Solutions to Chapter 14 Exercises

SOLVED EXERCISES

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S1. Some examples of incentive schemes that help induce more care on the part of policy holders:

1. The insurer can provide a multiyear contract or otherwise establish an ongoing relationship in which future insurance premiums depend on current outcomes. The premium would decrease after, say, a year of no (or low) claims but would increase after a year of more (or larger) claims.
2. The insurer can provide discounts for observable actions that reduce the risk that the insured-against outcome occurs, for example, installing burglar alarm systems, taking careful-driving courses, and so on.

Insurers can also implement practices that help them detect fraud:

1. Insurers can offer rewards to people who observe fraudulent behavior by neighbors or others (for example, reckless drivers) and report it. Companies could get together and set up a Web site, for example, so individuals who observe careless behavior can post the information without having to know which specific insurance company to contact. Such a scheme requires a system for filtering out false reports.
2. Insurers could increase the consequences associated with detected fraud. A firm can, of course, void the contract and not pay anything, but in some countries, penalties include criminal prosecution and fines or even jail time.

S2. (a) Here are several examples of discounts being offered for larger quantities:

(i) A single use of a news app, versus a month's subscription for $4.99, versus an annual subscription for $14.99.

(ii) A mobile-phone plan with a larger number of minutes per month, at a lower cost per minute.

(iii) A small (100-square-inch) pizza for $12, versus a large (256-square-inch) pizza for $16.

(iv) Buy two apples, get the third free.

(v) A single drop-in yoga class costs $15, whereas an unlimited monthly pass costs $100.

(vi) A 10-pound family pack of chicken thighs costs $1.99 per pound at the supermarket, whereas a regular 2-pound package costs $3.99 per pound.

Some quantity discounts involve bundling together different types of products:

(vii) Get a printer for $50 (regular price $100) when you buy a computer for $1,000.

(viii) Purchase a tour package of flight plus hotel at a lower price than you would pay for the flight and hotel separately.

(ix) Purchase Microsoft Excel, Word, and PowerPoint as a bundle for a lower price than the sum of the three individual prices.

(x) Purchase a service/repair contract bundled together with your new automobile at a price lower than you would expect to pay if you purchased your repairs later as needed.

(xi) Pay an extra fee to have breakfast included with your hotel room, but less than you would pay to purchase breakfast in your hotel restaurant as a separate item.

(b) Those who choose a large quantity reveal themselves to be willing to purchase more of the product, with the lower price inducing them to purchase more rather than less. For example, a large family may be willing to purchase a 10-pound package of chicken thighs at a discounted per-pound price, because they like chicken enough to be willing to eat it three meals in a row. By contrast, a small family knows it can’t eat that much chicken before it spoils, so they choose a 2-pound package at a higher price per pound.

Without offering the different package sizes, the grocery would not manage to separate the two types of customers. See the next exercise for a numerical example showing how this type of screening strategy may increase the firm’s profits.

S3. (a) Let’s consider the different prices P that OWL could choose for the 300-minute plan. Looking at the customers’ values, we can see who will purchase at each price:

$P \leq \$20$: Everyone will purchase

$P > \$20$ and $P \leq \$25$: Only the Regular users will purchase

$P > \$25$: No one will purchase

There’s no point charging $P < \$20$, because that would sell to the same number of people as $P = \$20$, but at a lower price. Similarly, there’s no point charging a price strictly between $20 and $25, because that would sell to the same number of customers as $P = \$25$, but at a lower price.

So we can immediately narrow the optimal price down to two choices: either $P = \$20$ or $P = \$25$. Let’s consider each of these in turn, to see which one is optimal.

P = \$25: Sell to 50% of potential customers

Profit per purchase = \$25 – \$10 = \$15

Expected profit per potential customer = (0.5)(\$15) = \$7.50

P = \$20: Sell to 100% of potential customers

Profit per purchase = \$20 – \$10 = \$10

Expected profit per potential customer = (1)(\$10) = \$10

Since \$10 > \$7.50, we know that the optimal price for the 300-minute plan (when offered alone) is P = \$20.

(b) Similarly, let's consider the different prices P that OWL could choose for the 600-minute plan, and which customers will purchase at each price:

$P \leq \$30$: Everyone will purchase

$P > \$30$ and $P \leq \$70$: Only the Regular users will purchase

$P > \$70$: No one will purchase

By an argument similar to that given in part (a), we know that the only two choices we need to consider are P = \$30 and P = \$70. Let's consider each of these in turn, to see which one is optimal.

P = \$70: Sell to 50% of potential customers

Profit per purchase = \$70 – \$10 = \$60

Expected profit per potential customer = (0.5)(\$60) = \$30

P = \$30: Sell to 100% of potential customers

Profit per purchase = \$30 – \$10 = \$20

Expected profit per potential customer = (1)(\$20) = \$20

Since \$30 > \$20, we know that the optimal price for the 600-minute plan (when offered alone) is P = \$70.

