Econ 221
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Chapter 1. Basic Ideas and Examples

1.1 What is a game of strategy

- Not all games are mainly about strategies.
- Not all strategizing happens in a game.
- Game of strategy: interactive decision making.
- Strategic thinking in a game of strategy.

1.2 Some examples and stories of games of strategy

- Economics (auctions), political science (voting), international relations (negotiation), biology (evolution).
- Sun Bin's horserace strategy.
- King Solomon's wisdom.
- Are animals capable of strategic thinking?

2.2 Classifying games

- Sequential or simultaneous moves.
- Zero-sum or win-win, or somewhere in between.
- One-time encounter or repeated interactions.
- Perfect or imperfect information, and further in the latter case, symmetric or asymmetric information.

2.3 Some terminologies and background assumptions

- Strategies
  - A strategy for a player in a given game is a complete plan of actions.
  - Same concept as any individual decision problem.
  - The collection of feasible strategies for a player may be too complex to fully describe.
  - An outcome of the game is determined once we specify a strategy for each player.

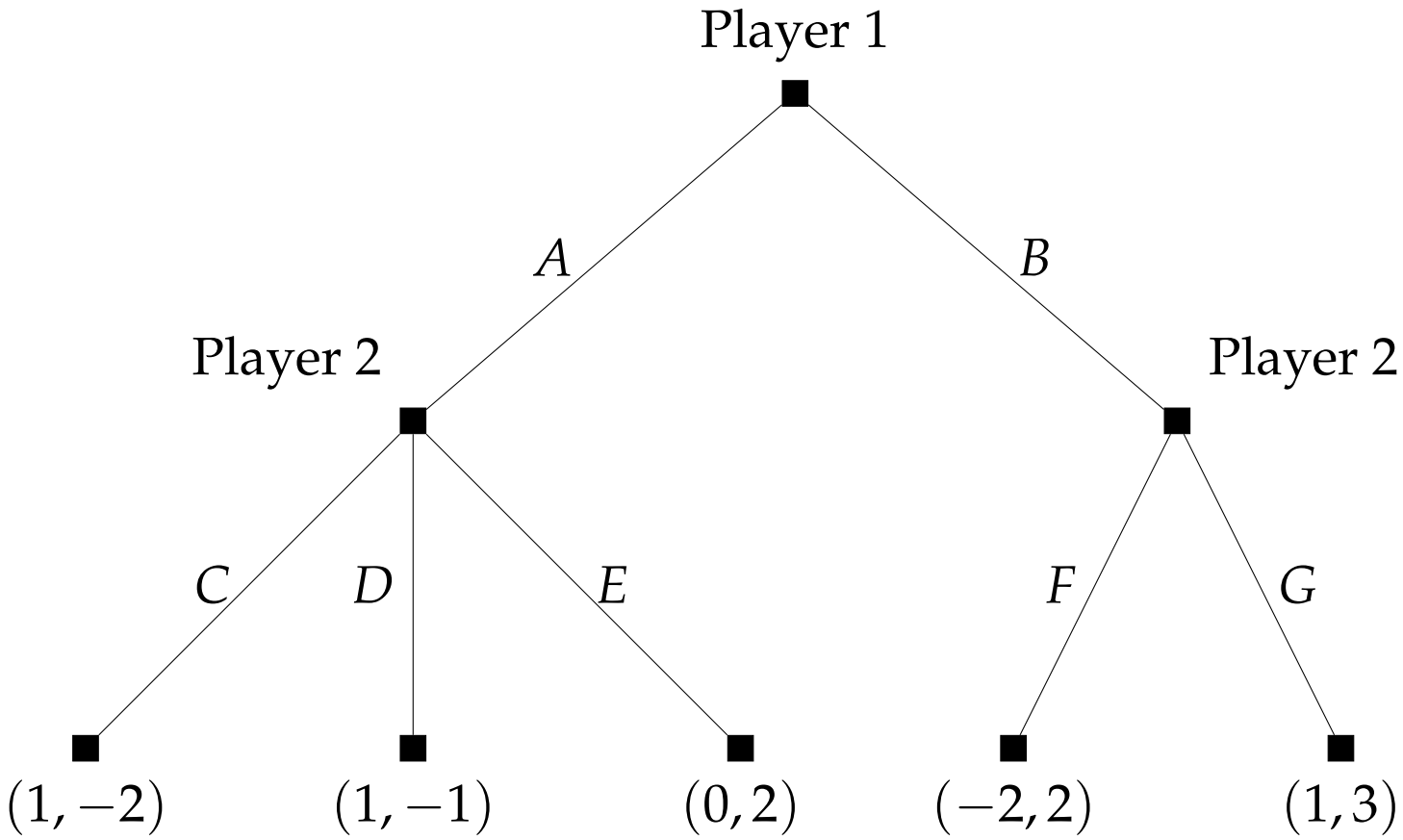
- Payoffs
  - To each outcome, a player attaches a number called payoff, with a higher payoff preferred to a lower one.
  - Maximizing one's payoff is the objective of the player in the game.
  - Expected payoff: players rank uncertain outcomes by computing mathematical expectation of their payoffs. (For example, the expected payoff from 20% probability of a payoff of 10 and 80% probability of a payoff of 20 is $.2 \times 10 + .8 \times 20 = 18$.)

- Rationality
  - Rational behavior: choose a strategy to maximize one's payoff given a belief about strategies other players choose.
  - Rationality is thus best responding to what one believes how others play.
  - Most of our analysis assumes not only rationality of each player but also common knowledge of each player's rationality among all players.

- Equilibrium
  - Equilibrium is Game Theory's answer to: what strategy will each player use in a given game?
  - We have an equilibrium if each player uses a strategy that best responds to strategies of other players.
  - Equivalently, an equilibrium is reached when no single player wishes to change strategy.
  - Two features of equilibrium: non-cooperative, correct beliefs.

