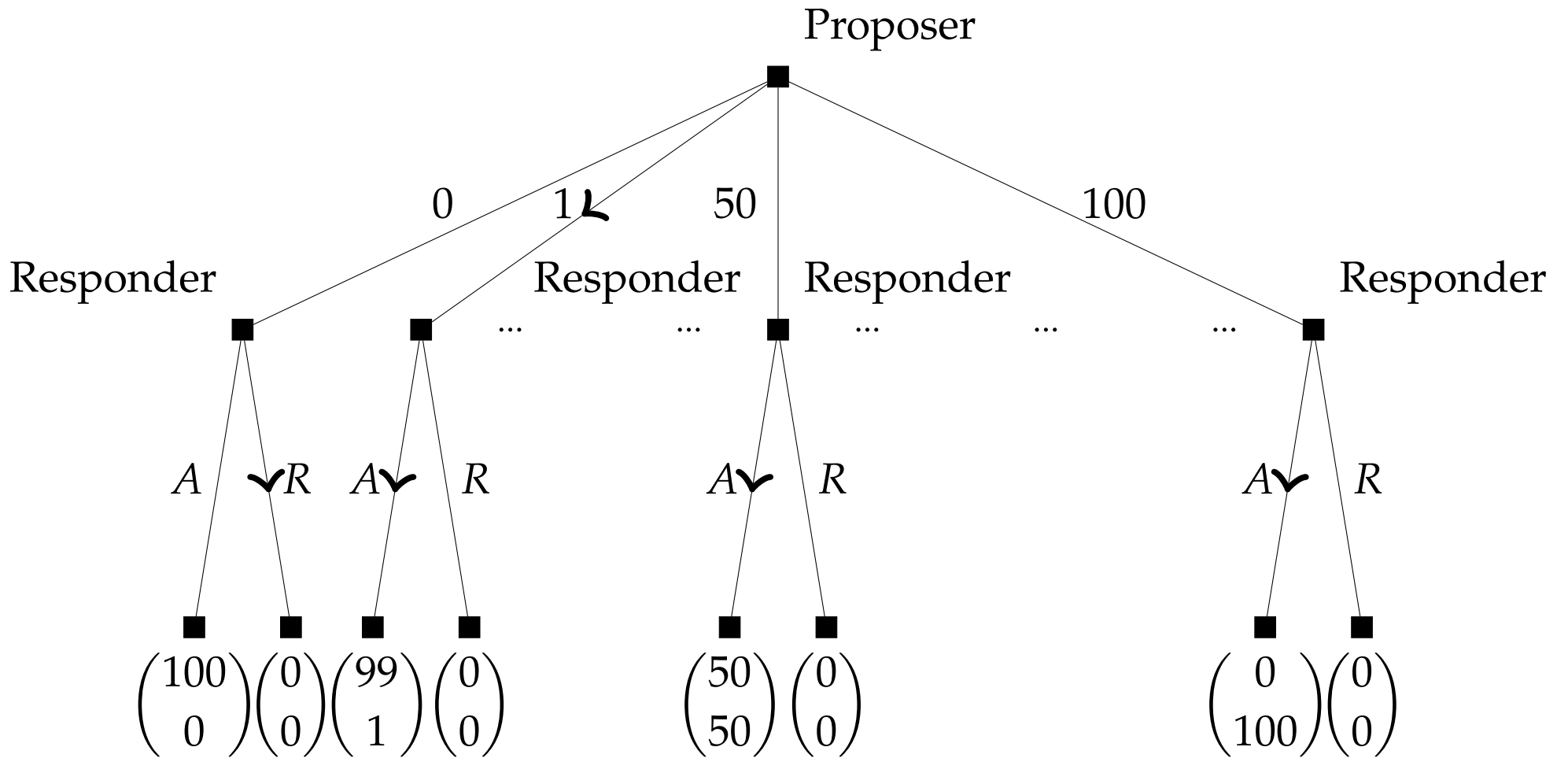


Figure 10. The Ultimatum Game: second rollback.