(c) The IC_L constraint will guarantee that a Light user chooses the 300-minute plan instead of the 600-minute plan. In other words, the net payoff (benefit minus price) should be higher for the Light user when buying 300 minutes than 600 minutes: IC_L: $\$20 – P_{300} \geq \$30 – P_{600}$

(d) The IC_R constraint will guarantee that a Regular user chooses the 600-minute plan instead of the 300-minute plan. In an analogue to the above inequality, we have IC_R: $\$70 – P_{600} \geq \$25 – P_{300}$.

(e) To maximize profits, OWL should charge as much as it can while still making sure the IC_L and IC_R constraints are satisfied. In addition, OWL should make sure the participation constraints are satisfied, so that each type of customer is willing to purchase a plan rather than no plan at all. The participation constraints are PC_L: $\$20 - P_{300} \geq 0$ and PC_R: $\$70 - P_{600} \geq 0$. These can be rewritten as: $P_{300} \leq \$20$ and $P_{600} \leq \$70$.

If all that mattered were the participation constraints, then OWL would prefer to set prices of $P_{300} = \$20$ and $P_{600} = \$70$, just as we discovered in parts (a) and (b) above. However, now we must also look at the incentive-compatibility constraints to make sure that each type purchases the right type of plan from the menu of two options.

IC_L can be rewritten by adding $P_{600} - \$20$ to both sides of the inequality: $P_{600} - P_{300} \geq \$10$. That is, the difference in price between the two plans must be at least \$10, or else the Light users will buy the larger plan instead of the smaller one. We can see that this constraint is automatically satisfied with the prices we chose above to satisfy the participation constraints.

Let's also rewrite IC_R, adding $P_{600} - \$25$ to both sides of the inequality: $\$45 \geq P_{600} - P_{300}$ or $P_{600} - P_{300} \leq \$45$. In other words, for Regular users to buy the 600-minute plan, the 600-minute plan must cost no more than \$45 more than the 300-minute plan: $P_{600} \leq P_{300} + \$45$.

Note that when we just satisfied the participation constraints, we wanted to charge \$70 for the 600-minute plan and \$20 for the 300-minute plan, which will not satisfy IC_R. In order to make sure the Regular users purchase the correct plan, we must charge no more than \$20 + \$45 = \$65 for it. So, the optimal prices are: $P_{300} = \$20$ and $P_{600} = \$65$.

The average profit per potential customer is then: expected profit = (0.5)(\$20 – \$10) + (0.5)(\$65 – 10) = \$32.50.

(f) Let's consider the three cases separately. First, for part (a): Part (a) is a pooling outcome, because when OWL charges the optimal price of \$20 for the 300-minute plan, both types of customers purchase, so we can't tell what type of customer someone is.

Part (b) is a separating outcome, because when OWL changes the optimal price of \$70, only the Regular users purchase. The Light users do not, so we can distinguish the two types of users based on their behavior.

Part (e) is a separating outcome, because when OWL charges the optimal prices of \$20 and \$65, the Light users buy 300 minutes, and the Regular users buy 600 minutes. Again, we can distinguish the two types of users by their behavior.

S4. (a) To casual users, Mictel offers to sell a low-end machine for a price of $x = 4$. The firm makes a profit of 3 by doing so; it would make only 2 by selling this user a high-end machine. To intensive users, Mictel offers to sell a high-end machine for a price of $y = 8$. The firm makes a profit of 5 by doing so; it would make only 4 by selling this user a low-end machine.

(b) If producing only low-end machines, Mictel can either set $x = 4$ and sell to everybody, or it can set $x = 5$ and sell only to intensive users. Selling at $x = 4$ produces an expected profit per user of (1)(4 – 1) = 3. Selling at $x = 5$ produces an expected profit per user of (4/5)(5 – 1) = 3.2. So the higher price produces the larger profit, and Mictel should charge $x = 5$ for the low-end machine.

(c) If producing only high-end machines, Mictel either sets $y = 5$ and sells to everybody, or sets $y = 8$ and sells only to intensive users. Selling at $y = 5$ produces an expected profit per user of (1)(5 – 3) = 2. Selling at $y = 8$ produces an expected profit per user of (4/5)(8 – 3) = 4. Thus, the higher price produces a larger profit, and Mictel should charge $y = 8$ for the high-end machine.

(d) The incentive-compatibility constraints are

For casual users: $4 - x \geq 5 - y$, which simplifies to $y - x \geq 1$.

For intensive users: $8 - y \geq 5 - x$, which simplifies to $y - x \leq 3$.

(e) The participation constraints are

For casual users: $4 - x \geq 0$, which simplifies to $x \leq 4$.

For intensive users: $8 - y \geq 0$, which simplifies to $y \leq 8$.

(f) We assume, as in part (d), that the firm wants casual users to buy the low-end machine and intensive users to buy the high-end machine. If the firm had to worry only about the participation constraints and not the incentive constraints, it would set $x = 4$ and $y = 8$ in order to maximize profits. The incentive constraint for casual users would be satisfied at these prices, because $8 - 4 \geq 1$. But the incentive constraint for intensive users would be violated, because $8 - 4 > 3$.