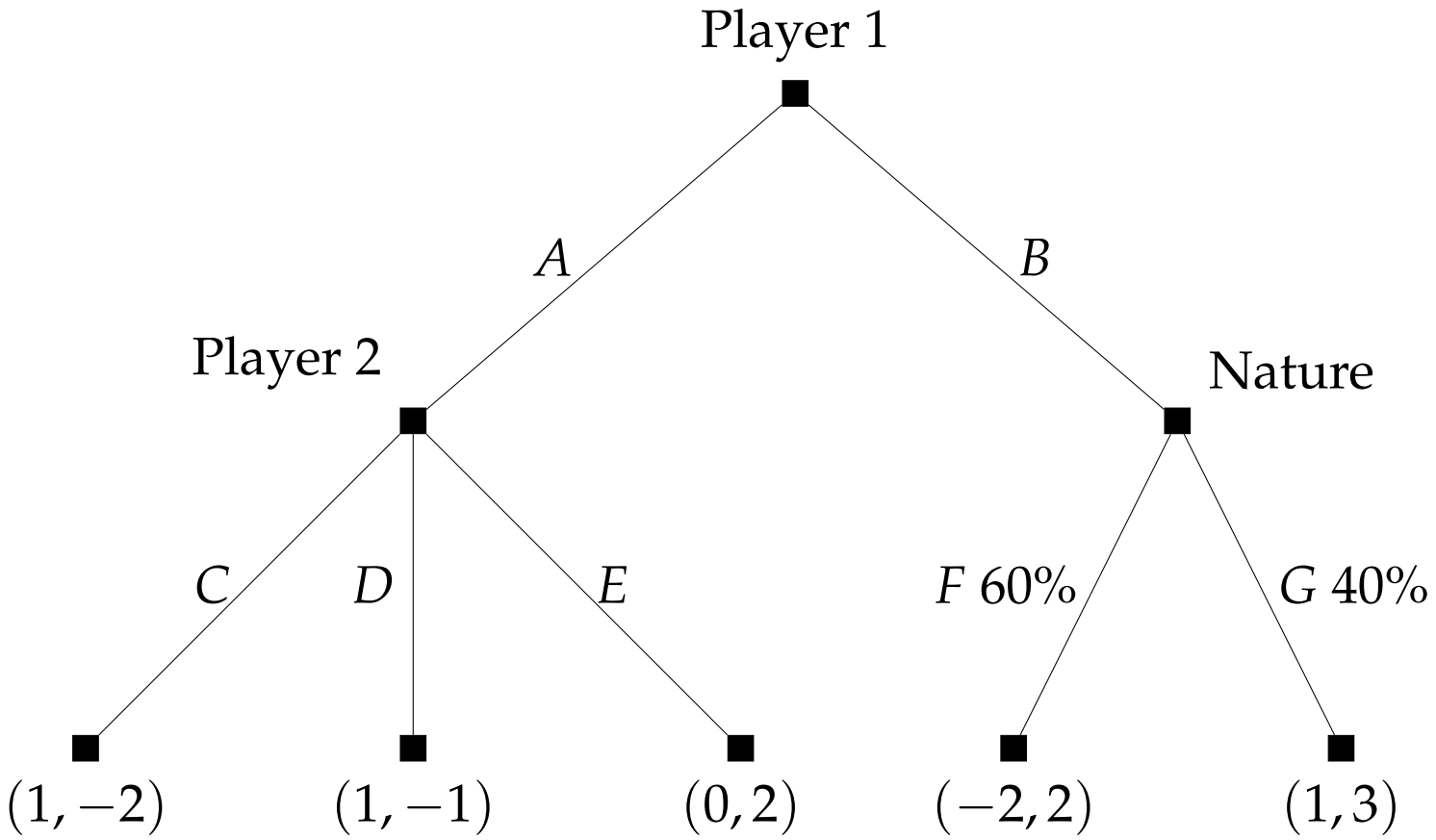
3.1 Game trees

- A game tree is a graphical representation of a sequential-move game.
  - Each decision node is marked with the player who makes the move, and the branches leading from the node, each representing a possible move by the player.
  - The initial node, or the root of the game tree, is the first decision node.



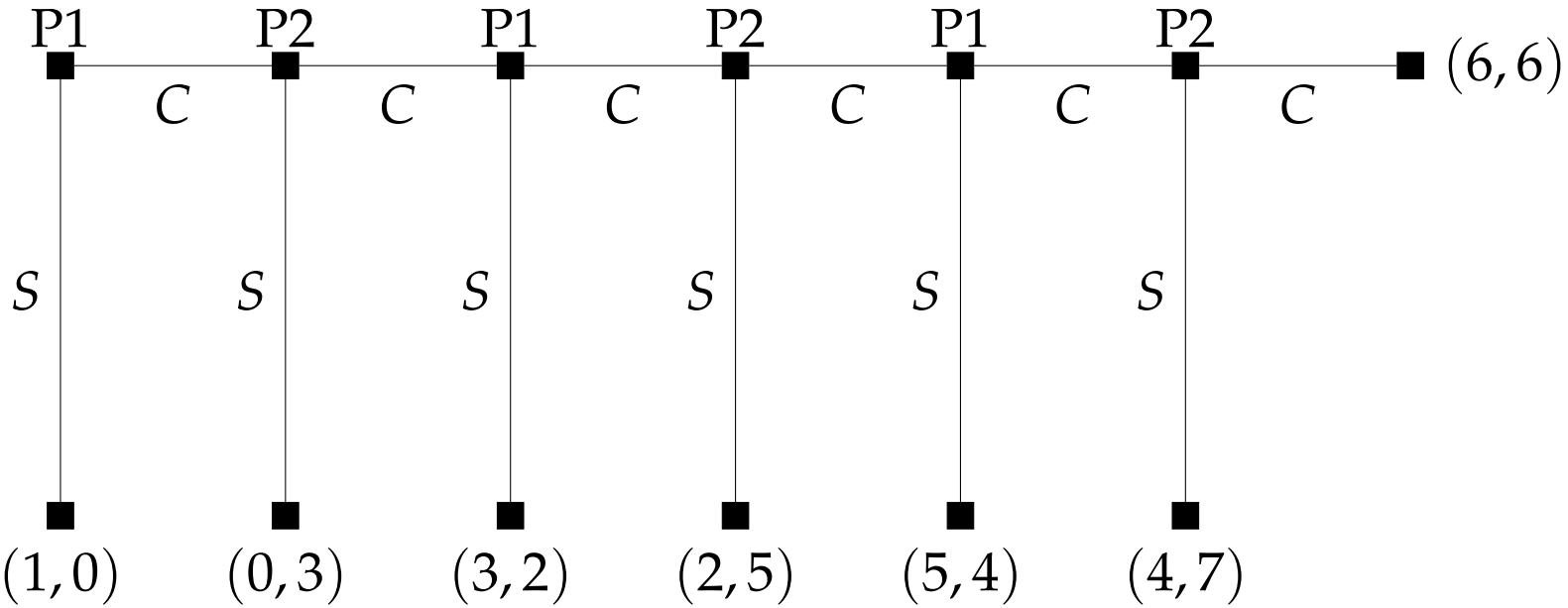
An example with 2 players and 3 decision nodes.

- Game tree continued.
  - Each terminal node is marked with the payoffs to the players, in the order of moves.
  - A decision node marked with "Nature" represents some external uncertainty outside the control of the players in the game, and each branch leading from such a node represents a possible resolution of the uncertainty and is marked with the corresponding probability.



An example with a Nature's move.

