

Solutions to Chapter 6 Exercises

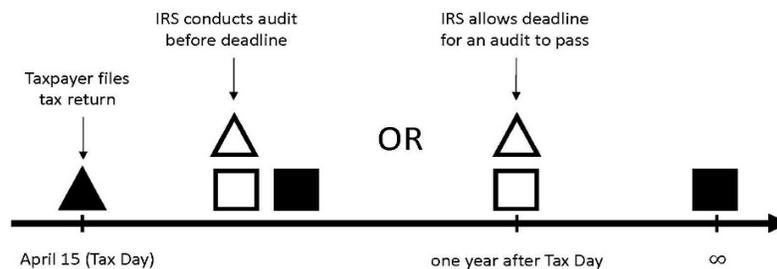
SOLVED EXERCISES

If you find a blank space where an equation or figure may appear, please select that area for it to appear.

S1. (a) The taxpayer's moment of irreversibility (when he "makes his move") is when he completes his tax return, which occurs prior to April 15th. What makes this game interesting (and a bit subtle) is that there are two ways that the game can proceed after tax submission, depending on whether the IRS subsequently conducts an audit.

Possibility #1 (shown to the left of "OR" in the figure): The IRS conducts an audit. The IRS's choice to conduct an audit becomes irreversible when it actually begins the work of auditing the taxpayer; this can occur at any time between Tax Day and one year after Tax Day. The IRS's move becomes observable to the taxpayer when IRS agents show up to conduct the audit, which is essentially simultaneous with the IRS's moment of irreversibility. This is why the unfilled triangle and unfilled square are shown as being on top of each other. The taxpayer's move then becomes observable to the IRS once the audit is completed, somewhat later.

Possibility #2: (shown to the right of "OR" in the figure): The IRS does not conduct an audit. Because the IRS can conduct its audit anytime until the yearlong window has passed, its choice not to conduct an audit does not become truly irreversible until the year has passed.¹ This is also the moment when the taxpayer knows that he will not be audited. So, the IRS's moment of irreversibility and moment of observability are both exactly one year after Tax Day. Finally, because the IRS never learns whether the taxpayer cheated, the taxpayer's moment of observability is at "infinity time."

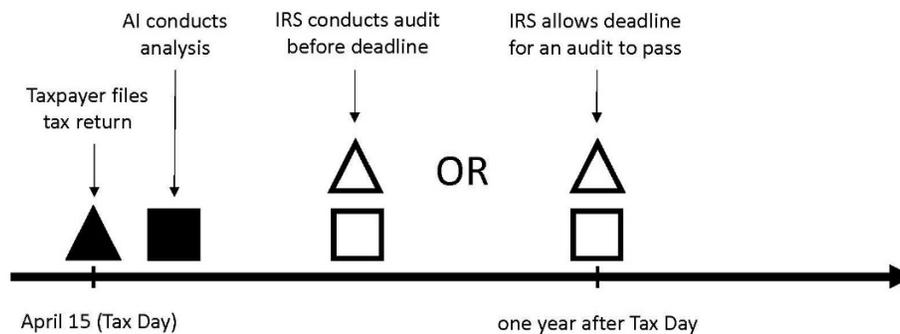


¹ If bureaucratic processes force IRS decision-makers to decide who to audit at some earlier point, then the IRS's moment of irreversibility will be earlier than the end of the yearlong window, leading to a slightly different order-of-moves diagram. However, this does not change the strategic order of moves in the game since the order of the key moments in the game remains the same.

(b) In either case, whether the IRS conducts an audit or not, the chronological order of the four key moments is the same: first, taxpayer irreversibility on Tax Day; second, IRS irreversibility and IRS observability together, either during the yearlong audit window (if an audit is conducted) or at the end of this window (if no audit is conducted); and finally, taxpayer observability, either at the end of the audit window or never (“infinity time”). Because each player’s moment of observability occurs after the other player’s moment of irreversibility, the game has simultaneous moves.

(c) The AI program informs the IRS whether the taxpayer has cheated before it needs to decide who to audit. The effect of the AI program is to move the taxpayer’s moment of observability (the black square) before the IRS’s moment of irreversibility, as shown in the next figure.

(d) The taxpayer’s moments of irreversibility and of observability now both occur before the IRS’s moment of irreversibility. The game has changed to have sequential moves with the taxpayer moving first.



S2. Second-mover advantage. In a sequential game of tennis, the second mover can best respond to the first mover’s chosen action. Put another way, the second mover can exploit the information she learns from the first mover’s action. However, since there is no Nash equilibrium in pure strategies, no outcome is the result of the players’ mutually best responding. The outcome reached will not be one that the first mover would prefer, given the action of the second mover.

