Solutions to Chapter 7 Exercises

SOLVED EXERCISES

If you find a blank space where an equation or figure may appear, please select that area for it to appear.

S1. (a) The game most resembles an assurance game because the two Nash equilibria occur when the players use symmetric moves. (Here they both use the same moves to arrive at the Nash equilibria; in other games, they might use exactly opposite moves to arrive at the Nash equilibria.) In an assurance-type game, both players prefer to make coordinated moves, but there is also a preferred Nash equilibrium with higher payoffs for both players. In this game, (Risky, Risky) is the preferred equilibrium because it has higher payoffs, but there is a chance that the players will play the worse Nash equilibrium with lower payoffs. Even worse, the players might not play an equilibrium at all. Without convergence of expectations, these results can occur, and this is characteristic of an assurance-type game.

(b) The two pure-strategy Nash equilibria for this game are (Risky, Risky) and (Safe, Safe). There is also a mixed-strategy Nash equilibrium in which each player chooses Safe with probability 2/5 and Risky with probability 3/5.

S2. (a) There is no pure-strategy Nash equilibrium here, hence the search for an equilibrium in mixed strategies. Rowena's p-mix (probability p on Up) must keep Colin indifferent and so must satisfy $16p + 20(1 - p) = 6p + 40(1 - p)$; this yields $p = 2/3 = 0.67$ and $(1 - p) = 0.33$. Similarly, Colin's q-mix (probability q on Left) must keep Rowena indifferent and so must satisfy $q + 4\,(1 - q) = 2q + 3(1 - q)$; the correct q here is 0.5.

(b) Rowena's expected payoff is 2.5. Colin's expected payoff = 17.33.

(c) Joint payoffs are larger when Rowena plays Down, but the highest possible payoff to Rowena occurs when she plays Up. Thus, in order to have a chance of getting 4, Rowena must play Up occasionally. If the players could reach an agreement always to play Down and Right, both would get higher expected payoffs than in the mixed-strategy equilibrium. This might be possible with repetition of the game or if guidelines for social conduct were such that players gravitated toward the outcome that maximized total payoff.

S3. The two pure-strategy Nash equilibria are (Don't Help, Help) and (Help, Don't Help). The mixed-strategy Nash equilibrium has the following equilibrium mixtures:

$2p + 2(1 - p) = 3p + 0$ $\quad p = 2/3$

$2q + 2(1 - q) = 3q + 0$ $\quad q = 2/3$

That is, each player helps two-thirds of the time and doesn't help one-third of the time.

S4. (a) On the one hand, Evert does worse when using DL than she did before. On the other hand, Navratilova does better against DL than she did before and Evert's p-mix must keep Navratilova indifferent. Importantly, the difference between Navratilova's DL versus her CC against Evert's DL has remained the same. This suggests that Evert's p-mix may not change.

(b) $70p + 10(1 - p) = 40p + 80(1 - p)$ $\quad p = 7/10$

$30q + 60(1 - q) = 90q + 20(1 - q)$ $\quad q = 2/5$

Thus, the mixed-strategy Nash equilibrium occurs when Evert plays 7/10(DL) + 3/10(CC) and Navratilova plays 2/5(DL) + 3/5(CC).

Evert's expected payoff is 30(2/5) + 60(1 – 2/5) = 48.

(c) Compared with the previous game, Evert plays DL with the same proportion, whereas Navratilova plays DL less, going from 3/5 to 2/5. Navratilova's q-mix changes because her mix is dependent on Evert's payoffs and Evert's relative success against each of Navratilova's choices has changed. On the other hand, Evert's p-mix doesn't change because Navratilova's relative success with her two strategies against each of Evert's strategies has remained unchanged.

S5. (a) $.7p + .85(1 - p) = .8p + .65(1 - p)$ $\quad p = 2/3$

$.3q + .2(1 - q) = .15q + .35(1 - q)$ $\quad q = 1/2$

The mixed-strategy Nash equilibrium is

Batter plays 2/3(Anticipate fastball) + 1/3(Anticipate curveball) and Pitcher plays 1/2(Throw fastball) + 1/2(Throw curveball).

(b) Batter's expected payoff = .3(1/2) + .2(1 – 1/2) = 0.25.

Pitcher's expected payoff = .7(2/3) + .85(1 – 2/3) = 0.75.

