Solutions to Chapter 10 Exercises

SOLVED EXERCISES

If you find a blank space where an equation or figure may appear, please select that area for it to appear.

S1. (a) Figure 10.4(b) in the text shows the players' payoffs when the mob only puts lone confessors on their hit list. The difference now is that both players go on the hit list, and hence get the equivalent of an extra 20 years in jail, when they both confess. As a result, both players now get 30 years in the (Confess, Confess) outcome. The resulting payoffs are shown here:

		Wife	
		Confess	Deny
Husband	Confess	30 yr, 30 yr	21 yr , 25 yr
	Deny	25 yr , 21 yr	3 yr, 3 yr

Each player has a dominant strategy to Deny, but both players are *better off* when they both play their dominant strategy. So, this game is not a prisoners' dilemma. The unique Nash equilibrium is (Deny, Deny).

(b) For a player who has confessed, WITSEC protection improves that player's payoff to 5 years. Under the policy of granting WITSEC protection to all confessors, this results in each player getting 5 years instead of 30 years in the outcome when both confess and getting 5 years instead of 21 years when only that player confesses. The resulting payoffs are shown here:

		Wife	
		Confess	Deny
Husband	Confess	5 yr , 5 yr	5 yr , 25 yr
	Deny	25 yr , 5 yr	3 yr , 3 yr

Applying our usual best-response analysis, each player prefers Confess if the other chooses Confess (5 yrs is better than 25 years) but prefers Deny if the other chooses Deny (5 years is worse than 3 years). Thus, neither player has a dominant strategy and this cannot be a prisoners' dilemma. There are two-pure strategy Nash equilibria, (Confess, Confess) and (Deny, Deny).

The exercise did not ask you to identify the game but notice that each player gets their best possible outcome in the Nash equilibrium (Deny, Deny). Thus, this is the assurance game, discussed in Chapter 4, Section 7.B.

(c) Under the policy of granting WITSEC protection only to lone confessors, each player gets 5 years instead of 21 years when only that player confesses but continues to get 30 years in the outcome when both confess. The resulting payoffs are shown here:

		Wife	
		Confess	Deny
Husband	Confess	30 yr , 30 yr	5 yr , 25 yr
	Deny	25 yr , 5 yr	3 yr , 3 yr

Because 25 < 30 and 3 < 5, each player has a dominant strategy to Deny. Exactly as in part (a), (Deny, Deny) is the unique Nash equilibrium and this is *not* a prisoners' dilemma because both players are better off when both player their dominant strategies.

S2. False. The players are not assured that they will reach the cooperative outcome. Rollback reasoning shows that the subgame-perfect equilibrium of a finitely played repeated prisoners' dilemma will entail constant defecting.

S3. (a) The payoffs are ranked as follows: high payoff from cheating (72) > cooperative payoff (64) > defect payoff (57) > low payoff from cooperating (20). This conforms to the pattern in the text so the game is a prisoners' dilemma, as can also be seen in the payoff table given below:

		Kid's Korner	
		High price	Low price
Child's play	High price	64, 64	20, 72
	Low price	72, 20	57, 57

If the game is played once, the Nash equilibrium strategies are (Low, Low) and payoffs are (57, 57).

(b) Total profits at the end of four years = 4 × 57 = 228. Firms know that the game ends in four years so they can look forward to the end of the game and use rollback to find that it's best to cheat in year 4. Similarly, it is best to cheat in each preceding year as well. It follows that it is not possible to sustain cooperation in the finite game.

(c) The one-time gain from defecting = 72 – 64 = 8. Loss in *every* future period = 64 – 57 = 7. Cheating is beneficial here if the gain exceeds the present discounted value of future losses or if $8 > 7/r$. Thus, $r > 7/8$ (or $d > 8/15$) makes cheating worthwhile, and $r < 7/8$ lets the grim strategy sustain cooperation between the firms in the infinite version of the game. If $r = 0.25$, cooperation can be sustained.