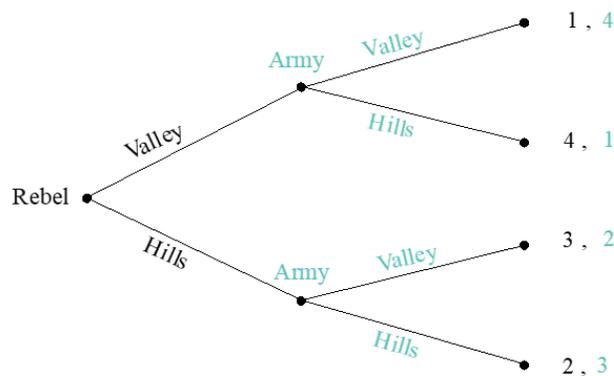
S3. (a) Yes, this is a zero-sum game. Any change in the outcome that makes the Rebel Force (Rebel) better off necessarily makes the Conventional Army (Army) worse off. To verify this, note that Rebel and Army have diametrically opposed payoffs: the best outcome for Rebel is the worst for Army, the second-best outcome for Rebel is the second-worst for Army, etc. *Note:* In a two-player game, the players have diametrically opposed payoffs when the ordinal payoffs in the four possible outcomes are

(4,1), (3,2), (2,3), (1,4). In such games, any change that makes one player better off must necessarily make the other player worse off.

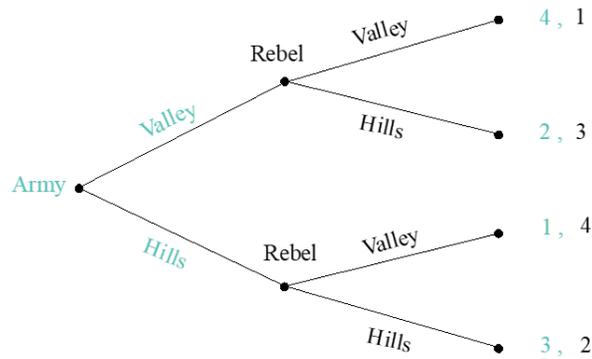
(b) This game does not have any pure-strategy Nash equilibrium. In each of the four possible outcomes, one of the players would prefer to have made a different move: in (Valley, Valley), Rebel would have preferred Hills; in (Hills, Valley), Army would have preferred Hills; in (Hills, Hills), Rebel would have preferred Valley; and in (Valley, Hills), Army would have preferred Valley.

Alternate solution: Rebel prefers that players be in different locations while Army prefers players to be in the same location. Consequently, both cannot simultaneously be playing their best response to the other's move.

(c) The game tree for the case when Rebel moves first is shown below. In the rollback equilibrium: as last-mover, Army will go wherever Rebel does; anticipating that as first-mover, Rebel compares the outcomes (Valley, Valley) and (Hills, Hills) and chooses Hills leading to an equilibrium outcome of (Hills, Hills). The rollback equilibrium strategies are (Hills; same as Rebels [Hills if Hills and Valley if Valley]), giving Rebel payoff 2 and Army payoff 3.



(d) The game tree for the case when Army moves first is shown below. In the rollback equilibrium: as last-mover, Rebel will go wherever Army does not; anticipating that as first-mover, Army compares the outcomes (Valley, Hills) and (Hills, Valley) and chooses Valley leading to a rollback equilibrium outcome of (Valley, Hills). The rollback equilibrium strategies are (Valley; opposite of Army [Hills if Valley and Valley if Hills]), giving Army payoff 2 and Rebel payoff 3.



(e) Yes, the order of moves matters. Moreover, both players prefer to move last, each getting ordinal payoff 3 when moving last versus ordinal payoff 2 when moving first; this game has a second-mover advantage.

(f) Mobility or nimbleness allows the Rebel Force to change its position on a moment's notice, compared with the Army, which takes much more time in making any move. In other words, the Army's decision where to go becomes irreversible *before* the Rebel's decision becomes irreversible. Recall that, by definition, a player is the "first-mover" if its decision is irreversible and observable to the other player before the other player's move is irreversible. The fact that Army's move must be made first (from a chronological point of view) therefore means that *Army cannot be the last-mover* (from a strategic point of view). This leaves two possibilities. First, if Army's decision is *observable* to Rebel ahead of time (e.g., any movement by Army creates a cloud of dust), then Army is the first-mover as in part (d). On the other hand, if Army's movements are unobservable until it is too late for the Rebels to escape (e.g., Army troops fly in on attack helicopters), then the game has simultaneous moves as in part (b).