3.4 Order advantages

- Compare two games of same two players and same move for each player that differ only in who moves first.
 - First-mover advantage: player gets a higher payoff when he moves first than when he moves second, due to a benefit from making commitment.
 - Second-mover advantage: player gets a higher payoff when he moves second than when he moves first, due to benefit from retaining flexibility.

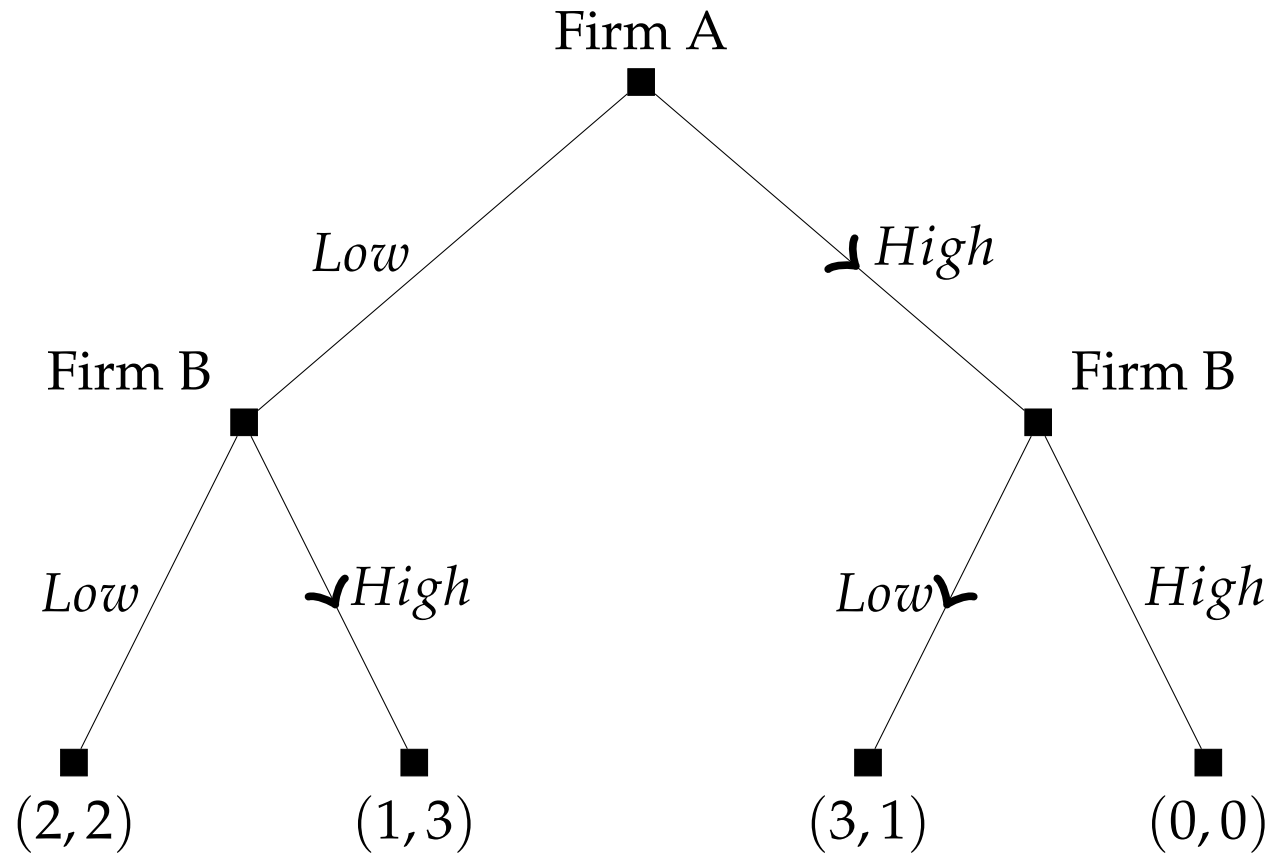


Figure 11. First-mover advantage.