(d) Total profits after four years = 4 × 64 = 256. With no known end of the world, the firms can sustain cooperation if $r < 7/8$ as in part (c). This answer is different from that in part (b) because the firms see no fixed end point of the game and can't use backward induction. Instead, they assume the game is infinite and use the grim strategy to sustain cooperative outcome.

(e) A 10% probability of bankruptcy translates into a 90% probability that the game continues, so $p = 0.9$. Then, for $r = 0.25$ ($d = 0.8$), $R = 39\%$. This rate would need to exceed 7/8 before cheating was worthwhile, so the firms will still cooperate in this case. For a 35% probability of bankruptcy, $p = 0.65$ and $R = 92\%$, so if bankruptcy becomes more certain, cheating becomes worthwhile.

S4. (a) Payoffs are in thousands of dollars of salary. Each manager has a dominant strategy to expend high effort. The Nash equilibrium is (High, High) with payoffs of 150 to each. This is not a prisoners' dilemma because the (Low, Low) outcome does not provide the managers with higher salaries than they receive in the Nash equilibrium:

		Manager 2	
		Low effort	High effort
Manager 1	Low effort	100, 100	80, 200
	High effort	200, 80	150, 150

(b)

Managers still have a dominant strategy to expend high effort, so the Nash equilibrium is still (High, High), but payoffs are now 90 to each in that equilibrium. The payoffs from the (Low, Low) outcome are now better than those achieved in the Nash equilibrium; this game *is* a prisoners' dilemma:

		Manager 2	
		Low effort	High effort
Manager 1	Low effort	100, 100	80, 140
	High effort	140, 80	90, 90

(c) In this version of the game, there are two Nash equilibria at (High, Low) and (Low, High). The game is now chicken rather than a prisoners' dilemma:

		Manager 2	
		Low effort	High effort
Manager 1	Low effort	100, 100	80, 120
	High effort	120, 80	70, 70

S5. (a) The game tree below shows payoffs in the order (You, Friend). The rollback equilibrium is shown by making the chosen branches thicker at each node; the equilibrium is (Don’t invest, Cheat if invest):

(b) In the repeated version, honesty could be sustained by a “grim trigger strategy,” where if your friend ever cheats you, you will never invest with him again. The friend can get an extra \$120 = 130 – 10 any one time but will lose \$10 every time thereafter. Then he will not cheat you if $10 > 120r$, or $r < 10/120 = 8.33\%$. Then $100(1 + r) < 120$ is also true, so it is optimal for you to invest in your friend’s business so long as he is following the equilibrium strategy, that is, not cheating first.

(c) If the rate of interest is 10%, the above agreement cannot be sustained. Suppose an alternative agreement is that your friend gets x and you get $130 – x$. By cheating you once, he can get an extra $130 – x$, but will lose x each period thereafter. He will not cheat you if $x > 0.1(130 – x)$, or $1.1x > 13$, or $x > 13/1.1$. So $x = 12$ will do. Note that this still leaves you a return of \$118 for your \$100, which is adequate when the rate of interest is 10%.

S6. (a) One manager is designated to choose High and the other Low. The High chooser makes a side payment to the Low chooser so that each gets [(200 – 60) + 80]/2 = 110 each period. The necessary side payment is 30 (thousand).

(b) Defection entails refusing to make the side payment, so the defector gets an extra 30 for one period. But then the game collapses to the single-shot Nash equilibrium in which payoffs are 90 to each manager, so the cost of defecting is 110 – 90 = 20 each subsequent period. Defection is beneficial if $30 > 20/r$, or if $r > 2/3 = 66.67\%$. This is unlikely to be the case.