S4. The strategic form, with best responses underlined, is shown below:

		Player 2			
		LL	LR	RL	RR
Player 1	U	2, <u>4</u>	<u>2</u> , <u>4</u>	<u>4</u> , 1	<u>4</u> , 1
	D	<u>3</u> , <u>3</u>	1, 2	3, <u>3</u>	1, 2

There are two Nash equilibria: (D, LL) with payoffs of (3, 3) and (U, LR) with payoffs of (2, 4). Only the first, (D, LL) is subgame perfect, however. Notice that LL weakly dominates LR for Player 2 so LR can be eliminated when searching for the subgame perfect equilibrium.

S5. The strategic form is shown below:

		Boeing	
		If In, then Peace	If In, then War
Airbus	In	\$300 m, \$300 m	-\$100 m, -\$100 m
	Out	0, \$1 b	0, \$1 b

There are two Nash equilibria: (In; If In, then Peace) and (Out; If In, then War). Only the first of these, (In; If In, then Peace), is subgame-perfect. The outcome (Out; If In, then War) is a Nash equilibrium but is not subgame-perfect; this equilibrium hinges on Airbus's belief that Boeing will start a price war on Airbus's entry into the market. However, Boeing lowers its own payoff by starting such a price war. Therefore, the threat to do so is not credible.

S6. (a) The strategic form follows:

		Tinman	
		If N, then t	If N, then b
Scarecrow	N	0, 2	2, 1
	S	1, 0	1, 0

(b) The only Nash equilibrium is (S; If N, then t) with payoffs of (1, 0).

		Tinman							
		nnn	nns	nsn	nss	snn	sns	ssn	sss
	NNN	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1
	NNS	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1
	NSN	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1
	NSS	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1	0, 1

Scarecrow	SNN	2, 3	2, 3	2, 3	2, 3	5, 4	5, 4	5, 4	5, 4
	SNS	2, 3	2, 3	2, 3	2, 3	5, 4	5, 4	5, 4	5, 4
	SSN	4, 5	4, 5	3, 2	3, 2	5, 4	5, 4	5, 4	5, 4
	SSS	1, 0	2, 2	3, 2	3, 2	5, 4	5, 4	5, 4	5, 4

S7. (a) The strategic form follows. The initials of the strategies indicate which action each player would take at his first, second, and third nodes, respectively.

Pure-strategy Nash equilibria are indicated by double borders. The unique subgame-perfect Nash equilibrium is (SSN, nns), with payoffs of (4, 5).

(b) The remaining Nash equilibria are not subgame-perfect because a player cannot credibly threaten to make a move that will give himself a lower payoff than he would otherwise receive. The Tinman would not play strategy nnn at his third node because $0 < 1$. Similarly, the 12 equilibria that arise when the Tinman plays S on his first node are not subgame-perfect because if he plays N at that node, he can expect the higher payoff of 5.

S8. (a) The strategic form follows. The initials of Scarecrow's strategies indicate which action he would take at his first, second, and third nodes, respectively.

<u>Lion = u</u>					<u>Lion = d</u>			
		Tinman					Tinman	
		t	b		t		b	
Scarecrow	NNN	1, 1, 1	2, 3, 2	Scarecrow	NNN	1, 1, 1	2, 3, 2	
	NNS	1, 1, 1	2, 3, 2		NNS	1, 1, 1	2, 3, 2	
	NSN	1, 1, 1	0, 0, 2		NSN	1, 1, 1	0, 0, 2	
	NSS	1, 1, 1	0, 0, 2		NSS	1, 1, 1	0, 0, 2	
	SNN	3, 3, 3	3, 3, 3		SNN	1, 2, 4	1, 2, 4	

SNS	3, 3, 3	3, 3, 3	SNS	0, 2, 0	0, 2, 0
SSN	3, 3, 3	3, 3, 3	SSN	1, 2, 4	1, 2, 4
SSS	3, 3, 3	3, 3, 3	SSS	0, 2, 0	0, 2, 0

Pure-strategy Nash equilibria are indicated by double borders. The unique subgame-perfect Nash equilibrium is (NNN, b, d), with payoffs of (2, 3, 2).

