Econ 221
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Chapter 3. Games with Sequential Moves

- Game trees.
  - Sequential-move games with finite number of decision notes.
  - Sequential-move games with Nature's moves.

- Strategies in sequential-move games.
  - A strategy as a complete plan.
  - List of all strategies for each player in a sequential-move games.
- Rollback method and rollback equilibrium.
  - Mark game tree to find rollback equilibrium (not just equilibrium outcome).
  - Find all rollback equilibria when there are ties.

- Game tables.
  - Simultaneous-move games with two players and finite number of strategies for each player.
- Strict dominance.
  - One strategy strictly dominates another strategy for a given player.
  - A strictly dominant strategy for a given player.

- Iterated elimination of strictly dominated strategies.
  - Games solvable through iterated elimination of strictly dominated strategies.
- Weak dominance.
  - One strategy weakly dominates another strategy for a given player.
  - A weakly dominant strategy for a given player.
  - Iterated elimination of weakly dominated strategies.

- Pure-strategy Nash equilibrium.
  - Find all Nash equilibria using best response analysis in game tables.
  - Only strategies surviving iterated elimination of strictly dominated strategies can be part of Nash equilibrium.
  - Players may use weakly dominated strategies in some Nash equilibria.

- Best response function.
  - Derive best response function.
  - Find all Nash equilibria by solving the simultaneous equations given by best response functions.
  - Illustrate in a diagram Nash equilibria as intersections of best response functions.

- Rationalizability.
  - Find rationalizable strategies in a simultaneous-move game through iterated elimination of never best responses.
  - Nash equilibrium strategies are rationalizable.
  - Strictly dominated strategies are never best responses, but never best responses may not be strictly dominated.

- Information set.
  - Use information set in game trees to represent games with both sequential and simultaneous moves.
  - Degenerate versus non-degenerate information sets.
  - Adapt strategy as a complete plan to information sets.

- Nash equilibrium.
  - Derive strategic form from extensive form (game tree) by listing all strategies, and find all Nash equilibria.
  - Verify Nash equilibrium without using strategic form.
- Subgame prefect equilibrium.
  - Subgames in games that combine sequential moves and simultaneous moves.
  - Subgame perfect equilibrium forms a Nash equilibrium in every subgame.

- Find all subgame perfect equilibria by combining rollback method with Nash equilibrium.
  - Find all subgame perfect equilibrium outcomes.
  - With multiple Nash equilibria in some subgames, each Nash equilibrium requires a separate rollback and yields a different subgame perfect equilibrium.

- Subgame perfect equilibrium and Nash equilibrium.
  - In sequential-move games, subgame perfect equilibrium is same as rollback equilibrium.
  - In simultaneous-move games, subgame perfect Nash equilibrium is same as Nash equilibrium.
  - Nash equilibrium is not subgame perfect if it does not form a Nash equilibrium in some subgame.

- Mixed strategies.
  - A mixed strategy of a given player assigns a probability number to each of the player's pure strategies, with the total probabilities equal to 1.
  - Pure strategy as a degenerate mixed strategy.
- Expected payoffs.
  - Compute expected payoff to each player from using a pure strategy against a mixed strategy of his opponent.

- Mixed-strategy Nash equilibrium.
  - For simultaneous-move games with two players and two pure strategies for each player, find all the Nash equilibria, in pure strategies and in mixed strategies, by finding each player's best response in mixed strategy against the opponent's mixed strategies, and illustrate in a diagram.

- General indifference principle for finding Nash equilibrium involving mixing, when some players have more than two pure strategies, and when there are more than two players.
  - A player uses a mixed strategy in a Nash equilibrium if and only if, against equilibrium strategies used by other players, the player is indifferent among all pure strategies used in the mixing, and weakly prefers each of them to any pure strategy not used in mixing.
  - Use general indifference principle to verify mixed-strategy Nash equilibrium.

- Uncertain income.
  - An uncertain income is a random number, represented by a probability assigned to each possible income, with probabilities summing up to 1.
  - Expected income of an uncertain income is probability-weighted sum of possible incomes.

- Risk-neutrality.
  - A player is risk-neutral if the expected payoff from any uncertain income depends only on the expected income.
  - For risk-neutral player, payoff from getting the expected income of any uncertain income with probability 1 equals expected payoff from uncertain income itself.
  - Expected income can be used to represent payoff of a risk-neutral player.

- Risk-aversion.
  - A player is risk-averse if payoff from getting expected income of any uncertain income with probability 1 exceeds expected payoff from uncertain income itself.
  - For a risk-averse player, the certain income that gives player same expected payoff as the uncertain income, is lower than expected income.

- Risk-aversion and concave payoff function.
  - A risk-averse player uses a concave function to convert incomes into payoffs.
  - For uncertain incomes with only two possible incomes, illustrate expected income and certainty equivalence under a concave payoff function.

- Adverse selection and other simultaneous-move games with asymmetric information.
  - Private information of an informed player is represented by this player having multiple possible types from the point of view of an uninformed player.
  - An uninformed player knows only the probability of each possible type.
  - Represent such games in a game tree using Nature's move at beginning of game and information sets.

- Bayesian Nash equilibrium.
  - Strategy of informed player specifies a choice for each possible type; strategy of uninformed player specifies a single choice.
  - Strategies of informed player and uninformed player form best responses to each other in a Bayesian Nash equilibrium: for an informed player, this requires best responding by each type, while for an uninformed player, this requires maximizing expected payoff.