S7. In the $k < 1$ case, (Swerve, Swerve) maximizes the player's joint payoff. Maintaining this type of cooperation, however, is essentially impossible. This game differs from a prisoners' dilemma because a defector in a prisoners' dilemma can rationally expect retaliation. When one player establishes a pattern of playing Defect, it is individually optimal for the other player also to play Defect. Therefore, a potential defector must compare the immediate gain from defecting with the future loss from the breakdown of cooperation.

In this chicken game, in contrast, if one driver succeeds in being the first to drive Straight (and will continue to do so), it is *not* rational for the other driver to retaliate; if James is going Straight, Dean's best response is to Swerve. James can thus lock in an outcome in which he achieves his most preferred result. Thus, any attempt to establish a pattern of (Swerve, Swerve) outcomes is likely to break down as each player tries to be the first to establish that he will choose Straight.

In the $k > 1$ case, a pattern in which each player alternates between Swerve and Straight is, once established, almost certain to last. Both (Swerve, Straight) and (Straight, Swerve) are Nash equilibria in a singleplay game. Thus, once the pattern of alternating actions has been established, neither driver can gain by deviating from it.

One difficulty that is likely to arise in this situation is in determining who gets the k payoff (and who gets the –1 payoff) in the first round. With either discounting or an uncertain end to the game (or an odd number of rounds), the player who drives Straight in the first round will have an advantage; if both players attempt to get this advantage, the alternating pattern may be hard to establish. Of course, either player would prefer to always drive Straight, while having the other driver respond by choosing Swerve. This Always Straight strategy, however, is not optimal if you expect the other driver to alternate.

S8. (a) The joint profit of South Korea and Japan is

$$\pi_{\text{Total}} = \pi_K + \pi_J = (P - c) \times Q = (180 - Q - 30) \times Q = 150Q - Q^2,$$

which is maximized at $Q = 75$.

(b) When each country i produces half of the Q found in part (a), each will earn

$$\pi_i = (P - c) \times q_i = (180 - 75 - 30) \times 37.5 = 2{,}812.5.$$

(c) Assume that Korea decides to defect. (Because the per-unit costs are the same, the answer will be identical if it is Japan that decides to defect.) Given that Japan is cooperating, Korea's profit function is

$$\pi_K = (P - c) \times q_K = (180 - 37.5 - q_K - 30) \times q_K = 112.5q_K - (q_K)^2,$$

which is maximized at $q_K = 56.25$. The resulting profit for Korea in that year is

$$112.5 * 56.25 - (56.25)^2 = 3{,}164.0625.$$

Japan's profits will be

$$(180 - 37.5 - 56.25 - 30) \times 37.5 = 2{,}109.375.$$

(d) Each year, Korea and Japan are playing the following game (rounding to the nearest tenth and assuming that when both defect each knows the other is also defecting, so that in that case they play their Nash strategies):

		Japan	
		Cooperate	Defect
Korea	Cooperate	2,812.5, 2,812.5	2,109.4, 3,164
	Defect	3,164, 2,109.4	2,500, 2,500

(e) The one-time gain from cheating is 3,164 – 2,812.5 = 351.5. The loss in each subsequent year is 2,812.5 – 2,500 = 312.5. If each country is using a grim-trigger strategy, to sustain cooperation the interest rate r needs to satisfy $351.5r < 312.5$, or $r < 88.9\%$.

S9. (a) Suppose that Player 1 decides to Roll (rather than Steal) with a prize pot of $\$X$. With probability 1/6, the die comes up 1 and everything is lost; the expected loss is $\$X/6$. With probability 5/6, however, the die comes up 2, 3, 4, 5, or 6 and increases the prize pot on average by \$4,000 and Player 1 has the chance to decide again whether to roll. So long as $\$X < \$20{,}000$, the expected benefit from rolling to increase the prize pot (5/6 × \$4,000) exceeds the expected loss from losing everything (1/6 × $\$X$); so, given any prize pot $\$X < \$20{,}000$, Player 1 should roll again. On the other hand, given any $\$X > \$20{,}000$, the expected increase in the prize pot is less than the expected loss from losing everything. (Moreover, should Player 1 continue to roll even though $\$X > \$20{,}000$, the expected loss from continuing to roll will only grow.) Overall, then, Player 1 finds it optimal to continue rolling so long as $\$X < \$20{,}000$ and to steal as soon as $\$X > \$\ 20{,}000$; if $X = 20{,}000$, player 1 will be indifferent between stealing and rolling one more time.

