Solutions to Chapter 5 Exercises

SOLVED EXERCISES

If you find a blank space where an equation or figure may appear, please select that area for it to appear.

S1. (a) R's best-response rule is given by $y = 10\sqrt{x} - x$. L spends \$16 million, so $x = 16$. Then R's best response is $y = 10\sqrt{16} - 16 = 10(4) - 16 = 40 - 16 = 24$, or \$24 million.

(b) R's best response is $y = 10\sqrt{x} - x$, and L's best response is $x = 10\sqrt{y} - y$. Solve these simultaneously:

$$x = 10(10\sqrt{x} - x)^{1/2} - 10\sqrt{x} + x$$

$$\Rightarrow \quad \sqrt{x} = (10\sqrt{x} - x)^{1/2}$$

$$\Rightarrow \quad x = 10\sqrt{x} - x$$

$$\Rightarrow \quad 2x = 10\sqrt{x}$$

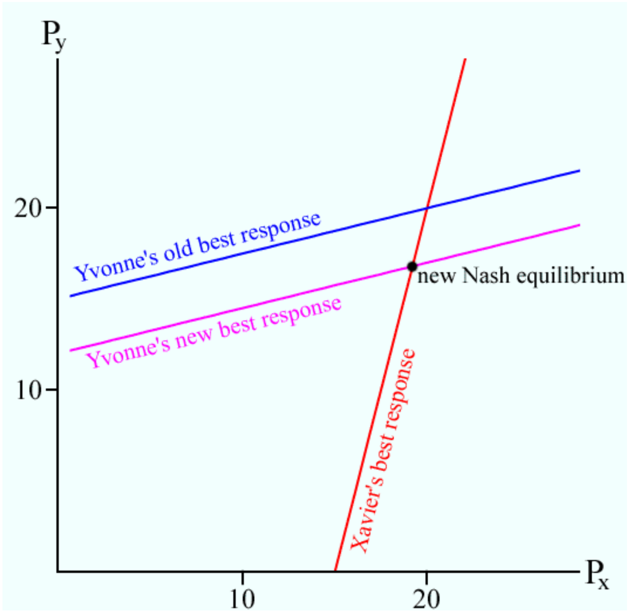
$$\Rightarrow \quad \sqrt{x} = 5$$

$$\Rightarrow \quad x = 25$$

$$y = 10\sqrt{25} - 25 = 25$$

S2. (a) Xavier's costs have not changed, nor have the demand equations, so Xavier's best-response rule is still the same as in Figure 5.1: $P_x = 15 + 0.25P_y$. Yvonne's new profit function is $B_y = (P_y - 2)Q_y = (P_y - 2)(44 - 2P_y + P_x) = -2(44 + P_x) + (4 + 44 + P_x)P_y - 2(P_y)^2$. Rearranging or differentiating with respect to P_y leads to Yvonne's new best-response rule: $P_y = 12 + 0.25P_x$. Solving the two response rules simultaneously yields $P_x = 19.2$ and $P_y = 16.8$.

(b) See the graph below. Yvonne's best-response curve has shifted down; it has the same slope but a new, lower intercept (12 rather than 15). Yvonne is able to charge lower prices due to lower costs. The new intersection point occurs at (19.2, 16.8), as calculated above.



S3. (a) La Boulangerie's profit is

$Y_1 = P_1Q_1 – Q_1 = P_1(14 – P_1 – 0.5P_2) – (14 – P_1 – 0.5P_2) = – P_1^2 + 15P_1 – 0.5P_1P_2 + 0.5P_2 – 14$.

To find the optimal P_1 without using calculus, we refer to the result in the appendix to Chapter 5, remembering that P_2 is a constant in this situation. Using the notation of the appendix, we have A = $0.5P_2 – 14$, B = $15 – 0.5P_2$, and C = 1, so the solution is

$P_1 = B/(2C) = 15 – 0.5P_2/(2)$, or $P_1 = 7.5 – 0.25P_2$.

This is La Boulangerie's best-response function. You get the same answer by setting $\partial Y_1/\partial P_1 = – 2P_1 + 15 – 0.5P_2 = 0$ and solving for P_1.

Similarly, La Fromagerie's profit is

$Y_2 = P_2Q_2 – 2Q_2 = P_2(19 – 0.5P_1 – P_2) – 2(19 – 0.5P_1 – P_2) = – P_2^2 + 21P_2 – 0.5P_1P_2 + P_1 – 38$.