To satisfy the incentive constraint for intensive users, Mictel would need to lower the price of the high-end machine in order to satisfy the intensive-user incentive constraint: $y \leq x + 3$. Thus, if Mictel charges $x = 4$ for the low-end machine, the maximum it can charge for the high-end machine is $y = 4 + 3 = 7$. We can verify that these two prices satisfy all four constraints.

The optimal prices are therefore $x = 4$ and $y = 7$. The company's expected profit from this policy is the weighted average between the profit earned from a casual user and the profit earned from an intensive user: (1/5)(4 – 1) + (4/5)(7 – 3) = 3.8. Note that this expected profit is lower than what the firm could get if it could costlessly identify the type of each user, as in part (a). Then the expected profit would be: (1/5)(4 – 1) + (4/5)(8 – 3) = 4.6.

(g) In part (b), we saw that the profit from selling only low-end machines is 3.2. In part (c), we saw that the profit from selling only high-end machines is 4. In part (f), we saw that the profit from selling both types of machines is 3.8. So the optimal policy is to sell the high-end machine at $y = 8$.

Note that all three possibilities give lower profit than the expected profit of 5 that the firm would get if it could identify the type of each user, as in part (a).

S5. (a) Same as the answer to Exercise S4 above. These prices don't depend on the proportion of casual users, only on users' willingness to pay.

(b) If producing only low-end machines, Mictel can either set $x = 4$ and sell to everybody, or it can set $x = 5$ and sell only to intensive users. Selling at $x = 4$ produces an expected profit per user of $(1)(4 - 1) = 3$. Selling at $x = 5$ produces an expected profit per user of $(1/2)(5 - 1) = 1.5$. So this time, the lower price produces the larger profit, and Mictel should charge $x = 4$ for the low-end machine.

(c) If producing only high-end machines, Mictel either sets $y = 5$ and sells to everybody, or sets $y = 8$ and sells only to intensive users. Selling at $y = 5$ produces an expected profit per user of $(1)(5 - 3) = 2$. Selling at $y = 8$ produces an expected profit per user of $(1/2)(8 - 3) = 2.5$. Thus, the higher price produces a larger profit, and Mictel should charge $y = 8$ for the high-end machine.

(d) Same as in Exercise S4.

(e) Same as in Exercise S4.

(f) As in Exercise S4, to satisfy the participation and incentive constraints with the highest possible prices, we set $x = 4$ and $y = 7$. The company's expected profit from this policy is the weighted average between the profit earned from a casual user and the profit earned from an intensive user: $(1/2)(4 - 1) + (1/2)(7 - 3) = 3.5$. Again, note that this expected profit is lower than what the firm could get if it could identify the type of each user, as in part (a). Then the expected profit would be: $(1/2)(4 - 1) + (1/2)(8 - 3) = 4$.

(g) In part (b), we saw that the profit from selling only low-end machines is 3. In part (c), we saw that the profit from selling only high-end machines is 2.5. In part (f), we saw that the profit from selling both types of machines is 3.5. So the optimal policy is to sell both machines at $x = 4$ and $y = 7$.

S6. (a) Same as the answer to Exercise S4 above.

(b) If producing only low-end machines, Mictel can either set $x = 4$ and sell to everybody, or it can set $x = 5$ and sell only to intensive users. Selling at $x = 4$ produces an expected profit per user of $(1)(4 - 1) = 3$. Selling at $x = 5$ produces an expected profit per user of $(1 - c)(5 - 1) = 4(1 - c)$. So the optimal price depends on whether $4(1 - c)$ is larger or smaller than 3.

The best price is $x = 5$ when $4(1 - c) \geq 3$, or $c \leq 1/4$. Otherwise, the best price is $x = 4$. In other words:

When $c \leq 1/4$, the best price is $x = 5$.

When $c > 1/4$, the best price is $x = 4$.

(c) If producing only high-end machines, Mictel either sets $y = 5$ and sells to everybody, or sets $y = 8$ and sells only to intensive users. Selling at $y = 5$ produces an expected profit per user of $(1)(5 - 3) = 2$. Selling at $y = 8$ produces an expected profit per user of $(1 - c)(8 - 3) = 5(1 - c)$. So the optimal price depends on whether $5(1 - c)$ is larger or smaller than 2.

The best price is $y = 8$ when $5(1 - c) \geq 2$, or $c \leq 3/5$. Otherwise, the best price is $y = 5$. In other words:

When $c \leq 3/5$, the best price is $y = 8$.

When $c > 3/5$, the best price is $y = 5$.

(d) Same as in Exercise S4.

(e) Same as in Exercise S4.