(c) The mixed-strategy Nash equilibrium is now

$.75p + .85(1 - p) = .8p + .65(1 - p)$ $\quad p = 4/5$

$.25q + .2(1 - q) = .15q + .35(1 - q)$ $\quad q = 3/5$.

The pitcher's new expected payoff is .75(4/5) + .85(1 – 4/5) = 0.77, which is indeed greater than his previous expected payoff. The pitcher can increase his expected payoff because the batter is forced to adjust his equilibrium strategy in a way that favors the pitcher.

S6. (a) $0p - 1(1 - p) = 1p - 10(1 - p)$ $\quad p = 9/10$

$0q - 1(1 - q) = 1q - 10(1 - q)$ $\quad q = 9/10$

In the mixed-strategy Nash equilibrium James plays 9/10(Swerve) + 1/10(Straight), and Dean plays 9/10 (Swerve) + 1/10(Straight). James and Dean play Straight less often than in the previous game.

(b) James's expected payoff = 9/10 – 10(1 – 9/10) = – 1/10.

Dean's expected payoff = 9/10 – 10(1 – 9/10) = – 1/10.

(c) If James and Dean collude and play an even number of games where they alternate between (Swerve, Straight) and (Straight, Swerve), their expected payoffs would be 0. This is better than the mixed-strategy equilibrium, because their expected payoffs are – 1/10.

(d) James's expected payoff = 1/2[(0 –1)/2] + 1/2[(1 – 10)/2] = – 5/2.

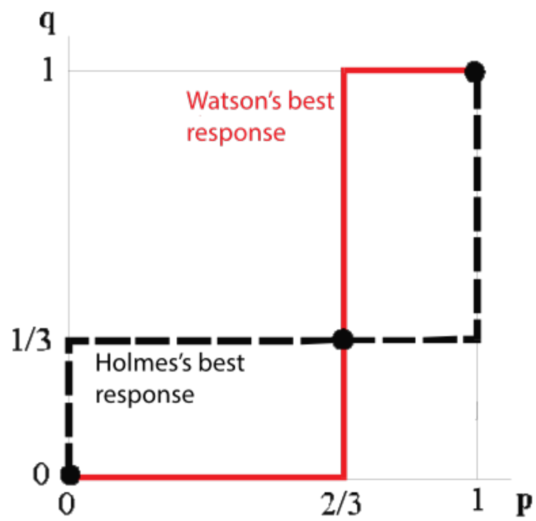
Dean's expected payoff = 1/2[(0 –1)/2] + 1/2[(1 – 10)/2] = – 5/2.

These expected payoffs are much worse than the collusion example or the mixed-strategy equilibrium. In this case both players are mixing with the wrong (that is, not the equilibrium) mixture. Neither player is best responding to the other's strategy, and in this situation—with the very real possibility of reaching the –10 payoff—the expected consequences are dire.

S7. Watson's expected payoff from choosing St. Bart's when Holmes is using his p-mix is p; Watson's expected payoff from choosing Simpson's when Holmes is mixing is $2 - 2p$. Similarly, Holmes's expected payoff from choosing St. Bart's when Watson is using his q-mix is $2q$; his expected payoff from choosing Simpson's when Watson is mixing is $1 - q$.

Watson's best response to Holmes's p-mix is to choose Simpson's ($q = 0$) for values of p below 2/3 and to choose St. Bart's ($q = 1$) for values of p above 2/3. Watson is indifferent between his two choices when $p = 2/3$. Similarly, Holmes's best response to Watson's q-mix is to choose Simpson's ($p = 0$) for values of q below 1/3 and to choose St. Bart's ($p = 1$) for values of q above 1/3. He is indifferent between his two choices when $q = 1/3$. The mixed-strategy equilibrium occurs when Holmes chooses St. Bart's two-thirds of the time and Simpson's one-third of the time ($p = 2/3$) and when Watson chooses St. Bart's one-third of the time and Simpson's two-thirds of the time ($q = 1/3$). Best-response curves are shown below:

There are three Nash equilibria indicated by the three large black dots on the diagram. At the bottom left and top right corners are the two pure strategy Nash equilibria: (Simpson's, Simpson's) where $p = q = 0$, and (St. Bart's, St. Bart's) where $p = q = 1$. The mixed strategy Nash equilibrium occurs in the center at $p = 2/3$, $q = 1/3$. Expected payoffs for Holmes and Watson are