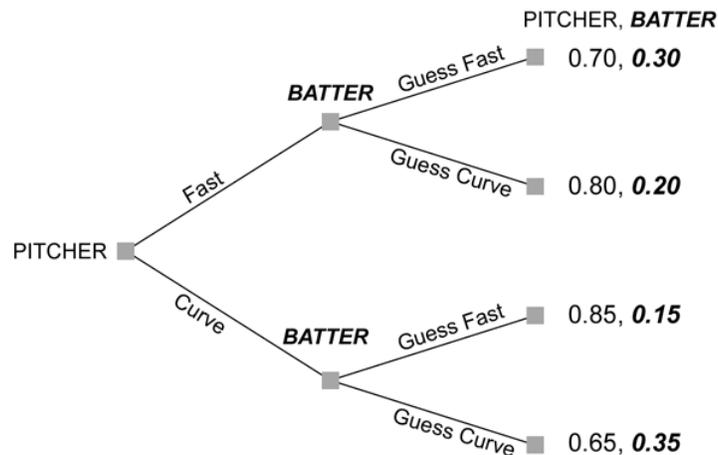
(b) Nash equilibria (SNS, t, u), (SNS, b, u), (SSS, t, u), and (SSS, b, u) are not subgame-perfect. Lion will not move u because he expects to earn 4 from moving d, knowing that Scarecrow will not move S at his third node.

(NSN, t, d) and (NSS, t, d) are not subgame-perfect because Tinman cannot expect Scarecrow to move S at his second node.

(SNN, t, d) and (SSN, t, d) are not subgame-perfect. Scarecrow cannot expect Tinman to move t because Tinman should expect to receive a higher payoff from playing b.

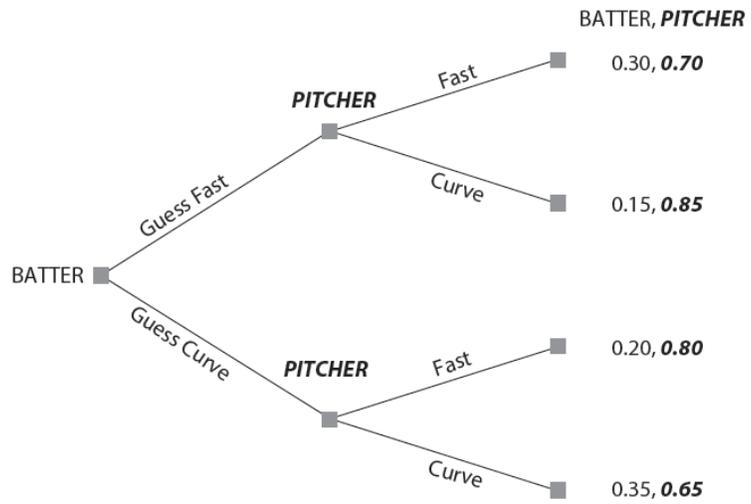
(NNS, b, d) is not subgame-perfect because no one can expect that Scarecrow would move S at his third node.

S9. (a) The game tree is shown below:



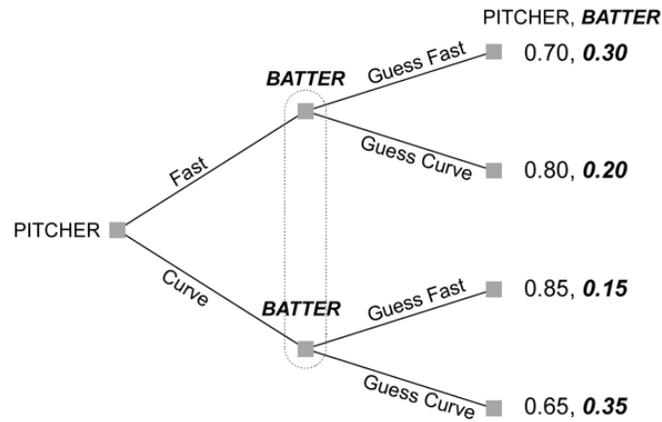
(b) The rollback
equilibrium for the game above is (Fast, Guess Fast/Guess Curve).

(c) The game tree is shown below:



The rollback equilibrium is (Guess Fast, Curve/Fast).

(d) The tree for the simultaneous game is shown below:



It can also be represented as the tree in part (c) with an information set between the pitcher's two nodes.

(e) The game table, with best responses underlined, follows:

		Batter	
		Guess fast	Guess curve
Pitcher	Fast	0.70, <u>0.30</u>	<u>0.80</u> , 0.20
	Curve	<u>0.85</u> , 0.15	0.65, <u>0.35</u>

There is no cell where both players are mutually best responding. There is no Nash equilibrium in pure strategies.