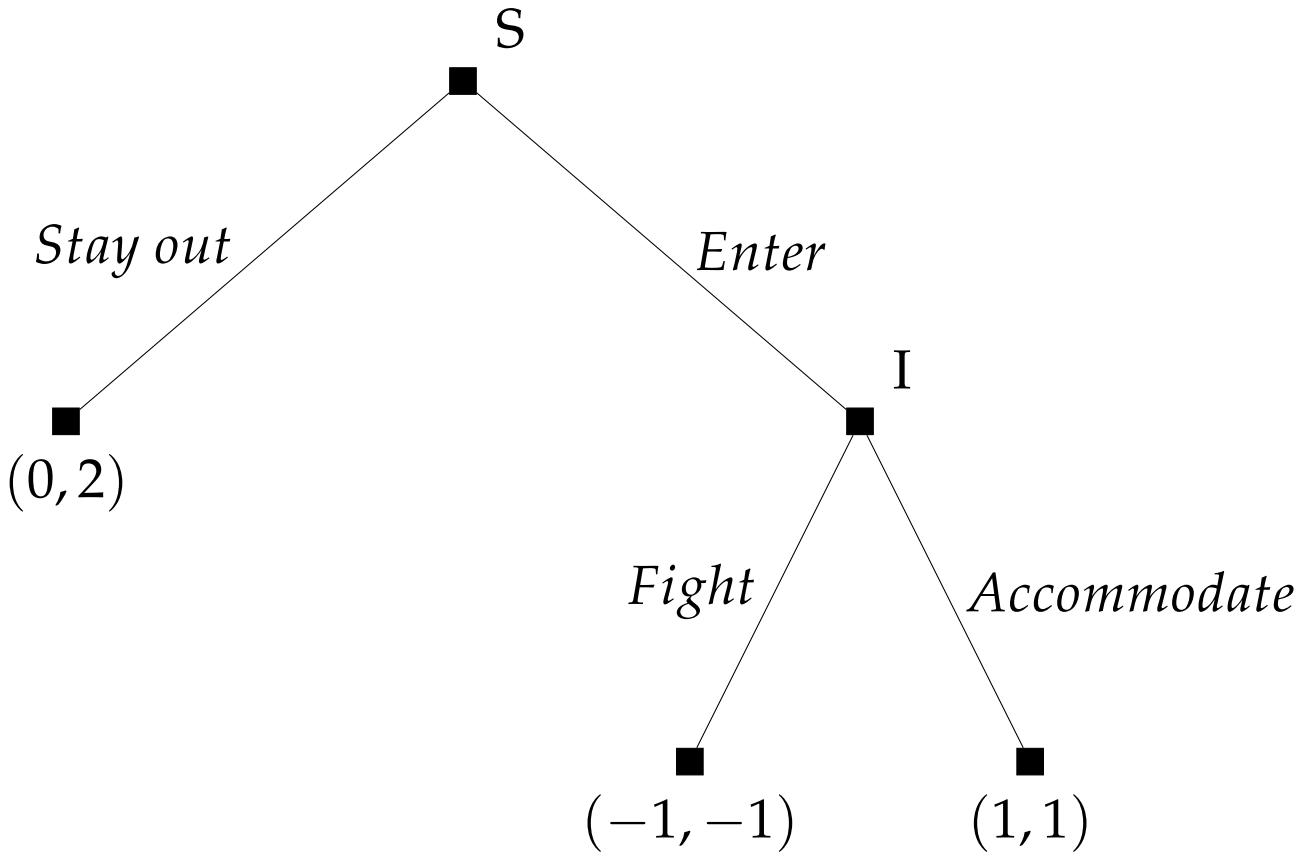
- A strategy of a player in a sequential-move game specifies a move for each decision node that belongs to the player.
  - A strategy of a player may specify moves at decision nodes that will not be reached if the player follows the strategy at earlier decision nodes.
  - This is not only required for a strategy to be complete, but also necessary for our analysis.



The Centipede Game: Player 1 has 8 strategies, 4 of which lead to $(1, 0)$.

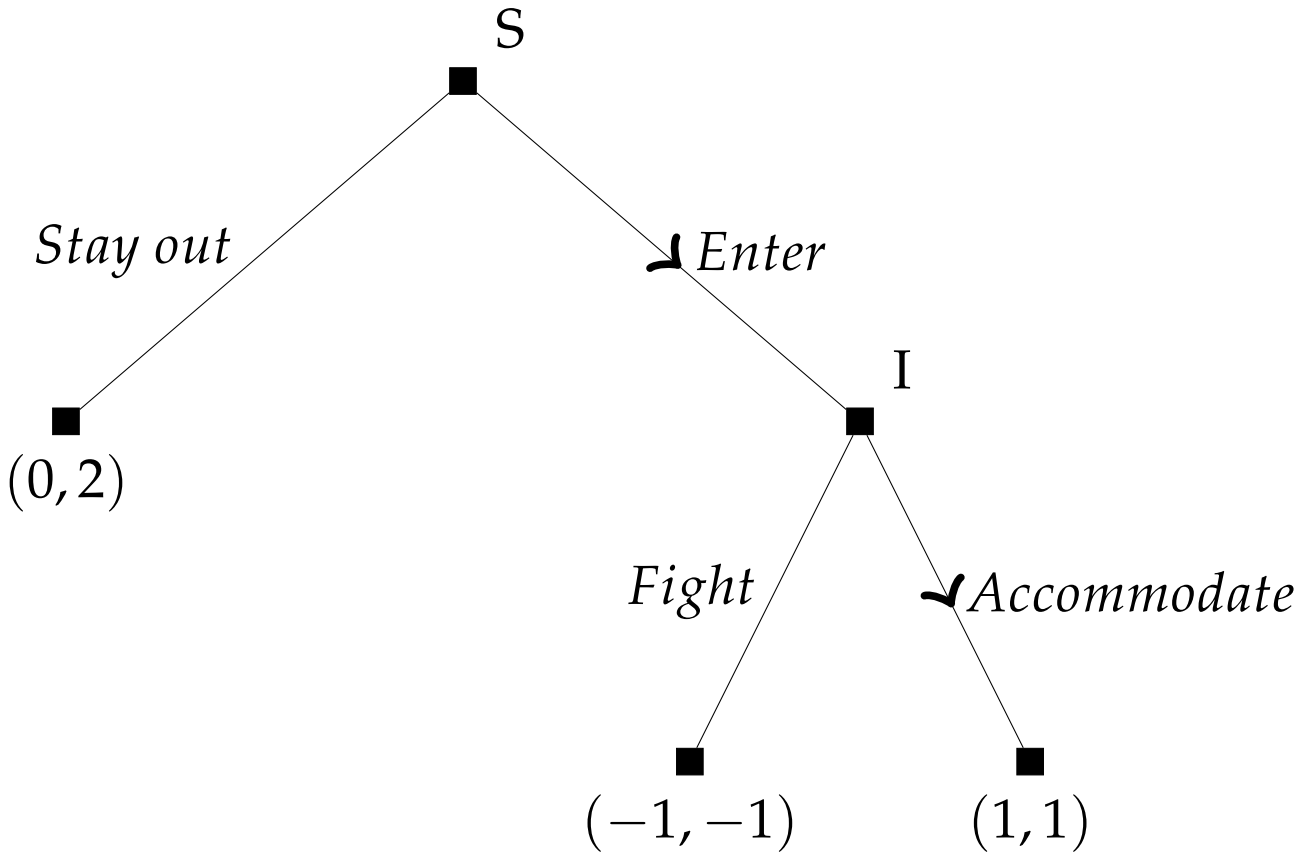
3.2 Solving games by using trees

- Entry Deterrence
  - A start-up car-sharing company S decides whether or not to enter a market monopolized by an incumbent company I. If S does not enter, nothing happens. If S enters, I has to decide whether to force S out (by cutting the price) or to accommodate S; in the first case both companies will lose money, while in the second case S will make positive profit and I will make a positive but smaller profit than if S does not enter.