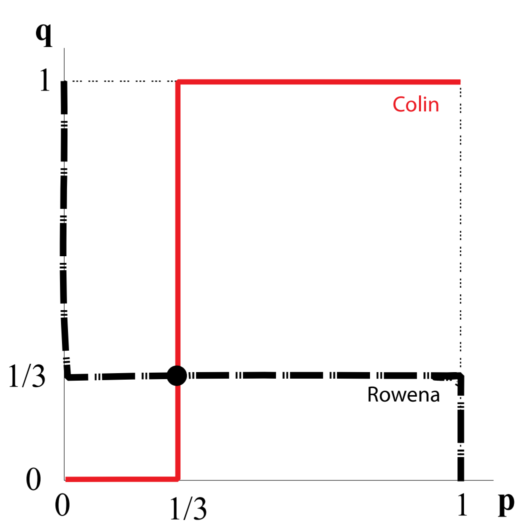


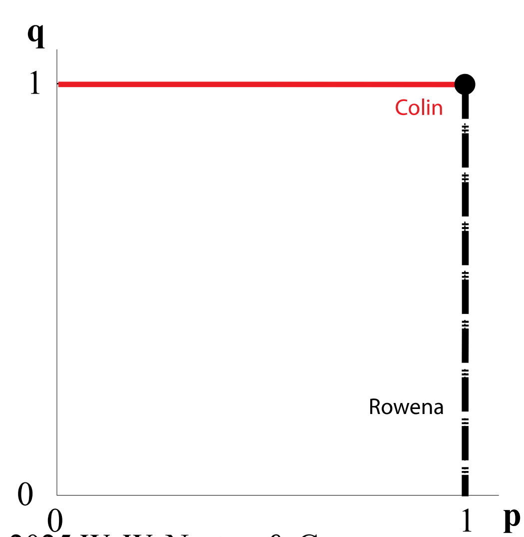
2/3 each in this last equilibrium. Both players would prefer either of the pure-strategy Nash equilibria. If they can coordinate their randomization in some way so as to alternate between the two pure-strategy Nash equilibria, they can achieve an expected payoff of 1.5 rather than the 2/3 that they achieve in the mixed-strategy equilibrium.

S8. (a) When $x < 1$, No is a dominant strategy for both players, so (No, No) is the unique Nash equilibrium; there is no equilibrium in mixed strategies.

(b) There is a mixed-strategy Nash equilibrium when $x > 1$. In that MSE, Yes will be played by both players with probability $1/x$. To solve for p, use $px = p + 1(1 - p)$ to find $x = 1/p$. Similarly, for q: $qx = q + 1(1 - q)$ gives $x = 1/q$. In the mixed-strategy Nash equilibrium, Rowena plays $1/x$(Yes) + $(1 - 1/x)$(No) and Colin plays $1/x$(Yes) + $(1 - 1/x)$(No).

(c) This is an example of an assurance-type game because there are two Nash equilibria in pure strategies where the players coordinate on the same strategy.

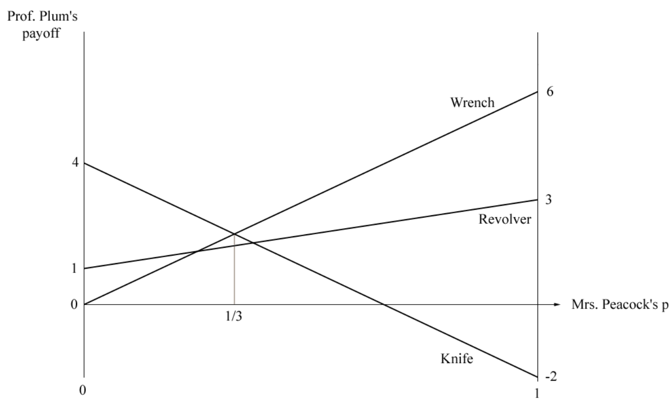




(d) The graph for $x = 3$ is at left, below. Note the unique mixed-strategy Nash equilibrium at $p = 1/3$, $q = 1/3$:

(e) Graph for $x = 1$ is at right, above. Note the unique *pure*-strategy Nash equilibrium at $p = 1$, $q = 1$ (Yes, Yes).

S9.