S10. (a) See table below. Best responses are underlined. There are 36 Nash equilibria shown by shading in the cells of the table:

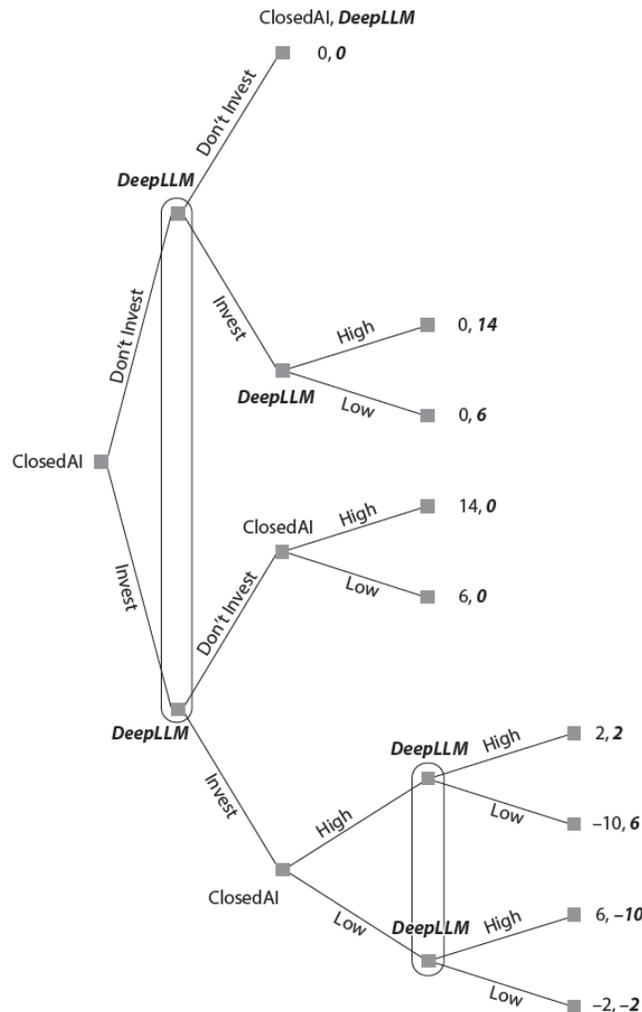
Emily								
Contribute					Don't			
Nina					Nina			
Talia	CC	CD	DC	DD	CC	CD	DC	DD
1: CCCC	3, 3, 3	3, 3, <u>3</u>	<u>3</u> , 4, 3	<u>3</u> , 4, <u>3</u>	<u>3</u> , <u>3</u> , 4	1, 2, 2	<u>3</u> , <u>3</u> , 4	1, 2, 2
2: CCCD	3, 3, 3	3, 3, <u>3</u>	<u>3</u> , 4, 3	<u>3</u> , 4, <u>3</u>	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, 2	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, 2
3: CCDC	3, 3, <u>3</u>	3, 3, <u>3</u>	<u>3</u> , 4, <u>3</u>	<u>3</u> , 4, <u>3</u>	2, 1, 2	1, <u>2</u> , 2	2, 1, 2	1, <u>2</u> , 2
4: CDCC	3, <u>3</u> , 3	3, <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	<u>3</u> , <u>3</u> , 4	1, 2, 2	<u>3</u> , <u>3</u> , 4	1, 2, <u>2</u>
5: DCCC	<u>4</u> , 3, 3	<u>4</u> , 3, <u>3</u>	<u>3</u> , 4, 3	<u>3</u> , 4, <u>3</u>	<u>3</u> , <u>3</u> , 4	1, 2, 2	<u>3</u> , <u>3</u> , 4	1, 2, 2
6: CCDD	3, 3, <u>3</u>	3, 3, <u>3</u>	<u>3</u> , 4, <u>3</u>	<u>3</u> , 4, <u>3</u>	2, 1, 2	<u>2</u> , <u>2</u> , 2	2, 1, 2	<u>2</u> , <u>2</u> , 2
7: CDDC	3, <u>3</u> , <u>3</u>	3, <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	2, 1, 2	1, <u>2</u> , 2	2, 1, <u>2</u>	1, <u>2</u> , <u>2</u>
8: DDCC	<u>4</u> , <u>3</u> , 3	<u>4</u> , <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	<u>3</u> , <u>3</u> , 4	1, 2, 2	<u>3</u> , <u>3</u> , 4	1, 2, <u>2</u>
9: CDCD	3, <u>3</u> , 3	3, <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, 2	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, 2
10: DCDC	<u>4</u> , 3, <u>3</u>	<u>4</u> , 3, <u>3</u>	<u>3</u> , 4, <u>3</u>	<u>3</u> , 4, <u>3</u>	2, 1, 2	1, 2, 2	2, 1, 2	1, <u>2</u> , 2
11: DCCD	<u>4</u> , 3, 3	<u>4</u> , 3, <u>3</u>	<u>3</u> , 4, 3	<u>3</u> , 4, <u>3</u>	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, 2	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, 2
12: CDDD	3, <u>3</u> , 3	3, <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	2, 1, 2	<u>2</u> , <u>2</u> , 2	2, 1, <u>2</u>	<u>2</u> , <u>2</u> , <u>2</u>
13: DCDD	<u>4</u> , 3, 3	<u>4</u> , 3, <u>3</u>	<u>3</u> , 4, <u>3</u>	<u>3</u> , 4, <u>3</u>	2, 1, 2	<u>2</u> , <u>2</u> , 2	2, 1, 2	<u>2</u> , <u>2</u> , 2
14: DDCC	<u>4</u> , <u>3</u> , 3	<u>4</u> , <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, 2	<u>3</u> , <u>3</u> , 4	<u>2</u> , 2, <u>2</u>
15: DDDC	<u>4</u> , <u>3</u> , <u>3</u>	<u>4</u> , <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	2, 1, 2	1, <u>2</u> , 2	2, 1, <u>2</u>	1, <u>2</u> , <u>2</u>
16: DDDD	<u>4</u> , <u>3</u> , <u>3</u>	<u>4</u> , <u>3</u> , <u>3</u>	2, 2, 1	2, 2, 1	2, 1, 2	<u>2</u> , <u>2</u> , 2	2, 1, <u>2</u>	<u>2</u> , <u>2</u> , <u>2</u>