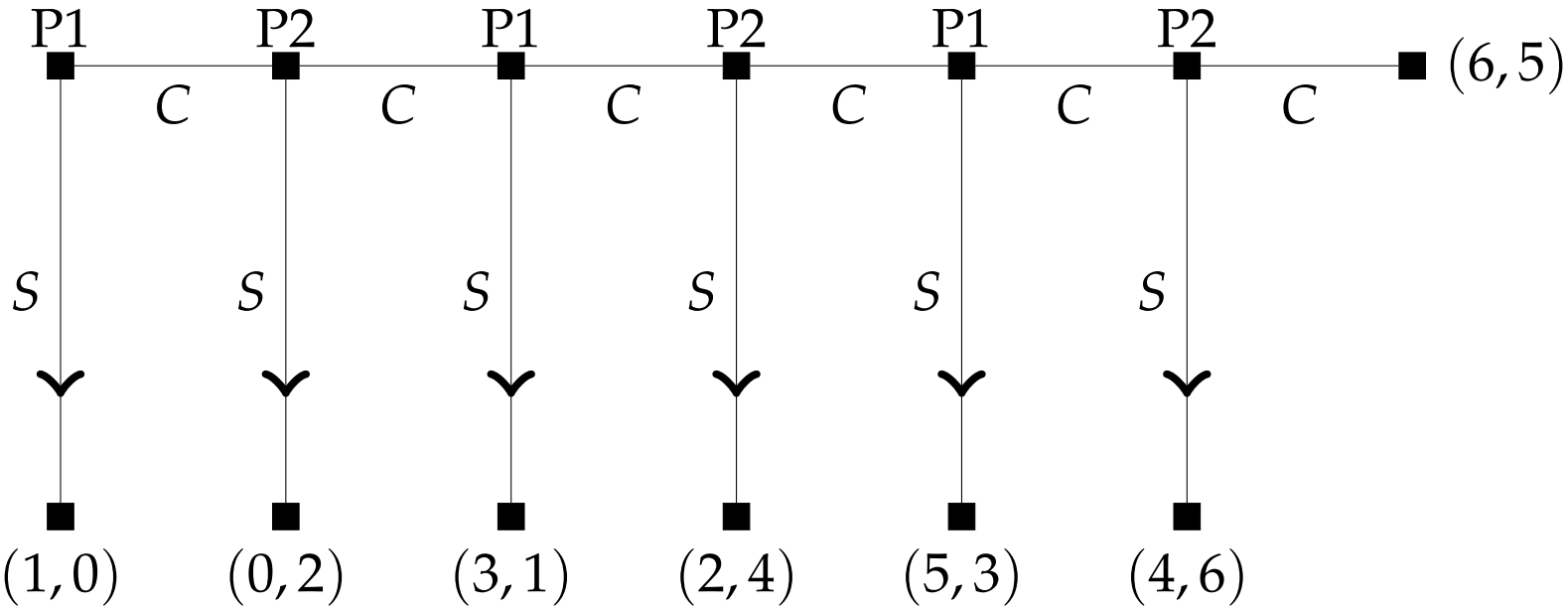


Entry Deterrence.

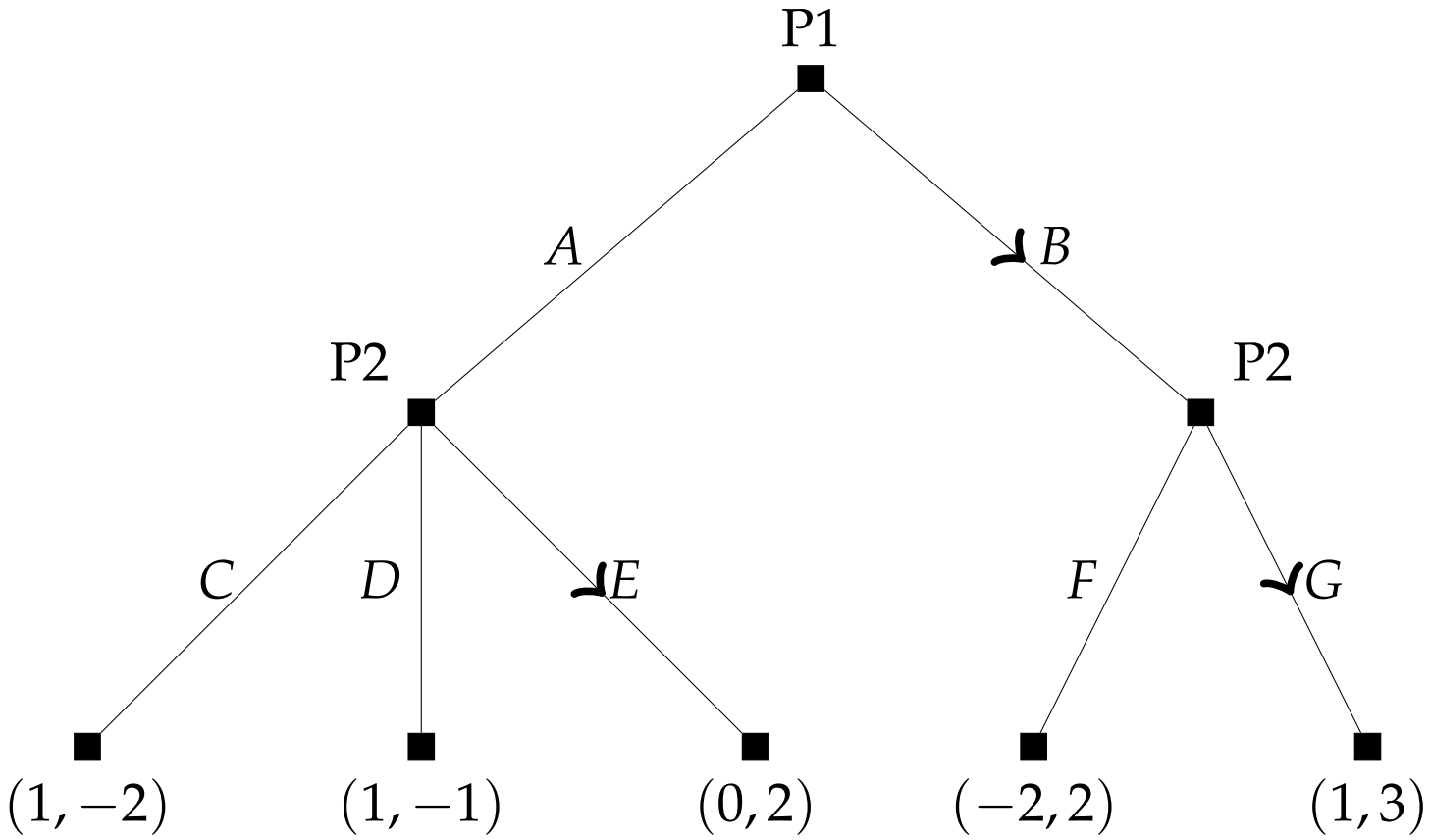
- Rollback: look ahead and reason back.
  - Rollback is a necessary implication of strategic thinking in a sequential-move game.
  - Rollback equilibrium is game theory's prediction of the strategies that will be used by players in such a game.
- Rollback: the arrowhead form.
  - Mark selected branches with arrows to find the rollback equilibrium and the equilibrium outcome.