(a)

(b) Revolver yields a higher expected payoff than Knife when

$1 + 2p > 4 - 6p \quad \Rightarrow \quad p > 3/8.$

(c) Revolver yields a higher expected payoff than Wrench when

$1 + 2p > 0 + 6p \quad \Rightarrow \quad p < 1/4.$

(d) Professor Plum will use only Knife and Wrench in his equilibrium mixture, because Revolver is dominated by a mixture of those two strategies. He can always get a higher payoff from either Knife or Wrench.

(e) Eliminating Revolver from consideration, we get the two-by-two table:

		Professor Plum	
		Knife	Wrench
Mrs. Peacock	Conservatory	2, –2	0, 6
	Ballroom	1, 4	5, 0

Let q be the probability that Professor Plum plays Knife. The mixed-strategy Nash equilibrium occurs where

$$-2p + 4(1 - p) = 6p + 0(1 - p) \quad \Rightarrow \quad p = 1/3$$

$$2q + 0(1 - q) = 1q + 5(1 - q) \quad \Rightarrow \quad q = 5/6.$$

Mrs. Peacock plays 1/3(Conservatory) +2/3(Ballroom), and Professor Plum plays 5/6(Knife) + 1/6(Wrench).

S10. (a) To find Bart's equilibrium mix, set Lisa's payoffs from each of her pure strategies, against Bart's p-mix, equal to one another. In his p-mix, Bart will play Rock with probability p_1, Scissors with probability p_2, and Paper with probability $1 - p_1 - p_2$. Lisa's payoff from Rock against this p-mix is $10p_2 - 10(1 - p_1 - p_2)$; her payoff from Scissors against the p-mix is $-10p_1 + 10(1 - p_1 - p_2)$; and her payoff from Paper against the p-mix is $10p_1 - 10p_2$. Equating the last two of these payoffs yields $-10p_1 + 10(1 - p_1 - p_2) = 10p_1 - 10p_2$, which simplifies to $-20p_1 - 10p_2 + 10 = 10p_1 - 10p_2$, or $10 = 30p_1$ or $p_1 = 1/3$. Then equating the first two payoffs yields $10p_2 - 10(1 - p_1 - p_2) = -10p_1 + 10(1 - p_1 - p_2)$ or $-10 + 20p_2 + 10p_1 = -20p_1 - 10p_2 + 10$. Rearranging and substituting 1/3 for p_1 yields $30p_2 = 10$, or $p_2 = 1/3$ also. Then we get $1 - p_1 - p_2 = 1/3$ as well. Bart's equilibrium mix entails playing each strategy 1/3 of the time or with probability 33 1/3. The symmetry of the game guarantees that Lisa's equilibrium mix is the same.

(b) Check Bart's payoffs from each of his pure strategies against Lisa's mix:

Payoff from Rock: (0)(0.4) + (10)(0.3) + (–10)(0.3) = 0

Payoff from Scissors: (–10)(0.4) + (0)(0.3) + (10)(0.3) = –1

Payoff from Paper: (10)(0.4) + (–10)(0.3) + (0)(0.3) = 1

Bart's expected payoff from using the pure-strategy Paper exceeds his expected payoffs from his other two strategies. He should play only Paper when Lisa uses the mix described. Presumably, Lisa has chosen a nonequilibrium mix, so Bart is not indifferent among his available pure strategies.

S11. As shorthand notation, we will use the letters R, P, S, F, and W, as algebraic symbols to denote the probabilities with which each of the five weapons is played. Moreover, we will use the notation (R, P, S, F, W) to denote a mixed strategy; so, for instance, (0.5, 0.5, 0, 0, 0) is the mixed strategy in which Rock and Paper are each played half the time. This notation allows us to express the expected payoff that a player gets from each of their pure strategies in an especially convenient way.

Let (R, P, S, F, W) denote Bob's mixed strategy. Because Rock beats Scissors and Water but loses to Paper and Fire, Ann's expected payoff from Rock is

ROCK: 10 $(\boldsymbol{S} + \boldsymbol{W} - \boldsymbol{P} - \boldsymbol{F}) = 10(S + W) - 10(P + F) + 0R$

Similarly, because Paper only beats Rock and Water, Ann's expected payoff from Paper is

PAPER: $10(\boldsymbol{R} + \boldsymbol{W} - \boldsymbol{S} - \boldsymbol{F})$

Because Scissors only beats Paper and Water, Ann's expected payoff from Scissors is