(b) Working with the normal form of the game, use iterated dominance of weakly dominated strategies to find the subgame-perfect equilibrium. For Talia, strategy 1 is weakly dominated by strategy 2, as are strategies 3, 4, 6, 7, 9, and 12. That leaves strategies 2, 5, 8, 10, 11, 13, 14, 15, and 16. Of these, 11 weakly dominates 2; 11 also weakly dominates the rest and must be Talia's subgame-perfect equilibrium strategy. Once you determined this, you can determine that Nina's DC weakly dominates her CC, CD, and DD and must be her subgame-perfect equilibrium strategy. Finally, Emily's D dominates her

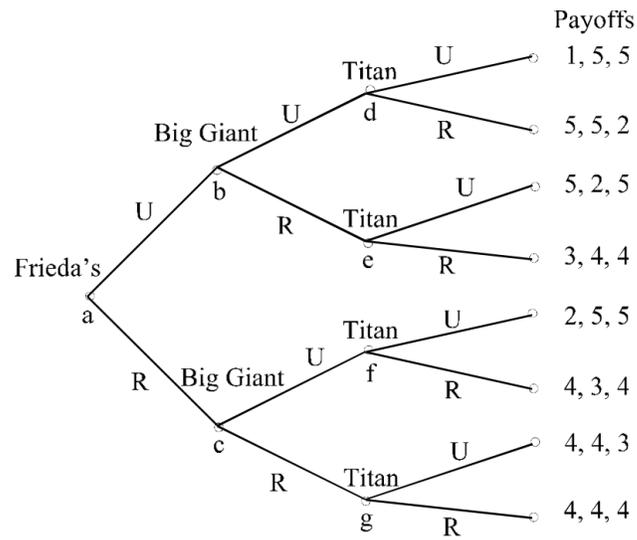
C. The subgame-perfect equilibrium is [DCCD, DC, D] with payoffs of (3, 3, 4) to Talia, Nina, and Emily; this cell has a double border in the table above.

The set of strategies that leads to the subgame-perfect equilibrium is the only set in which all three women use strategies that entail choosing rationally (that is, choosing the action that leads to the best possible outcome from every possible decision node). Another way to say this is that in 35 of the 36 Nash equilibria, someone must use a strategy that is *not* subgame-perfect. For example, there is a Nash equilibrium in row 1, column 5 of the first (left) table. This is not subgame-perfect, because Talia's strategy, CCCC, states that if she arrives at node d in Figure 6.9, she will choose C for a payoff of 3 rather than D for a payoff of 4. This strategy cannot be subgame perfect for Talia, and the equilibrium in that cell cannot be the subgame-perfect equilibrium.

S11. The larger tree follows:



S12. (a) The game tree is shown below:



Frieda's has two actions at one node, so it has two strategies.

Big Giant has two actions at each of three nodes, so it has $2 \times 2 = 4$ strategies.

Titan has two actions at each of four nodes, so it has $2 \times 2 \times 2 \times 2 = 16$ strategies.