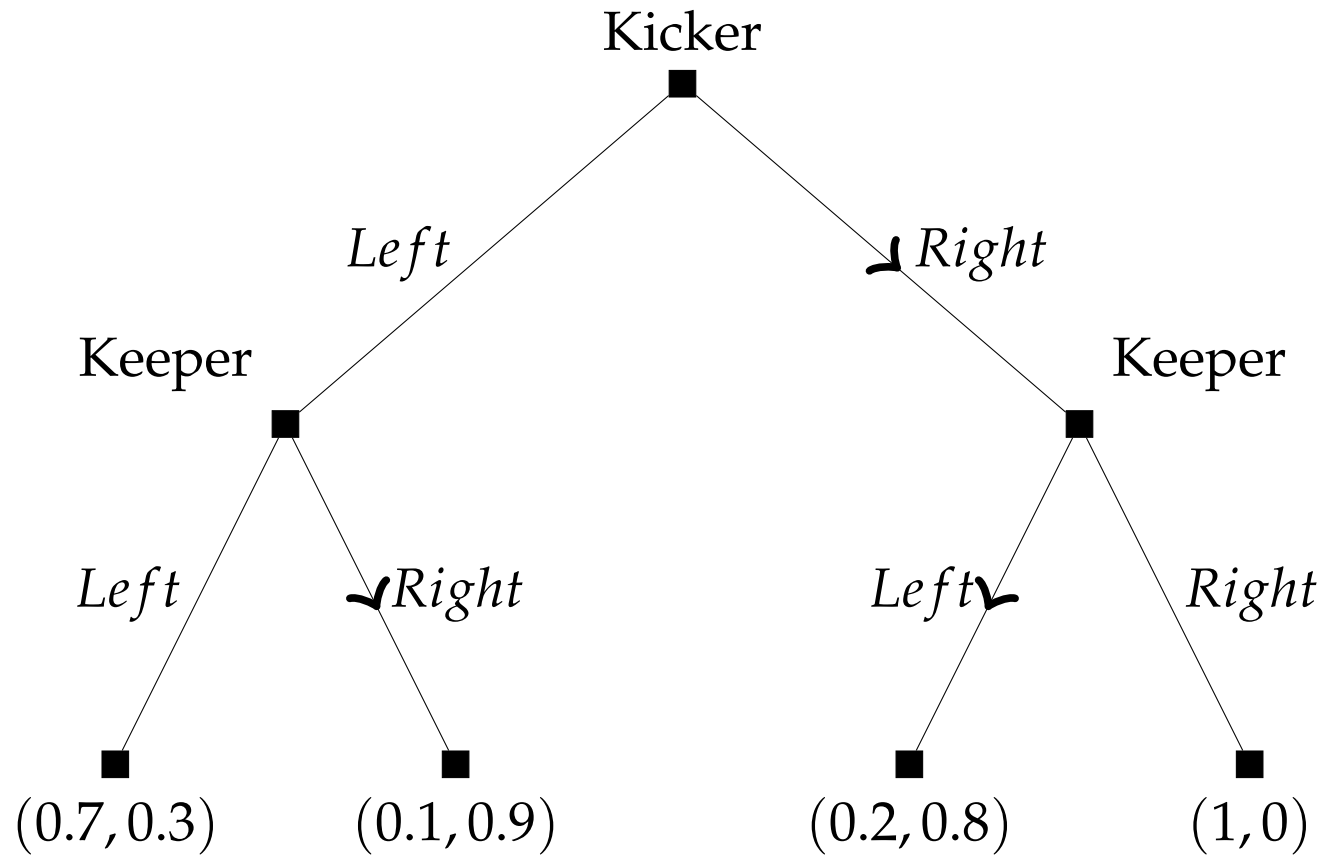


Figure 12. Second-mover advantage: Kicker moves first.