SCISSORS: $10(\boldsymbol{P} + \boldsymbol{W} - \boldsymbol{R} - \boldsymbol{F})$

Because Fire beats Rock, Paper, and Scissors, Ann's expected payoff from Fire is

FIRE: $10(\boldsymbol{R} + \boldsymbol{P} + \boldsymbol{S} - \boldsymbol{W})$

Finally, because Water only beats Fire, Ann's expected payoff from Water is

WATER: $10(\boldsymbol{F} - \boldsymbol{R} - \boldsymbol{P} - \boldsymbol{S})$

(a) When Bob uses the strategy (R, P, S, F, W) = (0, 0, 0, 0.5, 0.5), we can use the five formulas above to determine Ann's expected payoff from each of her five weapons. In particular: Rock, Paper, and Scissors each yield expected payoff 10(W - F) = 0, since they beat Water as often as they lose to Fire; Fire yields expected payoff -10(0.5) = -5 since Fire loses half the time and ties half the time. Finally, Water yields expected payoff 10(0.5) = 5 since Water wins half the time and ties half the time. We conclude that **Water is Ann's only optimal weapon**.

(b) Now suppose Bob's strategy (R, P, S, F, W) = (1/4, 0, 0, 5/12, 1/3). Plugging these numbers into our formulas for Ann's expected payoffs, and using the fact that 1/4 = 3/12 and 1/3 = 4/12, we get

ROCK: $10\,(\boldsymbol{S} + \boldsymbol{W} - \boldsymbol{P} - \boldsymbol{F}) =$ 10(4-5)/12 = -10/12

PAPER: $10\,(\boldsymbol{R} + \boldsymbol{W} - \boldsymbol{S} - \boldsymbol{F}) =$ 10(3+4-5)/12 = 20/12

SCISSORS: $10\,(\boldsymbol{P} + \boldsymbol{W} - \boldsymbol{R} - \boldsymbol{F}) =$ 10(4-3-5)/12 = -40/12

FIRE: $10\,(\boldsymbol{R} + \boldsymbol{P} + \boldsymbol{S} - \boldsymbol{W}) =$ 10(3-4)/12 = -10/12

WATER: $10\,(\boldsymbol{F} - \boldsymbol{R} - \boldsymbol{P} - \boldsymbol{S}) =$ 10(5-3)/12 = 20/12

We conclude that **Paper and Water are both optimal weapons for Ann**.

(c–d) There are two basic ways to derive the mixed-strategy equilibrium in a game.

Approach #1: Guess and verify. Sometimes you may have an intuition about what the equilibrium mixed strategies are likely to be. Given any particular guess for the players' mixed strategies, you can directly verify whether those strategies constitute an equilibrium. In particular, your guesses constitute a mixed-strategy equilibrium so long as (i) each player is indifferent between all the pure strategies in their mixture and (ii) each pure strategy *not* in their mixture does not give them a higher payoff than the pure strategies in their mixture. For instance, in the basic RPS game in which each weapon beats one other weapon, the symmetry of the game makes R = P = S = 1/3 a natural guess. Moreover, so long as the other player is following this mixed strategy (randomizing equally among the three weapons), Rock, Paper, and Scissors all generate zero expected payoff, satisfying the indifference condition; so, R = P = S = 1/3 is indeed an equilibrium strategy.

Part (c) asks you to make a guess about what the mixed-strategy equilibrium will be here. The asymmetry of the game, with Fire beating three things and Water only beating one, makes it hard to come up with a *confident* guess about what the equilibrium mixed strategy will be. That said, given that Rock, Paper, and Scissors continue to behave symmetrically, you might have guessed (correctly) that R = P = S.

Approach #2: Solve the system of indifference conditions. When guess-and-verify isn't working, you can still use the indifference conditions as a system of equations and solve directly for the equilibrium probabilities. For instance, in the basic RPS game, each of the three weapons generates expected payoff

ROCK: $10\,(\boldsymbol{S} - \boldsymbol{P})$

PAPER: $10\,(\boldsymbol{R} - \boldsymbol{S})$

SCISSORS: $10\,(\boldsymbol{P} - \boldsymbol{R})$

In any mixed-strategy equilibrium, all weapons that are sometimes used must generate the same expected payoff. Here, for Rock, Paper, and Scissors to generate the same expected payoff, we must have $S - P = R - S = P - R$, which is only possible if $R = P = S$. And since R, P, S are probabilities, they have to add up to one, confirming that $R = P = S = 1/3$ is a mixed-strategy equilibrium.