(f) As in Exercise S4, to satisfy the participation and incentive constraints with the highest possible prices, we set $x = 4$ and $y = 7$. Again, the company's expected profit from this policy is a weighted average between the profit earned from a casual user and the profit earned from an intensive user: $(c)(4 - 1) + (1 - c)(7 - 3) = 3c + 4 - 4c = 4 - c$.

Again, note that this expected profit is lower than what the firm could get if it could identify the type of each user, as in part (a). Then the expected profit would be $(c)(4 - 1) + (1 - c)(8 - 3) = 3 + 5 - 5c = 8 - 5c$. We can see that this expected profit is more than $4 - c$ because we can rewrite $8 - 5c = (4 - c) + (4 - 4c)$, and $(4 - 4c) > 0$ because the probability of c must be less than 1.

(g) Summarizing the results of parts (b), (c), and (f), we know that there are three possible cases we need to consider: $c \leq 1/4$, $1/4 < c \leq 3/5$, and $c > 3/5$.

(i) When $c \leq 1/4$:

Selling just the low-end machine earns a profit of $4(1 - c)$.

Selling just the high-end machine earns a profit of $5(1 - c)$.

Selling both machines earns a profit of $4 - c$.

Which of these three expected profits is highest? We can see immediately that $5(1 - c) > 4(1 - c)$, so Mictel should not sell just the low-end machine. Mictel should sell just the high-end machine if $5(1 - c) > 4 - c$, and both machines otherwise. This inequality simplifies to $c < 1/4$, which we know is true for the case we are considering. That is, when $c \leq 1/4$, we know Mictel should sell just the high-end machine.

(ii) When $1/4 < c \leq 3/5$:

Selling just the low-end machine earns a profit of 3.

Selling just the high-end machine earns a profit of $5(1 - c)$.

Selling both machines earns a profit of $4 - c$.

We can see readily that $4 - c > 3$, because $c < 1$, so Mictel should not sell just the low-end machine. Mictel should sell just the high-end machine if $5(1 - c) > 4 - c$, and both machines otherwise. Again, this inequality simplifies to $c < 1/4$, which we know is not true for this second case. That is, when c is between 1/4 and 3/5, we know Mictel should sell both machines.

(iii) When $c > 3/5$:

Selling just the low-end machine earns a profit of 3.

Selling just the high-end machine earns a profit of 2.

Selling both machines earns a profit of $4 - c$.

We see that 3 > 2, so Mictel should not sell just the high-end machine. Mictel should sell just the low-end machine if $3 > 4 - c$, and both machines otherwise. This inequality simplifies to $c > 1$, which is never true, because the proportion of casual users must be less than 1. That is, when $c > 3/5$, we know Mictel should sell both machines.

To summarize the three cases, we see that Mictel should do the following:

If $c \leq 1/4$, sell just the high-end machine at a price of $y = 8$.

If $c > 1/4$, sell both machines at prices of $x = 4$ and $y = 7$, respectively.

S7. (a) As in Exercise S3, part (a), we can conclude that there are only three sensible prices to consider for popcorn: \$1.50, \$3.50, or \$4.00. These three prices yield the following quantities sold, given individuals' values for popcorn:

P = \$1.50: Q = 3,000, so profit = (\$1.50)(3,000) = \$4,500

P = \$3.50: Q = 2,000, so profit = (\$3.50)(2,000) = \$7,000

P = \$4.00: Q = 3,000, so profit = (\$4.00)(1,000) = \$4,000

So the profit-maximizing price for popcorn is \$3.50.

Similarly, the best prices to choose for soda are \$2.50, \$3.00, or \$3.50. These three prices yield the following quantities sold:

P = \$2.50: Q = 3,000, so profit = (\$2.50)(3,000) = \$7,500

P = \$3.00: Q = 2,000, so profit = (\$3.00)(2,000) = \$6,000

P = \$3.50: Q = 3,000, so profit = (\$3.50)(1,000) = \$3,500

So the profit-maximizing price for soda is \$2.50.

To summarize, the best prices are $P_{popcorn}$ = \$3.50 and P_{soda} = \$2.50. This yields a total profit of \$7,000 + \$7,500 = \$14,500.

(b) We look at the table of valuations to determine who buys. At these prices, Cameron-type and Jessica-type customers buy both popcorn and soda. Sean-type consumers buy only soda, not popcorn.

(c) For a bundle, the valuations for each type of customer are

Cameron: \$3.50 + \$3.00 = \$6.50

Jessica: \$4.00 + \$2.50 = \$6.50

Sean: \$1.50 + \$3.50 = \$4.00

So there are just two sensible prices for Sticky Shoe to consider for a combo: \$4.00 or \$6.50.

P = \$4.00: Q = 3,000, so profit = (\$4.00)(3,000) = \$12,000

P = \$6.50: Q = 2,000, so profit = (\$6.50)(2,000) = \$13,000

Thus, the profit-maximizing price is P_{combo} = \$6.50. This yields a total profit of \$13,000, not so much as we found by selling the popcorn and soda separately as in part (a).