- Signaling: games with sequential moves and asymmetric information.
  - Private information of an informed player is represented by this player having multiple types; uninformed player knows only prior probability of each possible type.
  - Informed player moves first; uninformed player observed the move but not type of informed player.
  - Represent such games in a game tree using Nature's move at beginning of game and information sets.

- Beliefs of uninformed player about type of informed player.
  - To best respond, uninformed player must hold a belief after each possible first move of the informed player (information set).
  - A belief at an information set assigned a probability to each possible type of informed player.
  - Derive belief from strategy of informed player using Bayes' rule.

- Perfect Bayesian equilibrium specifies first move by each type of informed player, strategy of uninformed player, and belief at each information set, such that:
  - Each type of informed player best responds.
  - Uninformed player best responds at each information set given the belief.
  - For information set reached with positive probability according to strategy of informed player (on the path), belief is derived from informed player's strategy using Bayes' rule.

- Types of perfect Bayesian equilibrium.
  - Separating equilibrium: different types of informed make different first moves, and belief of uninformed at any information set on the path puts all the probability on corresponding type.
  - Pooling equilibrium: different types of informed make same first move, and belief of uninformed at the only information set on the path is same as prior.
  - Semi-separating equilibrium: some types of informed make different first moves while others make same move.

- Repeated Prisoners' Dilemma.
  - General formulation of stage game: *Defect* is strictly dominant, but both players are better off if both instead choose *Cooperate*.
  - Discounting: each player's payoff is discounted sum of stage game payoffs, with same discount factor between 0 and 1.

- Finite repetition.
  - Use rollback method to show unique subgame perfect equilibrium is each player playing *Defect* in each round regardless of the outcomes in previous rounds.
  - Repetition alone does not induce cooperation.
- Infinite repetition.
  - Rollback method no longer applies.
  - Compute payoff from a discounted sum of an infinite sequence of payoffs from stage games.

- Sustain cooperation through a pair of trigger strategies in infinitely repeated Prisoners' Dilemma.
  - Trigger strategy.
  - Find the critical value of discount factor above which cooperation can be sustained.

- General formulation.
  - Simultaneous-move games by many identical players, each choosing between participation and shirking.
  - Two payoff functions represent payoff to each player from each choice as function of number of participants.

- Nash equilibrium is a number of participants such that, given the number, each participant weakly prefers to participate and each non-participant weakly prefers to shirk.
  - One of the two Nash equilibrium conditions is absent if all participate, or if all shirk.
  - Find all Nash equilibria using the two payoff functions, and illustrate in a diagram.

- Social optimum.
  - Social payoff function is sum of payoffs to participants and non-participants, as a function of the number of participants.
  - Social optimum is the number of participants which maximizes social payoff function.

- Spillovers.
  - Spillovers are difference between marginal private gain, which is change in a player's payoff when the player switches to participation, and marginal social gain, which is change in social payoff function when the number of participants increases by 1.
  - Use the two payoff functions to derive an expression for spillovers.
  - Spillovers explain why Nash equilibria can differ from social optimum.

- Mixed-strategy Nash equilibria in collective-action games.
  - There is no need for coordination in mixed-strategy Nash equilibria.
  - An example: use the indifference principle to find the mixed-strategy Nash equilibrium in collective Game of Chicken.

- Mechanism design in general adverse selection problems (hidden information).
  - Principal wishes to separate different types of agent.
  - Incentive condition: each type of agent is willing to choose the option intended for that type.
  - Participation condition: each type of agent weakly prefers to participate in the mechanism instead of opting out.

- Price discrimination with two types of buyer.
  - Price-quality menu when seller observes buyer's type (full-information mechanism) does not satisfy incentive condition for type with a greater marginal valuation for quality (rich type).
  - Seller can satisfy incentive condition for rich type by reducing price for rich type in full-information menu, but can achieve the same with a smaller reduction, and thus increase profits, by reducing quality and price for the other type (poor type).

- Mechanism design under moral hazard (hidden action).
  - Principal wants agent to take particular action.
  - Incentive condition: agent is wiling to take the desired action.
  - Participation condition: agent must weakly prefer to participate in the mechanism instead of opting out.

- Wage contract with a risk-averse worker and two possible outputs.
  - Full-insurance for worker when firm observes whether worker exerts effort or not, but it violates the incentive condition because effort is costly to worker.
  - Firm can satisfy incentive condition, and thus increase profits, by linking wage to output.

- Auction versus pricing.

- Auction formats: First price versus second price.

- Auction environments: private value versus common value.

- Reserve price in auctions.

- Second price, sealed bid auctions with private values.
  - Bidding one's own valuation is weakly dominant.
- First price, sealed bid auctions with private values.
  - Bidding one's own valuation is weakly dominated by bidding below it, so is bidding above it, but there is no weakly dominant bidding strategy.
  - Equilibrium bidding strategy involves shading the bid, i.e., bidding below one's own valuation.

- Verify Bayesian Nash equilibrium in two-bidder, sealed-bid, first-price auction with private values.
  - Each bidder's type is own private valuation, which the other bidder believe is uniformly distributed on $[0, 1]$.
  - Equilibrium: bid half of own valuation.
  - Compute the expected payoff to a bidder with a fixed valuation as a function of any bid between 0 and 1, against equilibrium bidding strategy of the opponent, and show this expected payoff is maximized by a bid equal to half of the valuation.

- Winner's curse.
  - Common values: each bidder's valuation is sum of one's own estimate and estimates of opponents, which the bidder does not know.
  - Winner's curse: in any Bayesian Nash equilibrium of the auction where equilibrium bid increases with one's own estimate, winning the auction means rival bidders have low estimates, which reduces bidder's valuation upon winning the auction.