To derive the equilibrium mixture used by both players in RPSFW, we leverage the fact that every action in a player's equilibrium mixture must give the same expected payoff. In particular, the five probabilities (for R, P, S, F, and W) must be chosen in such a way that the five expressions for expected payoffs (labelled **ROCK**, ... , **WATER** derived above) are all equal, while also satisfying the property that R + P + S + F + W = 1 since these are probabilities. There are a number of ways to solve the resulting system of equations, but we will do so through a series of simple observations and small derivations:

1. **ROCK** = **PAPER** implies that $S = \frac{P + R}{2}$

Because players must be indifferent between Rock and Paper, we have

$$10\,(S + W - P - F) = 10\,(R + W - S - F)$$

which, after dividing by 10 and cancelling the W and F terms, implies that $S - P = R - S$ or, equivalently, 2S = P + R or $S = \frac{P + R}{2}$.

2. **PAPER = SCISSORS** implies that $R = \frac{S + P}{2}$

Because players must be indifferent between Paper and Scissors, we have

$$10\,(R + W - S - F) = 10\,(P + W - R - F)$$

which implies that $R - S = P - R$ or, equivalently, $R = \frac{S + P}{2}$.

3. **SCISSORS = ROCK** implies that $P = \frac{R + S}{2}$

Because players must be indifferent between Scissors and Rock, we have

$$10\,(P + W - R - F) = 10\,(S + W - P - F)$$

which implies that $P - R = S - P$ or, equivalently, $P = \frac{R + S}{2}$.

4. Points 1–3 imply that R = P = S.

Point 1 says that S must be equal to the *average* of P and R, while Points 2–3 similarly require that P be the average of S and R and that R be the average of P and S. The only way this is possible if all three numbers R, P, and S are equal. This allows us to simplify the five expected payoff expressions:

ROCK: $10\,(\boldsymbol{W} - \boldsymbol{F})$

PAPER: $10\,(\boldsymbol{W} - \boldsymbol{F})$

SCISSORS: $10\,(\boldsymbol{W} - \boldsymbol{F})$

FIRE: $10\,(3\boldsymbol{R} - \boldsymbol{W})$

WATER: $10\,(\boldsymbol{F} - 3\boldsymbol{R})$

The expressions **PAPER** and **SCISSORS** are now redundant and henceforth ignored.

5. **FIRE** = **WATER** implies that $3R = \frac{F + W}{2}$

Because players must be indifferent between Scissors and Rock, we have

$$10\,(3R - W) = 10\,(F - 3R)$$

which implies that $3R - W = F - 3R$ or, equivalently, $3R = \frac{F + W}{2}$.

6. **ROCK = FIRE** implies that $W = \frac{3R + F}{2}$

Because players must be indifferent between Rock and Fire, we have

$$10(\mathrm{W} - \mathrm{F}) = 10\,(3R - W)$$

which implies that $W - F = 3R - W$ or, equivalently, $W = \frac{3R + F}{2}$.

7. **ROCK = WATER** implies that $F = \frac{3R + W}{2}$

Because players must be indifferent between Rock and Water, we have

$$10(\mathrm{W} - \mathrm{F}) = 10\,(F - 3R)$$

which implies that $W - F = F - 3R$ or, equivalently, $F = \frac{3R + W}{2}$.

8. Points 5–7 imply that 3R = F = W.

Point 5 says that 3R must be equal to the *average* of F and W, while Points 6–7 similarly require that W be the average of 3R and F and that F be the average of 3R and W. The only way this is possible if all three numbers 3R, F, and W are equal.

9. Points 4 and 8 imply that $R = P = S = \frac{1}{9}$ and $F = W = \frac{1}{3}$

R + P + S + F + W = 1 because these are probabilities (and probabilities always add up to 1, or 100%) and R = P = S by Point 4, we know that 3R + F + W = 1. But because these three quantities are all equal by Point 8, we know that 3R = F = W = $\frac{1}{3}$, which in turn implies that R = P = S = $\frac{1}{9}$.

In equilibrium, then, each player plays (R, P, S, F, W) = (1/9, 1/9, 1/9, 1/3, 1/3).

S12. (a) Table is below.