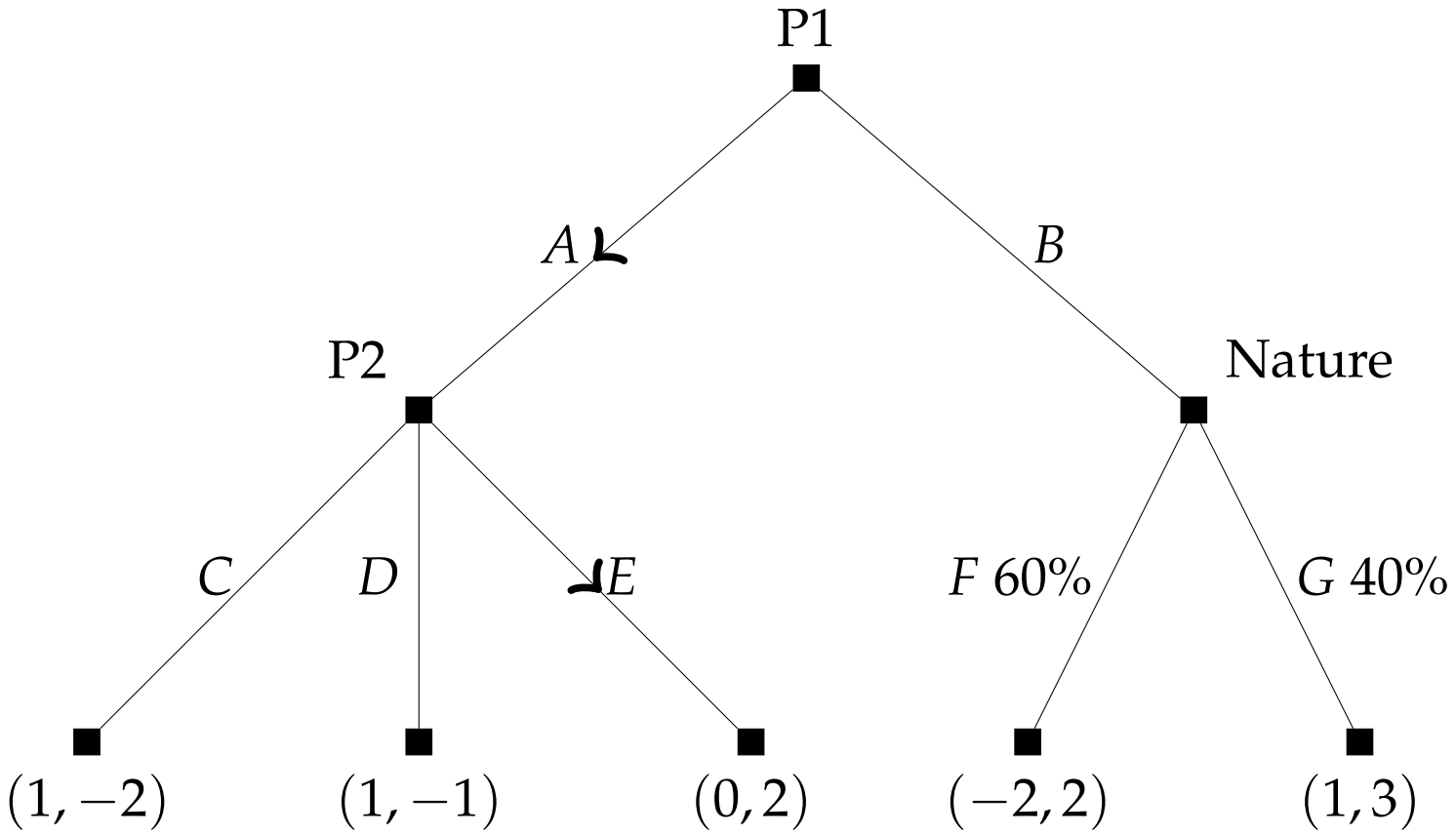
Rollback in Entry Deterrence.



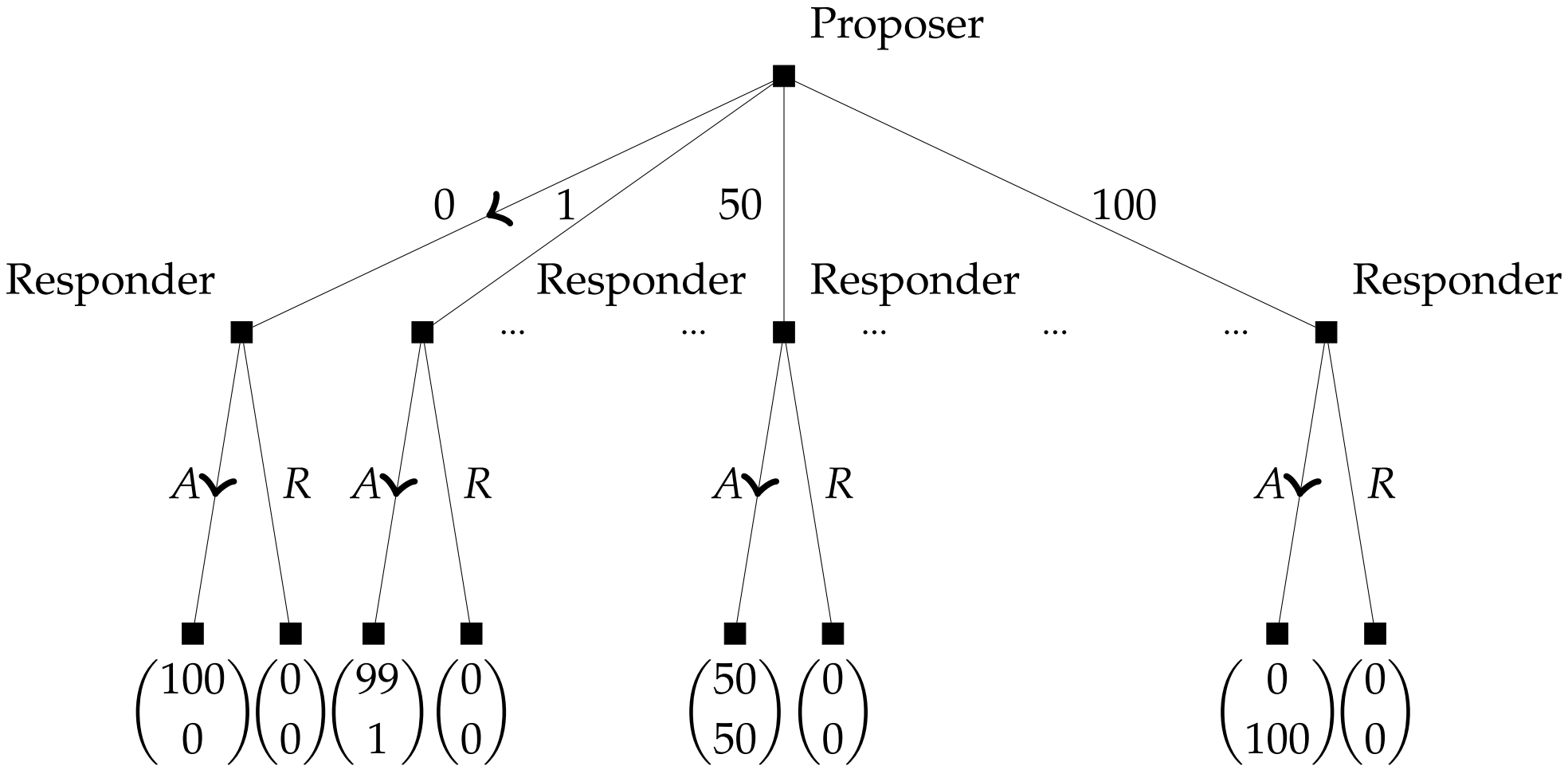
Rollback in the Centipede Game.