(d) With the combo, Cameron and Jessica buy popcorn and soda, but Sean buys nothing. The difference from part (b) is that now Sean no longer buys soda.

(e) With separate pricing, Cameron values the popcorn and soda at a total of \$6.50 and pays a total price of \$3.50 + \$2.50 = \$6.00. With the combo pricing, he pays \$6.50 for the same items, so he prefers separate pricing.

The same is true for Jessica. She buys both popcorn and soda in either case, but with separate pricing she pays \$6.00 instead of \$6.50. So she also prefers separate pricing.

Sean buys nothing with combo pricing. He buys soda at a price of \$2.50 under separate pricing, when he would be willing to pay up to \$3.50. Since he gets value from purchasing the soda, he prefers separate pricing as well.

(f) With the combo, Sticky Shoe can get Cameron and Jessica to pay their maximum willingness for popcorn and soda, earning \$6.50 per customer instead of the mere \$6.00 it earns through separate pricing. So, combo pricing produces superior profits from Cameron and Jessica. But it loses profits on Sean-type customers, because it fails to sell them soda.

Note that Sticky Shoe could offer soda separately at a price of up to \$3.50 and get Sean-type customers to purchase it. This would provide extra profit of (\$3.50)(1,000) = \$3,500, in addition to the profit it makes on the combo from Cameron- and Jessica-type customers.

Also note that neither Cameron nor Jessica is willing to buy soda at a price of \$3.50. So, the incentive-compatibility constraint is automatically satisfied for these two types of consumers. They would not consider switching from buying the combo to buying just soda, because this would make them strictly worse off.

So Sticky Shoe should sell soda separately (for \$3.50) to Sean-type consumers while selling the combo (for \$6.50) to Cameron- and Jessica-type consumers.

Should Sticky Shoe consider selling popcorn separately to Sean-type customers as well? The answer turns out to be no. The maximum price Sticky Shoe could charge the Sean-type customers is \$1.50. But at that price, both Cameron and Jessica would want to buy popcorn instead of a combo, because they are both willing to pay much more than \$1.50 for popcorn. Since they are both just indifferent between buying the combo at \$6.50 and not buying it, they would prefer buying popcorn alone to buying the combo. Sticky Shoe would then be giving up \$6.50 in revenue on each customer in order to earn just $1.50.

How can Sticky Shoe get the customers to buy the products it intends? By setting prices appropriately. In the language of this chapter, Sticky Shoe wants to satisfy incentive-compatibility constraints to get its customers to buy the intended products. As we saw above, Sticky Shoe wants to constrain Cameron and Jessica to prefer purchasing the combo to purchasing popcorn alone. If Sticky Shoe offers popcorn at only $1.50, these incentive-compatibility constraints will be violated.

Therefore, Sticky Shoe wants to sell combos to Cameron- and Jessica-type customers, and soda alone to Sean-type customers. Sticky Shoe will get the customers to select the intended products by setting prices appropriately.

(g) We saw in part (f) that the optimal price for the combo is $6.50, and the optimal price for the soda is $3.50. Sean does not want to buy the combo at these prices, and Cameron and Jessica do not want to buy the soda. If Sticky Shoe wants to offer popcorn individually as well, it must set a price high enough so that nobody wants to buy it, thus satisfying incentive-compatibility constraints for all three customer types. Since Jessica has the greatest willingness to pay for popcorn at $4.00, this means charging an individual popcorn price greater than $4.00. So, for example, the prices could be

$P_{popcorn} = \$4.50$

$P_{soda} = \$3.50$

$P_{combo} = \$6.50$

At these prices, Sticky Shoe will sell 2,000 combos (to Cameron- and Jessica-type customers), and 1,000 sodas. Its profit will be

Profit = ($6.50)(2,000) + ($3.50)(1,000) = $16,500.

(h) Separate pricing produces more profit than combo pricing, but less profit than combo-and-separate pricing (also known as “mixed bundling”). The combo allows Sticky Shoe to earn more profits on Cameron and Jessica than separate pricing does, but offering the combo alone will cause it to lose profits on Sean. Introducing the option of buying soda separately gives Sticky Shoe the opportunity to keep high profits on Cameron and Jessica without having to give up on Sean entirely. That is why mixed bundling produces the highest possible profit in this case.

S8. (a) If the company wants only low effort from the manager, it doesn’t need to worry about an incentive-compatibility constraint, because the manager prefers low effort to high effort. All the firm needs is to set the salary, s, to satisfy the manager’s participation constraint: $s \geq 0.12$ (in millions of dollars, so $120,000). No bonus is required. To maximize its profit, the firm wants to pay the manager as little as possible, so the salary is $s = 0.12$.

(b) The expected profit is the expected revenue under low effort minus the cost of compensating the manager: Profit = $0.25 \times 1 + 0.75 \times 0 - 0.12 = 0.13$ (or $130,000).