(b) By part (a), Player 1 prefers to steal now even if she believes that Player 2 is certain to roll in all future periods. Moreover, Player 1's incentive to steal is even higher if Player 2 may steal now or in the next period. (If Player 2 steals now, Player 1 gains nothing unless she steals now. If Player 2 steals in the next period, Player 1 only gets half of the prize pot when stealing next period, increasing the relative attractiveness of stealing now.) Thus, Player 1 must steal; by the same reasoning, so must Player 2.

(c) To show that this is a prisoners' dilemma, we need to show that both players have a dominant strategy (to steal now) and that they are both worse off when they both play their dominant strategies than when they both play another strategy (to roll one more time and then steal). By (a), both players are better off rolling one more time and then splitting the pot, compared to splitting \$18,000. However, both players have a dominant strategy to steal right away. To see why, suppose that Player 1 believes that Player 2 is stealing now with probability p. Stealing now gives Player 1 a payoff of $\$9,000 \times p + \$18,000 \times (1 - p) = \$18,000 - \$9,000\, p$. By contrast, since the best possible outcome after rolling is to split a prize pot of \$24,000, rolling now gives Player 1 an expected payoff less than $0 \text{ x } p + \$24,000/2 \times (1 - p) = \$12,000 - \$12,000\, p$, which is less than $\$18,000 - \$9,000\, p$ for any value of p.

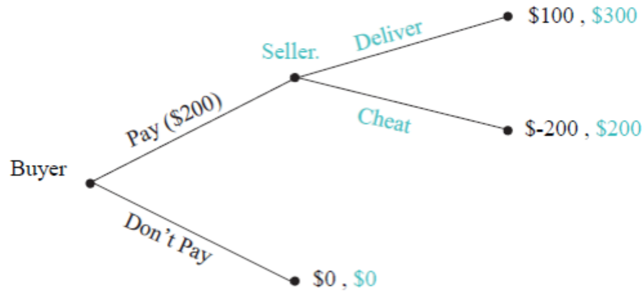
(d) Yes! Even though both players must steal when the prize pot gets sufficiently large, they have an incentive not to steal at first, if they don't believe that the other player is stealing. To see why, suppose that the prize pot is \$1,000 and that Player 1 believes that Player 2 is certain to roll. Player 1's payoff from stealing now is \$1,000. However, rolling now (and stealing next time!) gives a payoff of at least $1/6 \times \$0 + 1/6 \times (\$3,000/2) + 1/6 \times (\$4,000/2) + 1/6 \times (\$5,000/2) + 1/6 \times (\$6,000/2) + 1/6 \times (\$7,000/2) = 5/6 \times 2,500$, slightly more than \$2,000. So, this is Player 1's best response when believing that Player 2 is going to roll herself, and vice versa for Player 2. Consequently, there exists an equilibrium in which both roll at least once.

(e) Let X^* be the highest prize pool given which both players choose to roll in some rollback equilibrium. Because the players are certain to steal given any larger prize pool, they are certain to split the pool next period if they do roll now; so, if they roll, each player expects to get payoff $1/6 \times 0 + 1/6 \times (X^* + \$2,000)/2 + 1/6 \times (X^* + \$3,000)/2 + 1/6 \times (X^* + \$4,000)/2 + 1/6 \times (X^* + \$5,000)/2 + 1/6 \times (X^* + \$6,000)/2 = 5/6 \times (X^*/2 + \$2,000)$. For each player to resist the temptation to steal, it must therefore be true that $X^* < 5/6 \times (X^*/2 + \$2,000)$ or, after re-arranging, that $7/12 \times X^* < 5/6 \times \$2,000$ or $X^* < \$20,000/7$, which is about \$2,850.