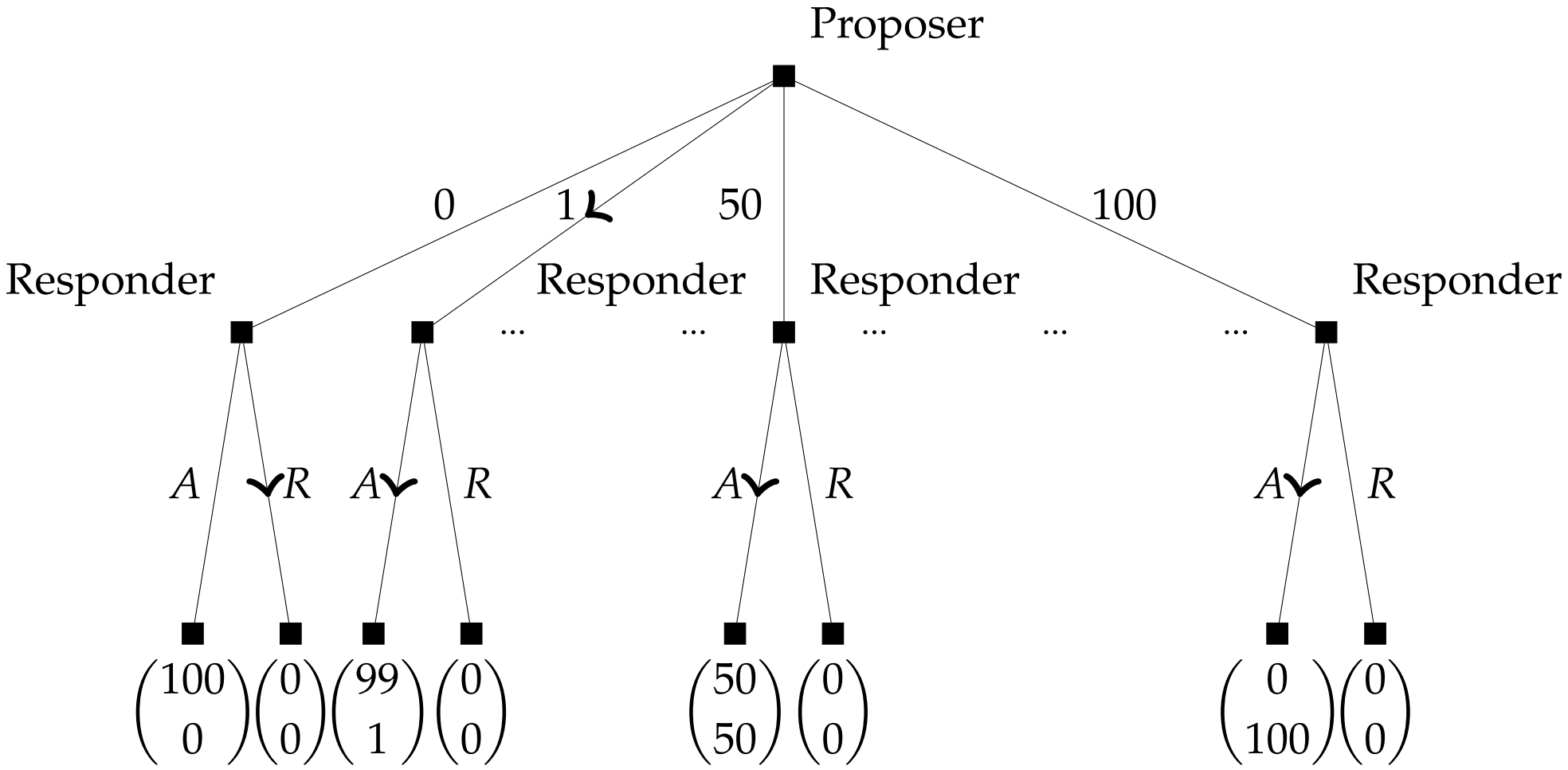
Rollback in the example with 2 last moves.



Rollback in the example with Nature's move.