(c) Now there is both a participation constraint and an incentive-compatibility constraint to worry about, because the company wants the manager to prefer high effort. Suppose it pays the manager $x > 0.12$ if the project succeeds, but $y < 0.12$ if it fails. The manager values y at less than its monetary value because of his loss aversion: losses get twice as much weight as gains. Suppose the manager regards y as equivalent to z where $0.12 – z = 2 \times (0.12 – y)$, or $z = 2y – 0.12$. Then the manager expects $0.5 \times x + 0.5 \times z$ if the project succeeds and $0.25 \times x + 0.75 \times z$ if it fails. To incentivize the manager's effort, the firm needs $0.5x + 0.5z - 0.06 \geq 0.25x + 0.75z$, or $x \geq z + 0.24$, or $x \geq 2y - 0.12 + 0.24 = 2y + 0.12$, or $x - 2y \geq 0.12$. This is the incentive compatibility constraint.

The firm must also pay the manager enough, when he exerts the high effort, to ensure that he receives compensation to cover his previous salary plus the cost of his effort. This is the participation constraint: $0.5x + 0.5z \geq 0.12 + 0.06$, or $x + z \geq 0.36$, or $x + 2y - 0.12 \geq 0.36$, or $x + 2y \geq 0.48$.

Adding the two constraints gives $2x \geq 0.6$, or $x \geq 0.3$. The cheapest way to satisfy the two constraints is then to set $x = 0.3$ and $y = 0.09$, that is, to pay the manager $90,000 if the project fails, and $300,000 if it succeeds.

(d) With high effort, the firm's expected profit is $0.5 \times (1 – 0.3) + 0.5 \times (0 – 0.09) = 0.305$ ($305,000).

(e) The expected profit of $305,000 in part (d), when the firm induces high effort from the manager, is greater than the $130,000 expected profit with low effort from the manager. The firm prefers the incentive contract that induces high effort. (Note that if the manager's effort was verifiable, the firm would offer a contract that pays the manager 0.18 [$180,000] and requires high effort. The firm's expected profit would be $0.5 \times 1 – 0.18 = 0.32$ [$320,000]. Therefore, the firm has to give up $15,000 in expected profit to incentivize the loss-averse manager.)

S9. (a) The insurance company would like the manufacturer to implement the fire-prevention program, because it would save ($300,000)(0.01) – ($300,000)(0.001) = $2,700 in expected fire losses. The insurance company would be more than willing to cover the $80 cost of the program by offering the manufacturer a discounted insurance premium.

However, moral hazard arises because the manufacturer has an incentive to save money by not actually implementing the program, knowing that the insurer cannot tell the difference and will cover its losses anyway. The source of the moral hazard is the unobservability of the manufacturer's decision.

(b) The insurer can try to mitigate the moral-hazard problem by making the manufacturer incur some of the costs in the event of a loss. The fire-prevention program decreases the probability of loss from 0.01 to 0.001. Suppose the insurer were to make the manufacturer pay some deductible amount *x* in case of a fire loss. Then the manufacturer would implement the fire-prevention program so long as its expected savings exceeded the cost of $80.

For example, suppose the insurer specified a deductible of $10,000, so that the insurer would pay only $290,000 in case of a loss. Then the manufacturer loses $10,000 in the event of a fire, making its expected loss with the prevention program ($10,000)(0.01) = $10 and its expected loss without the prevention program ($10,000)(0.01) = $100. Thus, the expected benefits of the fire-prevention program to the manufacturer are $100 – $10 = $90, which exceeds the cost. This would solve the moral-hazard problem.

Of course, if the manufacturer wants to purchase insurance in the first place, it may well be risk averse, caring not just about expected losses but also about the utility of its expected losses. Faced with a loss of $10,000, a risk-averse manufacturer would find it even more beneficial than a risk-neutral manufacturer to reduce the probability of the loss with a fire-prevention program. Risk aversion will not reduce the insurer's ability to use a deductible to solve the moral-hazard problem. A large deductible reduces the benefits of the insurance to a risk-averse manufacturer, but $10,000 is still a much smaller loss than $300,000, and the deductible helps solve the moral-hazard problem in a way that increases the gains from trade. As noted in part (a), the insurer could split these expected gains of the fire-prevention program with the manufacturer by offering lower insurance premiums along with the $10,000 deductible.

S10. The most obvious asymmetric information in this case is the amount of salary that Mozart would be willing to accept in order to work for the emperor. If Mozart applied for a position, then he would signal himself to be eager to work for the emperor, and thus willing to work for less pay. If instead he waited for the emperor to seek him out, he would be demonstrating himself to be less interested, and thus the emperor might have to offer a higher salary in order to get him interested in the job.

Does this make sense from the point of view of the theory of signaling? Recall that for a signal to be credible, it must be somewhat costly, and less costly to the type that can signal than to the type that can't signal. The ideal situation for Mozart is one in which he knows he has other very good employment options because he is so tremendously talented, so it is not very costly to go without the employment contract with the emperor, at least for a time. Even better would be if the emperor believes that a lesser musician would be eager to apply for the position. Then the emperor would take Mozart's waiting as a credible sign that Mozart will need a high salary in order to be willing to come work for him.