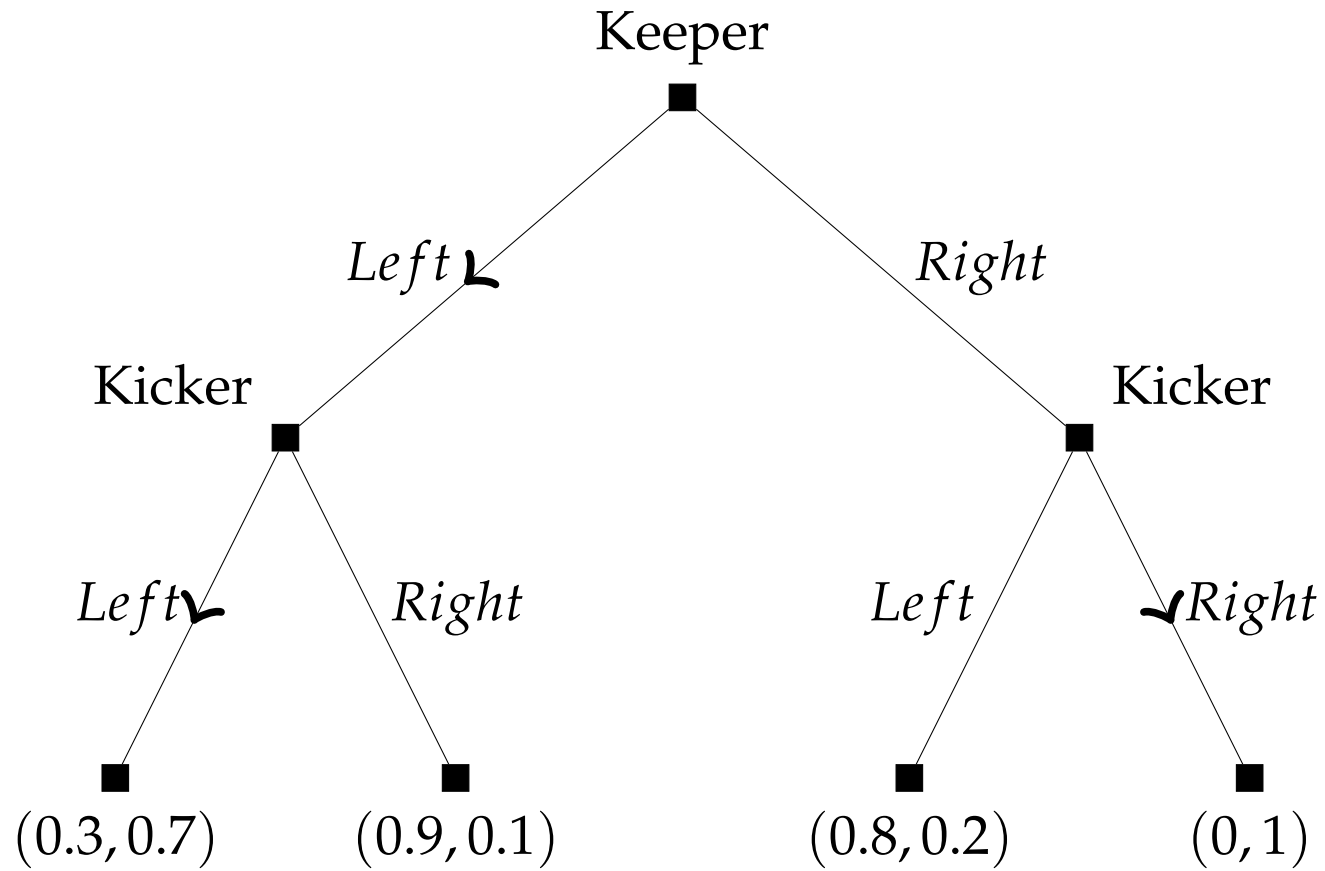


Figure 13. Second-mover advantage: Keeper moves first.

3.3 Adding more players

- Rollback method and rollback equilibrium generalize to more than two players.
- Chain Store Game
 - An incumbent firm I operates in K cities. In each city $k = 1, \dots, K$, a start-up firm S_k plays Entry Deterrence with I , after observing the outcome of previous $k - 1$ cities. Each S_k cares only about the outcome in city k , while I cares about sum of its payoffs across K cities.