(b) The strategic form is given below:

		Frieda's							
		Urban				Rural			
		Big Giant		Big Giant		Big Giant		Big Giant	
Titan		UU	UR	RU	RR	UU	UR	RU	RR
UUUU		5, 5, 1	5, 5, 1	5, 2, 5	5, 2, 5	5, 5, 2	3, 4, 4	5, 5, 2	3, 4, 4
UUUR		5, 5, 1	5, 5, 1	5, 2, 5	5, 2, 5	5, 5, 2	4, 4, 4	5, 5, 2	4, 4, 4
UURU		5, 5, 1	5, 5, 1	5, 2, 5	5, 2, 5	4, 3, 4	3, 4, 4	4, 3, 4	3, 4, 4
URUU		5, 5, 1	5, 5, 1	4, 4, 3	4, 4, 3	5, 5, 2	3, 4, 4	5, 5, 2	3, 4, 4
RUUU		2, 5, 5	2, 5, 5	5, 2, 5	5, 2, 5	5, 5, 2	3, 4, 4	5, 5, 2	3, 4, 4
UURR		5, 5, 1	5, 5, 1	5, 2, 5	5, 2, 5	4, 3, 4	4, 4, 4	4, 3, 4	4, 4, 4
URRU		5, 5, 1	5, 5, 1	4, 4, 3	4, 4, 3	4, 3, 4	3, 4, 4	4, 3, 4	3, 4, 4
RRUU		2, 5, 5	2, 5, 5	4, 4, 3	4, 4, 3	5, 5, 2	3, 4, 4	5, 5, 2	3, 4, 4
URUR		5, 5, 1	5, 5, 1	4, 4, 3	4, 4, 3	5, 5, 2	4, 4, 4	5, 5, 2	4, 4, 4
RURU		2, 5, 5	2, 5, 5	5, 2, 5	5, 2, 5	4, 3, 4	3, 4, 4	4, 3, 4	3, 4, 4
RUUR		2, 5, 5	2, 5, 5	5, 2, 5	5, 2, 5	5, 5, 2	4, 4, 4	5, 5, 2	4, 4, 4
URRR		5, 5, 1	5, 5, 1	4, 4, 3	4, 4, 3	4, 3, 4	4, 4, 4	4, 3, 4	4, 4, 4
RURR		2, 5, 5	2, 5, 5	5, 2, 5	5, 2, 5	4, 3, 4	4, 4, 4	4, 3, 4	4, 4, 4
RRUR		2, 5, 5	2, 5, 5	4, 4, 3	4, 4, 3	5, 5, 2	4, 4, 4	5, 5, 2	4, 4, 4
RRRU		2, 5, 5	2, 5, 5	4, 4, 3	4, 4, 3	4, 3, 4	3, 4, 4	4, 3, 4	3, 4, 4
RRRR		2, 5, 5	2, 5, 5	4, 4, 3	4, 4, 3	4, 3, 4	4, 4, 4	4, 3, 4	4, 4, 4

The eight pure-strategy Nash equilibria are indicated by shaded cells.

(c) Strategy UUUR for Titan weakly dominates strategy UUUU, and it also weakly dominates every other strategy for Titan; therefore, UUUR is Titan's subgame-perfect equilibrium strategy. Then for Big Giant, UU is weakly dominant, and Frieda's choice is R. The subgame-perfect equilibrium is [UUUR, UU, R], with payoffs of (5, 5, 2) to Titan, Big Giant, and Frieda's.

Note that the possible equilibria produce two possible outcomes. The first four equilibria produce an outcome in which Frieda's is alone in the rural mall (payoffs are [5, 5, 2]); the last four find all three stores in the rural mall (payoffs are [4, 4, 4]). The subgame-perfect equilibrium is reasonable because

each of the three stores makes the decision that is in its own best interest at every decision node that could possibly arise during the game (even those nodes that do not arise when the equilibrium is played). In the other seven equilibria, one store (or more) uses a strategy in which, at some possible decision node, it makes a choice that lowers its own payoff. Of course, these self-defeating choices do not arise when all three stores use their equilibrium strategies. In other words, these seven equilibria are supported by beliefs about off-the-equilibrium-path behavior.

S13. (a) In the simultaneous version of the game each store has only two strategies: Urban and Rural. The payoff table, with best responses underlined, follows:

		Frieda's = U		Frieda's = R			
		Big Giant		Big Giant			
		U	R	U	R		
Titan	U	<u>5</u> , <u>5</u> , 1	<u>5</u> , 2, <u>5</u>	Titan	U	<u>5</u> , <u>5</u> , <u>2</u>	3, 4, 4
	R	2, <u>5</u> , <u>5</u>	4, 4, 3		R	4, 3, 4	<u>4</u> , <u>4</u> , <u>4</u>

The U, U, R equilibrium (with payoffs of [5, 5, 2]) are likely focal for Titan and Big Giant. Frieda's would prefer the other equilibrium, but the big stores don't care if Frieda's deviates from the U, U, R equilibrium—they receive a payoff of 5 each no matter what Frieda's does. There is some risk for the big stores: if the other big store chooses R, the one that plays U will receive only 3.