		Vendor 2				
		A	B	C	D	E
Vendor 1	A	85, 85	100, 170	125, 195	150, 200	160, 160
	B	170, 100	110, 110	150, 170	175, 175	200, 150
	C	195, 125	170, 150	120, 120	170, 150	195, 125
	D	200, 150	175, 175	150, 170	110, 110	170, 100

	E	160, 160	150, 200	125, 195	100, 170	85, 85

(b) For both vendors, locations A and E are dominated. Thus, for a fully mixed equilibrium, we need only consider each vendor's choice among locations B, C, and D. Let Vendor 1's mixture probabilities be p_B, p_C, and $(1 - p_B - p_C)$. Similarly, let Vendor 2's mixture probabilities be q_B, q_C, and $(1 - q_B - q_C)$. After simplifying, the *p*-mix and *q*-mix payoffs are as shown below.

		Vendor 2			
		B	C	D	*q*-mix
Vendor 1	B	110, 110	150, 170	175, 175	$175 - 65q_B - 25q_C$, $175 - 65q_B - 5q_C$
	C	170, 150	120, 120	170, 150	$170 - 50q_C$, $150 - 30q_C$
	D	175, 175	150, 170	110, 110	$110 + 65q_B + 40q_C$, $110 + 65q_B + 60q_C$
	p-mix	$175 - 65p_B - 5p_C$, $175 - 65p_B - 25p_C$	$150 - 30p_C$, $170 - 50p_C$	$110 + 65p_B + 60p_C$, $110 + 65p_B + 40p_C$	

(c) To find the equilibrium p, set Vendor 2's payoffs equal:

$175 - 65p_B - 25p_C = 170 - 50p_C = 110 + 65p_B + 40p_C$

$65p_B - 25p_C = 5$ and $60 - 90p_C = 65p_B$

$60 - 90p_C - 25p_C = 5$

Then $p_C = 55/115 = 11/23$, $p_B = (25p_C + 5)/65 = (275/23 + 115/23)/65 = (390/23)65 = 6/23$, and $p_D = (1 - p_B - p_C) = 1 - 11/23 - 6/23 = 6/23$. Similarly, $q_B = 6/23$, $q_C = 11/23$, and $q_D = 6/23$.

One way to explain why A and E are unused in the equilibrium is to point out that they are (as noted above) dominated. This also implies that A and E are unused because they result in a payoff against the opponent's equilibrium mixture that is lower than produced by choices B, C, and D. Specifically, when Vendor 2 uses the equilibrium mixture probabilities of (6/23, 11/23, 6/23), Vendor 1's payoff from choosing

A is 100(6/23) + 125(11/23) + 150(6/23) = 2,875/23.

B is 110(6/23) + 150(11/23) + 175(6/23) = 3,360/23.

C is 170(6/23) + 120(11/23) + 170(6/23) = 3,360/23.

D is 175(6/23) + 150(11/23) + 110(6/23) = 3,360/23.

E is 150(6/23) + 125(11/23) + 100(6/23) = 2,875/23.

Clearly, A and E are inferior choices.

An alternative possibility is a partially mixed equilibrium in which one player plays pure C and the other player mixes using strategies B and D with probabilities 1/13 = 0.076 and 12/13 = 0.923, respectively. The expected payoff to the player using only C (the pure player) is 170; the expected payoff to the player using a mixture of B and D (the mixer) is 150. The equilibrium can arise in the following way: If Vendor 2 is playing pure C, then Vendor 1 gets equal highest payoffs from B and D, and therefore is willing to mix between them in any proportions. Suppose Vendor 1 chooses B with probability p and D with probability $(1 - p)$. To make this an equilibrium, pure C should be Vendor 2's best response to this mixture. A and E are clearly bad for Vendor 2, as we established above. Vendor 2 does not switch to B if

$110p + 175(1 - p) = 170$, or $5 = 65p$, or $p = 1/13 = 0.07692$.

Similarly, Vendor 2 does not switch to D if

$175p + 110(1 - p) = 170$, or $65p = 60$, or $p = 12/13 = 0.9231$.

Thus, there is really a whole continuum of mixed-strategy equilibria, in which Vendor 2 plays pure C and Vendor 1 mixes between B and D in any proportions between 1/13 and 12/13. The answer just above describes the equilibrium that results at the extreme points of this range.