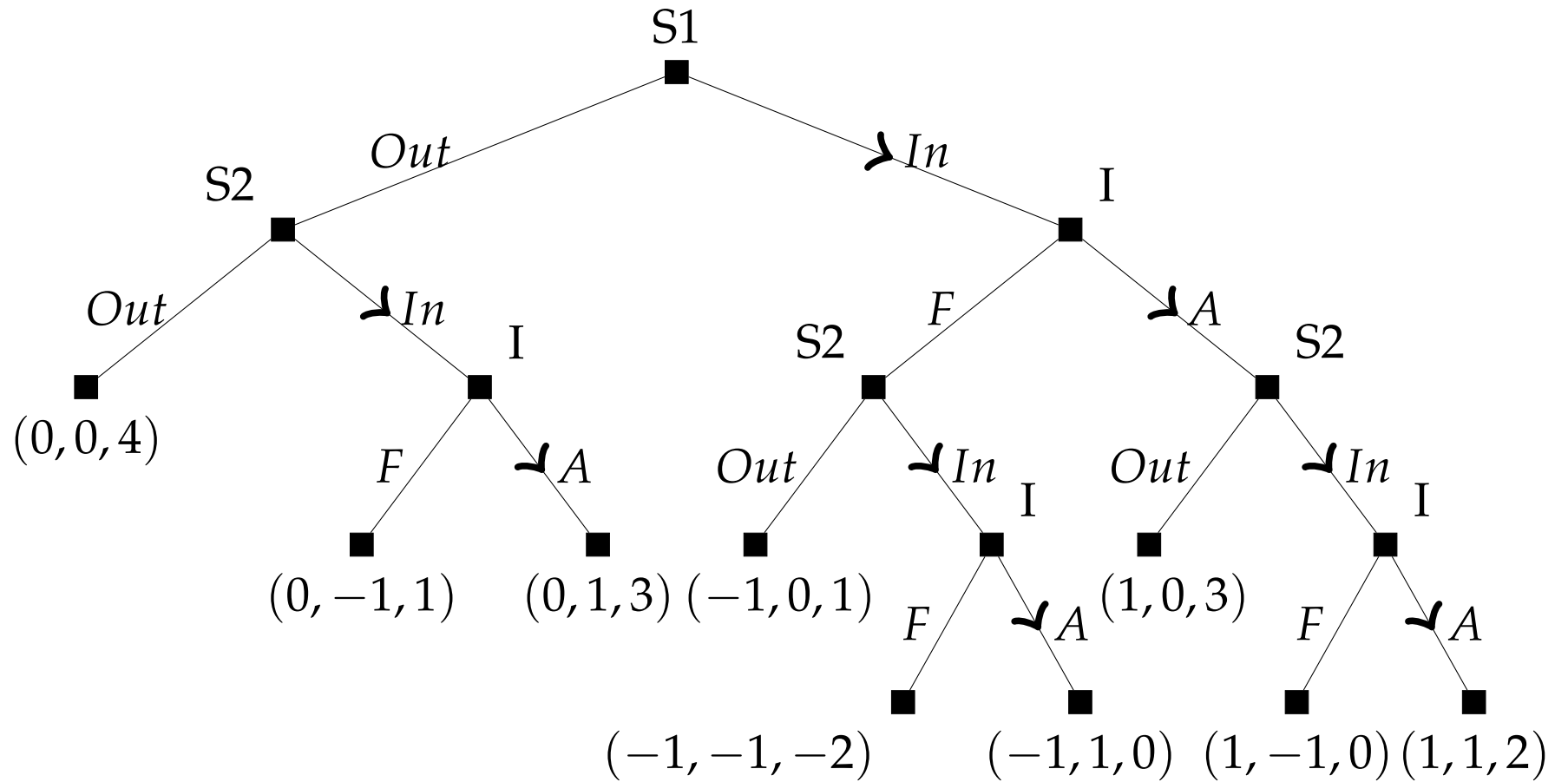


Figure 14. Chain Store Game with $K = 2$.