Because prize-pot amounts are always multiples of \$1,000, we can conclude that both players will always steal in any rollback equilibrium as soon as the prize pot reaches or exceeds \$3,000. In particular, while a rollback equilibrium exists in which the players roll once, they never roll twice!

S10. (a) Each transaction generates profit of \$100 = (\$200-\$100). The present value of this profit stream is PV = $100 + 100 \times \delta + 100 \times \delta^2 + \cdots = 100/(1-\delta)$. Given $\delta = 2/3$, PV = \$300.

(b) The game tree when Buyer moves first is shown below. To explain the payoffs: In the outcome (Pay, Deliver), Buyer gets \$100 net benefit because \$200 is paid for something of \$300 value, while Seller gets \$300 as shown in part (a); these include the present value of future transactions. On the other hand, in the outcome (Pay, Cheat), Buyer loses \$200 and Seller gains \$200 because the money is stolen and there are no future transactions (since, by presumption, Seller will be kicked off the forum and not be allowed to sell usernames in the future).



The benefit of cheating a Buyer is that the Seller does not need to pay \$100 to create a ready-to-sell eBay username. The cost of cheating is that all *future* transactions will be lost; the present value of these lost future transactions is $100 \times \delta + 100 \times \delta^2 + \ldots = 100/(1-\delta) - 100$ = \$200. Since \$200 > \$100, the Seller will choose not to cheat if the Buyer pays; anticipating this outcome, the Buyer pays, and each player enjoys a net benefit of \$100. In the unique rollback equilibrium, Seller plays the strategy "Deliver if Buyer chooses Pay"; Buyer chooses Pay; and the rollback equilibrium outcome is (Pay, Deliver).

(c) The benefit of cheating a Buyer remains \$100, but now the cost of lost future business is only $20 \times \delta + 20 \times \delta^2 + \cdots = 20/(1-\delta) - 20$ = \$40, where \$20 = (\$120-\$100) is the profitability of each transaction. Since \$40 < \$100, the Seller will choose to cheat any Buyer who pays; anticipating this, the Buyer will not pay, and no transaction will occur. Activity on the Aspkin Forum comes to a complete halt!

(d) There are three basic ways in which the Aspkin Forum could "change the game" to incentivize Sellers not to cheat.

First, the forum could create a penalty in addition to loss of future sales that cheating Sellers would have to pay. For instance, Sellers could be required to post a bond of \$100 that they will lose if they ever cheat a Buyer. This increases the cost of cheating from \$40 (derived in part (c)) to \$140. Since \$140 > \$100, Sellers now prefer not to cheat.

Second, the forum could take steps to increase the value of future business. For instance, reduce the number of Sellers who are allowed to participate, thereby increasing the volume of sales for each remaining Seller. If each remaining Seller expects to sell (say) three usernames per year, being expelled from the site now entails losing $3 \times 20 \times \delta + 3 \times 20 \times \delta^2 + \cdots = 3 \times 20/(1 - \delta) - 3 \times 20 = \$120$ in future-year profits, enough to deter them from cheating today.

Finally, the forum could reduce Seller's temptation to cheat in the first place. For instance, suppose that the forum shared "best practices" with its Sellers, helping them to create fake eBay usernames most easily. If such efforts drove down the cost of creating such usernames, Sellers would have less to gain by cheating Buyers. For example, if the cost of creating a ready-to-sell eBay username falls from $100 to $60, the profitability of each trade will increase from $20 to $60, and the present value of future business will rise to $60 \times \delta + 60 \times \delta^2 + \cdots = 60/(1 - \delta) - 60 = \$120$, again enough to deter cheating.