Another potential source of asymmetric information in this situation is that Mozart may not truly know how much the emperor really wants him to be a court musician. By waiting for the emperor to approach him, Mozart is designing a bit of a screening mechanism—an emperor who really wants Mozart will be willing to make him an unsolicited offer. Of course, in this case, Mozart may also learn that the emperor doesn't want him so much after all, in which case he can later decide to apply for a position.

S11. (a) $\text{Profit}_L = M - 0.1Q$

$\text{Profit}_H = M - 0.16Q$

(b) IC_1: $M_1 - 0.1Q_1 \geq M_2 - 0.1Q_2$

IC_2: $M_2 - 0.16Q_2 \geq M_1 - 0.16Q_1$

(c) PC_1: $M_1 - 0.1Q_1 \geq 0$

PC_2: $M_2 - 0.16Q_2 \geq 0$

(d) If BMA has low cost, then the benefit would be $B_L = 2\sqrt{Q_1} - M_1$. If BMA has high cost, then the benefit would be $B_H = 2\sqrt{Q_2} - M_2$. The expected benefit is:

$$B = 0.4B_1 + 0.6B_2$$

$$= 0.4(2\sqrt{Q_1} - M_1) + 0.6(2\sqrt{Q_2} - M_2)$$

(e) First assume that IC_1 binds. This means that the low-cost type gets just enough payoff from choosing contract 1 in order to induce it to choose contract 1 over contract 2: $M_1 - 0.1Q_1 = M_2 - 0.1Q_2$. Next, assume that PC_2 binds. This means that the high-cost type gets just enough payoff to convince it to accept contract 2 instead of no contract at all: $M_2 - 0.16Q_2 = 0$, so $M_2 = 0.16Q_2$.

The previous calculation gives a lower bound for M_2. Next, substituting the PC_2 equation into the IC_1 equation, we have: $M_1 - 0.1Q_1 = 0.16Q_2 - 0.1Q_2$ or $M_1 = 0.1Q_1 + 0.06Q_2$. Our minimum required payments are, in terms of Q_1 and Q_2: $M_1 = 0.1Q_1 + 0.06Q_2$ and $M_2 = 0.16Q_2$.

(f) First, let's consider PC_1: $M_1 - 0.1Q_1 \geq 0$. Substituting for M_1 from part (e), we have: $0.1Q_1 + 0.06Q_2 - 0.1Q_1 \geq 0$, or $0.06Q_2 \geq 0$. Since we know the quantity Q_2 is not negative, we see that PC_1 is automatically satisfied.

Next, let's consider IC_2: $M_2 - 0.16Q_2 \geq M_1 - 0.16Q_1$. Substituting for M_1 and M_2 using the results derived in part (e), we have:

$$0.16Q_2 - 0.16Q_2 \geq 0.1Q_1 + 0.06Q_2 - 0.16Q_1$$

$$0 \geq 0.06\,Q_2 - 0.06Q_1$$

$$Q_1 - Q_2 \geq 0.$$

Thus, as long as the final contracts specify that the low-cost firm should produce a higher quantity than the high-cost firm, which we should expect, then the constraint IC_2 will be automatically satisfied.

(g) Substituting our results from part (e) into our results from part (d):

$$B = 0.4(2\sqrt{Q_1} - M_1 + 0.6(2\sqrt{Q_2} - M_2)$$

$$= 0.4(2\sqrt{Q_1} - [0.1Q_1 + 0.06Q_2]) + 0.6(2\sqrt{Q_2} - 0.16Q_2)$$

$$= 0.8\sqrt{Q_1} - 0.04Q_1 - 0.024Q_2 + 1.2\sqrt{Q_2} - 0.096Q_2$$

$= 0.8\sqrt{Q_1} - 0.04Q_1 + 1.2\sqrt{Q_2} - 0.12Q_2$ (h) This function must be maximized with respect to both Q_1 and Q_2. The first-order condition for Q_1 can be solved for Q_1 as follows:

$$\frac{dB}{dQ_1} = 0$$

$$0 = 0.8(1/2)Q_1^{-1/2} - 0.04$$

$$0.04 = 0.4Q_1^{-1/2}$$

$$0.1 = \frac{1}{\sqrt{Q_1}}$$

$$\sqrt{Q_1} = 10$$

$Q_1 = 100$And the first-order condition for Q_2 can be solved for Q_2 as follows:

$$\frac{dB}{dQ_2} = 0$$

$$0 = 1.2(1/2)Q_2^{-1/2} - 0.12$$

$$0.12 = 0.6Q_2^{-1/2}$$

$$0.2 = \frac{1}{\sqrt{Q_2}}$$

$$\sqrt{Q_2} = 5$$

$Q_2 = 25$

So the optimal menu of contracts will specify $Q_1 = 100$ and $Q_2 = 25$. Note that this satisfies the assumption we made in part (f), that $Q_1 \geq Q_2$.