The R, R, R equilibrium (with payoffs of [4, 4, 4]) may be more attractive to a big store if it is risk averse; if *one* of the other stores (either Frieda's or the other big store) deviates by choosing U, the big store choosing R is still guaranteed a payoff of 4.

(b) When all three stores request Urban, there is a one-third chance that Titan and Big Giant will be in Urban while Frieda's is alone in Rural, a one-third chance that Titan and Frieda's will be in Urban while Big Giant is alone in Rural, and a one-third chance that Big Giant and Frieda's will be in Urban while Titan is alone in Rural. The expected payoff when all three choose Urban is thus

$$(1/3)(5, 5, 1) + (1/3)(5, 2, 5) + (1/3)(2, 5, 5) = (4, 4, 11/3).$$

The payoff table is the same as in part (a), with the exception of the Urban, Urban, Urban cell:

		Frieda's = U		Frieda's = R			
		Big Giant		Big Giant			
		U	R	U	R		
Titan	U	<u>5</u> , <u>5</u> , 1	<u>5</u> , 2, <u>5</u>	Titan	U	<u>5</u> , <u>5</u> , <u>2</u>	3, 4, 4
	R	2, <u>5</u> , <u>5</u>	4, 4, 3		R	4, 3, 4	<u>4</u> , <u>4</u> , <u>4</u>

Titan	U	<u>4, 4, 11/3</u>	<u>5, 2, 5</u>	Titan	U	<u>5, 5, 2</u>	3, 4, 4
	R	2, <u>5, 5</u>	4, 4, 3		R	4, 3, 4	<u>4, 4, 4</u>

The Nash equilibria are U, U, U (with expected payoffs of [4, 4, 11/3]) and R, R, R (with payoffs of [4, 4, 4]). The R, R, R equilibrium seems more likely to be played not only because it represents a Pareto improvement (Frieda's expected payoff is more while the expected payoff of the big stores remains the same) but also because with the R, R, R equilibrium each store is guaranteed a payoff of 4 even if one of the other stores deviates and plays U.

(c) The change in the payoff table causes an important change in the equilibria of the games found in parts (a) and (b). The randomized allocation of the two Urban slots when all three stores choose Urban in part (b) greatly benefits the underdog Frieda's. In fact, the increased expected payoff for Frieda's to play U even when the other two stores play U is so great that it becomes Frieda's best response when the other two stores play U. Thus U, U, R ceases to be a Nash equilibrium, and U, U, U becomes a new one.

S14. (a) As seen in Exercise S10 of Chapter 5, Nancy's best response to Monica's choice m is

$$n^* = 1 + m/4.$$

Monica knows Nancy's best-response rule, so when she chooses her m to maximize her profit she can plug in $(1 + m/4)$ for n :

$$\pi_M = [4m + 4(1 + m/4) + m(1 + m/4)]/2 - m^2$$

$$\pi_M = 3m + 2 - (7/8)m^2$$

The m that maximizes π_M is $m^* = 12/7$.

Nancy's best response is then $n^* = 1 + (12/7)/4 = 10/7$.

(b) When $m = 12/7$ and $n = 10/7$ the profits are

$$\pi_M = (4m + 4n + mn)/2 - m^2 = [4(12/7) + 4(10/7) + (12/7) * (10/7)]/2 - (12/7)^2 = 169/49 \approx 3.449 \text{ and}$$

$$\pi_N = (4m + 4n + mn)/2 - n^2 = [4(12/7) + 4(10/7) + (12/7) * (10/7)]/2 - (10/7)^2 = 213/49 \approx 4.347.$$

In Exercise S10 of Chapter 5, when Monica and Nancy choose their effort levels simultaneously, their profits are

$$\pi_M = \pi_N = 8/3 \approx 2.667.$$

Both Monica and Nancy make higher profits when Monica commits to an effort level first, but Nancy experiences a greater increase in her profits. This game thus has a second-mover advantage.

S15. (a) The game table for the first-stage game (with best responses underlined) follows:

		Nancy	
		Yes	No
Monica	Yes	2.667, 2.667	<u>3.449</u> , <u>4.347</u>
	No	<u>4.347</u> , <u>3.449</u>	2.667, 2.667

(b) There are two pure-strategy Nash equilibria: (Yes, No) and (No, Yes).