S13. (a) The kicker's expected payoffs playing against the goalie's mixed strategy of L 42.2%, R 42.2%, and C 15.6% are as follows:

HL 42.2(0.50) + 15.6(0.85) + 42.2(0.85) = 70.23

LL 42.2(0.40) + 15.6(0.95) + 42.2(0.95) = 71.79

HC 42.2(0.85) + 15.6(0) + 42.2(0.85) = 71.74

LC 42.2(0.70) + 15.6(0) + 42.2(0.70) = 59.08

HR 42.2(0.85) + 15.6(0.85) + 42.2(0.50) = 70.23

LR 42.2(0.95) + 15.6(0.95) + 42.2(0.40) = 71.79

(b) Thus, the kicker should be using low, side shots (LL and LR) and high, centered shots (HC) because these strategies give him the highest expected payoff given the goalie's mix.

(c) For the goalie playing against the kicker using LL 37.8%, HC 24.4%, and LR 37.8% of the time, payoffs from each of his possible strategies are

L .378(0.40) + .244(0.85) +.378(0.95) = 71.77

C .378(0.95) + .244(0) +.378(0.95) = 71.82

R .378(0.95) + .244(0.85) +.378(0.40) = 71.77

(d) All three strategies give almost the same expected payoff, so the goalie could play all of them effectively in his mix. He will use all of L, C, and R, in his equilibrium mixture.

(e) These mixed strategies are Nash equilibria, because each player's strategy is a best response given the other player's strategy.

(f) The equilibrium payoff to the kicker is 71.77.

S14. (a) Suppose Rowena plays Up with probability p and Down with probability (1 – p). Her p-mix must keep Colin indifferent between his two pure strategies, Left and Right. Thus it must be true that $p\text{A} + (1 - p)\text{C} = p\text{B} + (1 - p)\text{D}$, or $p = (\text{D} - \text{C})/[(\text{A} - \text{B}) + (\text{D} - \text{C})]$.

(b) Colin must keep Rowena indifferent between her two pure strategies, Up and Down. Therefore: $qa + (1 - q)b = qc + (1 - q)d$, or $q = (d - b)/[(a - c) + (d - b)]$.

(c) In each case, the player's equilibrium mixture probabilities are totally independent of his or her own payoffs. Rowena's optimal p depends only on Colin's payoffs and Colin's equilibrium q depends only on Rowena's payoffs.

(d) To guarantee that a mixed-strategy Nash equilibrium exists, the values for p and q cannot be 0 or 1. Thus, we need C ≠ D, (A – B) ≠ (C – D), and A ≠ B. Similarly, $b \neq d$, $(a - c) \neq (b - d)$, and $a \neq c$.

S15. (a) Consider any one of the young men. If he chooses to go after a Brunette, he gets a guaranteed payoff of 5. If he chooses to go after the Blonde, he gets 10 if none of the other (n – 1) men choose Blonde, and 0 otherwise. Since each of the other (n – 1) chooses Blonde with probability p, and they are choosing independently, the probability that none of them choose Blonde is $(1 - p)^{n-1}$. Therefore, the expected payoff to any one man from choosing Blonde is

$$10(1 - p)^{n-1} + 0[1 - (1 - p)^{-1}] = 10(1 - p)^{n-1}.$$

In the mixed-strategy equilibrium, this man must be indifferent between his two pure strategies. Therefore, it must be true that

$$10(1 - p)^{n-1} = 5, \quad \text{or} \quad (1 - p)^{n-1} = 1/2 = 2^{-1}, \quad \text{so} \quad p = 1 - 2^{-1/(n-1)}.$$

(b) The same logic can be applied to the m young men who are choosing Blonde, so $q = 1 - 2^{1/(n-1)}$.

Consider any one of the $(n - m)$ choosing Brunette. If he switched to Blonde, his payoff will be 10 if all the m-mixers happen to choose Brunette, and 0 otherwise. Therefore, his expected payoff is

$$10(1 - q)^m.$$

So this pure Brunette chooser does not want to switch to a pure Blonde strategy if $10(1 - q)^m < 5$. For this Brunette chooser, the payoff from any mixture will be an average between the two sides of this inequality, so the same inequality also rules out his wanting to switch to any mixed strategy.

But we already have the condition of the mixed-strategy equilibrium

$$10(1 - q)^{m-1} = 5.$$

Therefore, $10(1 - q)^m = 10(1 - q)^{m-1}(1 - q) = 5(1 - q) < 5$. So, the condition holds.