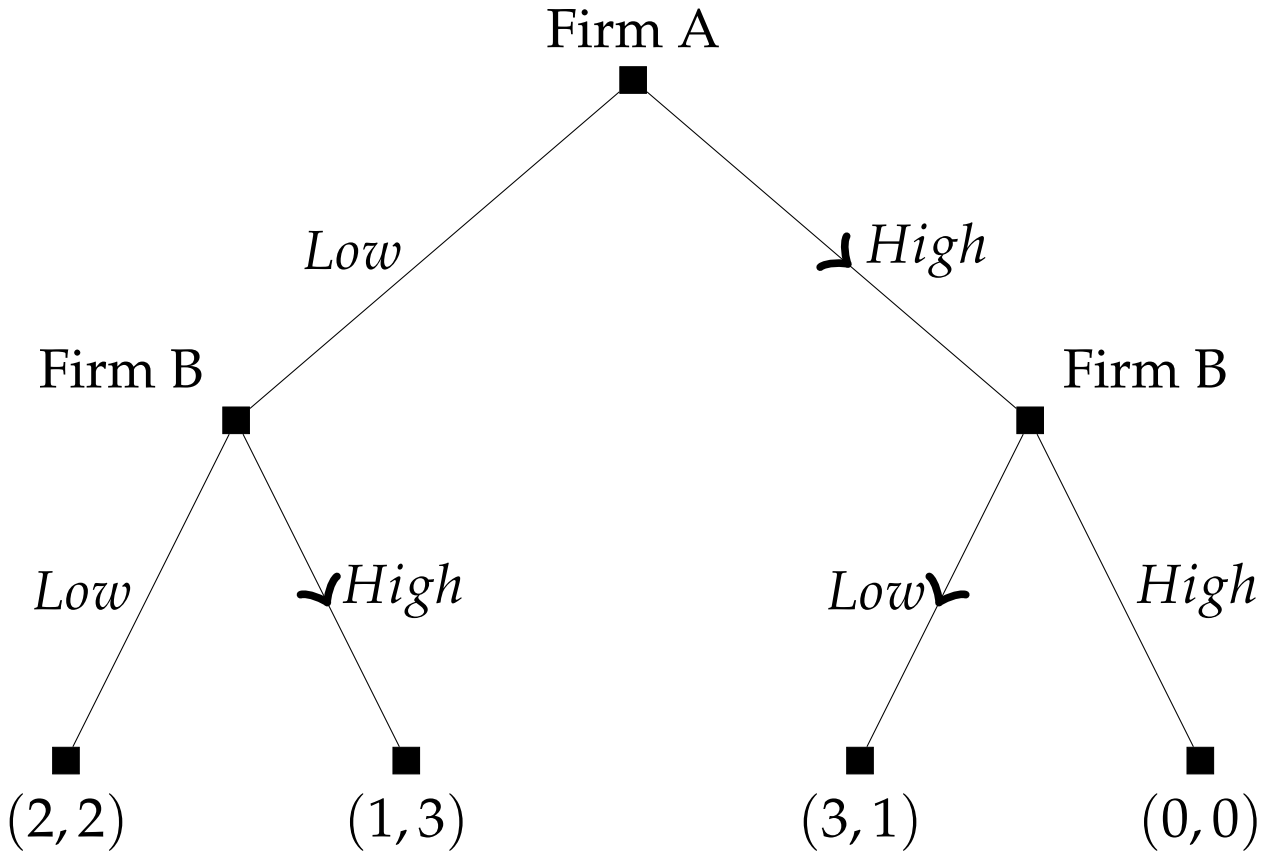
The Ultimatum Game: first rollback.



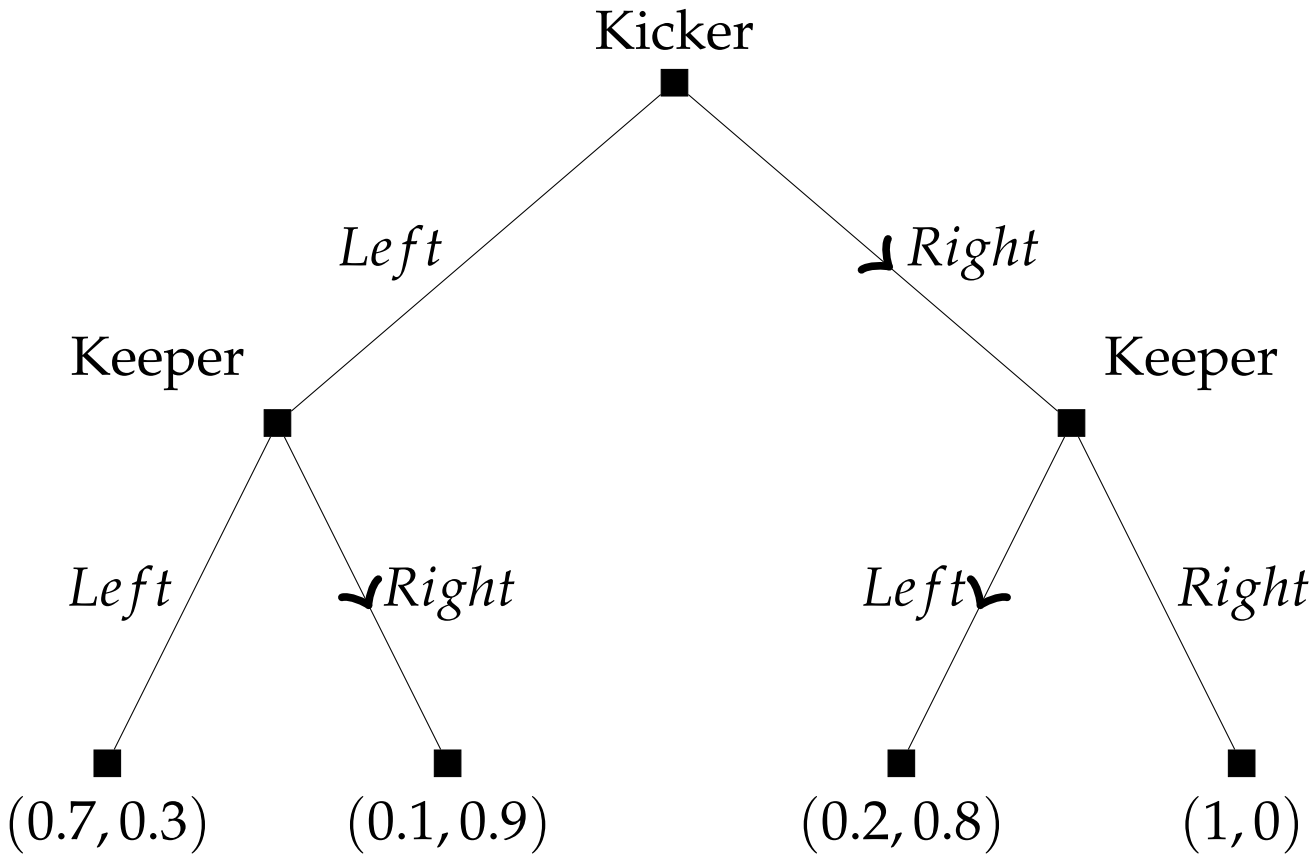
The Ultimatum Game: second rollback.