(i) $M_1 = 0.1Q_1 + 0.06Q_2 = 0.1(100) + 0.06(25) = 11.5$
$M_2 = 0.16Q_2 = 0.16(25) = 4$

(j) The expected net benefit is, from part (d):

$$B = 0.4(2\sqrt{Q_1} - M_1) + 0.6(2\sqrt{Q_2} - M_2)$$

$$= 0.4(2[10] - 11.5) + 0.6(2[5] - 4)$$

$= 0.4(8.5) - 0.6(6) = 7$ (k) The main issue to be addressed by our screening mechanism is that the low-cost firm may wish to pretend to be the high-cost type, in order to try to earn higher profit.

An important principle of screening is that the low-cost contract has to be designed with extra profit to induce it not to choose the high-cost contract; we see this in the IC_1 constraint. The low-cost firm earns more profit than would be necessary if its type could be observed directly; this is the "carrot" referred to in the chapter.

A second feature is to use a "stick" to make the high-cost contract less attractive to the low-cost firm. As we will see in part (d) of Exercise S12 below, if Oceania knew that BMA had high cost, it would offer a contract for a quantity of just over 39. The high-cost contract designed above for the screening game has a smaller quantity of 25, which helps to make the high-cost contract less attractive to the low-cost type.

S12. (a) If Oceania only offers a contract designed for the low-cost type of firm, then we have no incentive-compatibility constraint to worry about. We only need to worry about the participation constraint, which is to make sure that the firm earns at least zero profit. This happens when $M - 0.1Q = 0$, or $M = 0.1Q$.

To find the optimal levels of Q and M for this contract, we find the public benefit from this contract:
$b = 2\sqrt{Q} - M = 2\sqrt{Q} - 0.1Q$

Now maximize the benefit by taking the derivative and setting it equal to zero:

$$\frac{db}{dM} = (2)(0.5)Q^{-0.5} - 0.1$$

$$0 = \frac{1}{\sqrt{Q}} - 0.1$$

$$Q = 100$$

$$M = 0.1(100) = 10$$

This gives benefit of $b = 2\sqrt{100} - 10 = 10$ when the firm accepts the contract. Notice that we have the same quantity level $Q = 100$ as in the low-cost contract for the asymmetric-information case in Exercise S11. But the payment is lower, at $M = 10$ instead of $M = 11.5$. This extra payment offered in Exercise S11 is a "carrot" offered to BMA to reveal its private information that it is a low-cost firm.

(b) With $Q = 100$, a high-cost firm would have a cost of $(100)(0.16) = 16$, but would earn payment of only $M = 10$. This would result in losses, making the firm worse off than no contract at all. So no, a high-cost firm would not want to accept the contract in part (a).

(c) The expected net benefit is 0.4 times the benefit from purchasing from the low-cost firm:

$B = 0.4(2\sqrt{Q} - M) = 0.4(2[10] - 10) = 4$

(d) We proceed as in part (a). The high-cost firm will be just barely willing to accept a contract (its participation constraint will be just barely satisfied) when $M - 0.16Q = 0$, or $M = 0.16Q$. The benefit from offering this contract will be $b = 2\sqrt{Q} - M = 2\sqrt{Q} - 0.16Q$.

This benefit will be maximized by setting its derivative equal to zero:

$$\frac{db}{dM} = (2)(0.5)Q^{-0.5} - 0.16$$

$$0 = \frac{1}{\sqrt{Q}} - 0.16$$

$$Q = 39.0625$$

$M = 0.16(39.0625) = 6.25$ (e) With $Q = 39.0625$ and $M = 6.25$, the low-cost firm would earn a profit of $6.25 - 0.1(39.0625) = 2.34375$. So yes, the low-cost firm would accept the contract.

(f) When the firm accepts the contract, the benefit is:

$b = 2\sqrt{Q} - M = 2\sqrt{39.0625} - 6.25 = 6.25$

Because both low-cost and high-cost firms will accept this contract, the benefit in part (c) will be realized with probability 1. Thus, the expected net benefit to this contract is $B = 6.25$. This is less than the benefit of $B = 7$ from offering a menu of two contracts, as in Exercise S11, part (j) above.

(g) If Oceania knew BMA's type and could offer the single best contract based on this knowledge, then it would offer the contract ($Q = 100$, $M = 10$) to the low-cost type, and the contract ($Q = 39.0625$, $M = 6.25$) to the high-cost type. This would produce an expected net benefit of

$B = 0.4(10) + 0.6(6.25) = 7.75.$

This is larger than the benefits of the other three contracts we have considered: offering an optimal contract for the low-cost type ($B = 4$), offering an optimal contract for the high-cost type ($B = 6.25$), or offering an optimal menu of two contracts ($B = 7$). The optimal two-contract menu performs better than either of the single contracts, but not so well as could be done if BMA could eliminate the asymmetric information entirely.