3.4 Order advantages

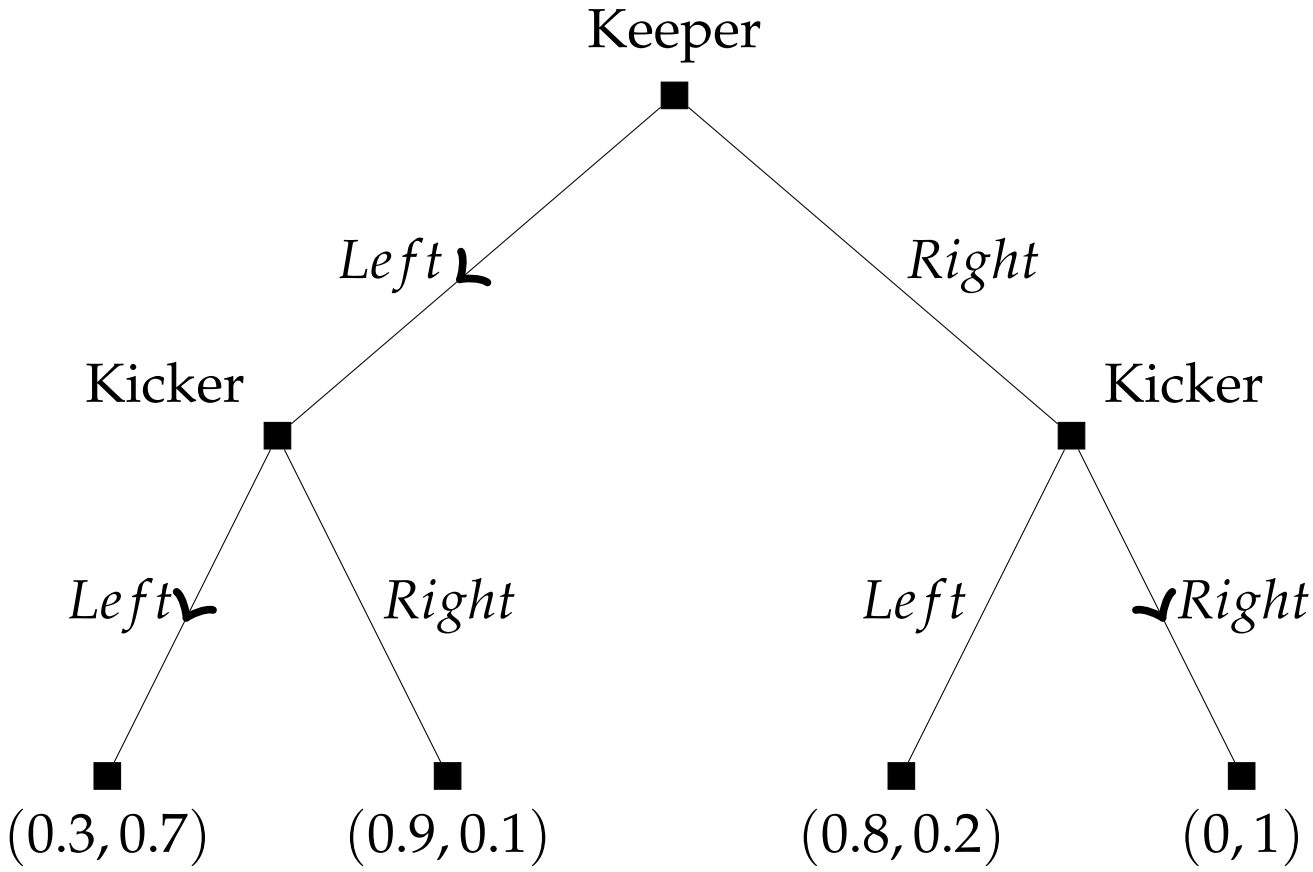
- Compare two games of same two players and same move for each player that differ only in who moves first.
  - First-mover advantage: player gets a higher payoff when he moves first than when he moves second, due to a benefit from making commitment.
  - Second-mover advantage: player gets a higher payoff when he moves second than when he moves first, due to benefit from retaining flexibility.



First-mover advantage.



Second-mover advantage: Kicker moves first.



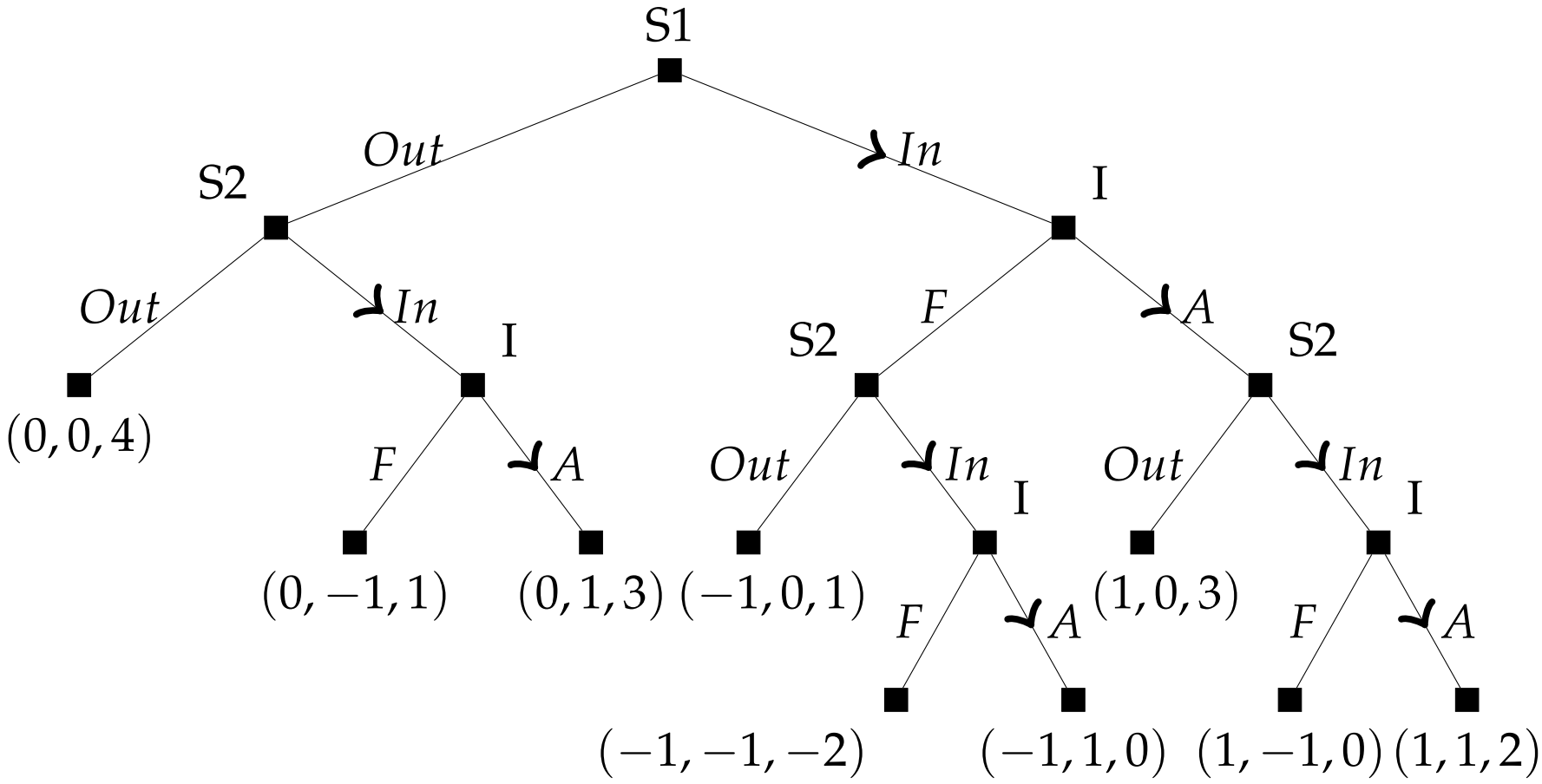
Second-mover advantage: Keeper moves first.

3.3 Adding more players

- Rollback method and rollback equilibrium generalize to more than two players.

- Chain Store Game

  – An incumbent firm I operates in K cities. In each city $k = 1, \ldots, K$, a start-up firm S_k plays Entry Deterrence with I, after observing the outcome of previous $k-1$ cities. Each S_k has cares only about the outcome in city k, while I cares about sum of its payoffs across K cities.



Chain Store Game with $K = 2$.