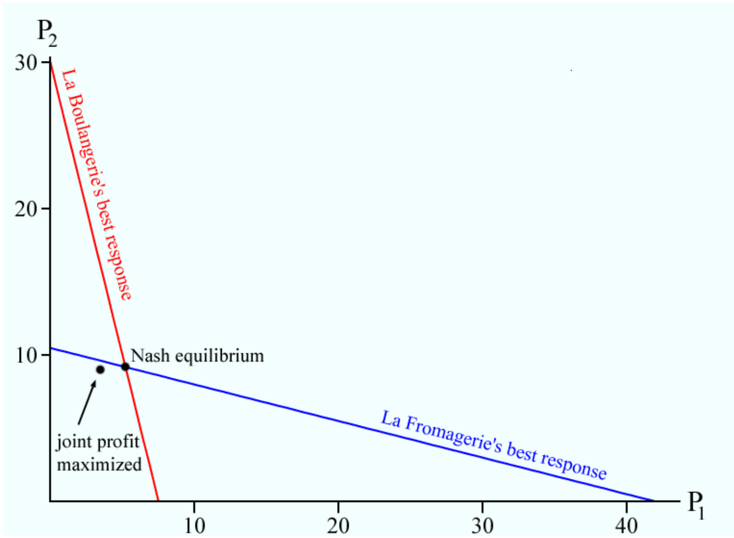
Again, using the notation in the appendix, A = $P_1 – 38$, B = $21 – 0.5P_1$, and C = 1, which yields

$P_2 = B/(2C) = 21 – 0.5P_1/(2)$, or $P_2 = 10.5 – 0.25P_1$.

This is La Fromagerie's best-response function. You get the same answer by setting $\partial \boldsymbol{Y}_2/\partial \boldsymbol{P}_2 = – 2P_2 + 21 – 0.5P_1 = 0$ and solving for P_1.

To find the solution for the equilibrium prices analytically, substitute La Fromagerie's best-response function for P_2 into La Boulangerie's best-response function. This yields $P_1 = 7.5 – 0.25(10.5 –$

0.25P_1), or P_1 = 5.2. Given this value for P1, you can find P2 = 10.5 – 0.25(5.2) = 9.2. The best-response curves are shown in the diagram below:



(b) Colluding to set prices to maximize the sum of profits means that the firms maximize the joint-profit function:

$$Y = Y_1 + Y_2 = 16P_1 + 21.5P_2 - P_1^2 - P_2^2 - P_1P_2 - 52$$

To answer without calculus, use the result in the appendix. Using that notation to solve for P_1, A = $21.5P_2 - P_2^2 - 52$, B = 16 – P_2, and C = 1, so the solution is

$$P_1 = B/(2C) = 16 - P_2/(2), \text{ or } P_1 = 8 - 0.5P_2.$$

Similarly, solving for P_2, A = $16P_1 - P_1^2 - 52$, B = 21.5 – P_1, and C = 1, so the solution is

$$P_2 = B/(2C) = 21.5 - P_1/(2), \text{ or } P_2 = 10.75 - 0.5P_2.$$

Solving these two equations simultaneously yields the solution P_1 = 3.5 and P_2 = 9.

You can get the same answer by partially differentiating the joint-profit function with respect to each price. Profits must be maximized with respect to both P_1 and P_2, so we need $\partial Y/\partial P_1 = 16 - 2P_1 - P_2 = 0$ and $\partial Y/\partial P_2 = 21.5 - P_1 - 2P_2 = 0$.

(c) When firms choose their prices to maximize joint profit, they act as a single firm and ignore any individual incentives that they might have to deviate from the joint profit goal. However, given their partner's collusive price, each company can reap more profit individually by charging more. For instance, plugging the joint-profit-maximizing value of La Boulangerie's price into La Fromagerie's individual best-response rule will not yield La Fromagerie's joint profit-maximizing price:

$P_2 = 10.5 - 0.25(3.5) = 9.625 \neq 9$

Likewise, plugging the joint-profit-maximizing value of La Boulangerie's price into La Fromagerie's individual best-response rule gives

$P_1 = 7.5 - 0.25(9) = 4.75 \neq 3.5.$

Thus, the two joint profit-maximizing prices are not best responses to each other; that is, they do not form a Nash equilibrium.

(d) When firms produce substitutes, a drop in price at one store hurts the sales of the other. Thus, as your rival drops her price, you also want to drop yours to attempt to maintain sales (and profits). In the bistro example in the text and in Exercise 1 above, this result led to best-response curves that were positively sloped and Nash equilibrium prices that were lower than the joint-profit-maximizing prices. Here, the firms produce complements, so a drop in price at one store leads to an increase in sales at the other. In this case, as one store drops its price, the other can safely increase its price somewhat and still maintain sales (and profits). Thus, the best-response curves are negatively sloped, and the Nash equilibrium prices are higher than the joint-profit-maximizing prices.

S4. To rationalize the nine possible outcomes, you need a separate argument for each one. We offer just one example, leaving you to construct the rest. Note that you need not consider the strategy combination (A, A) since that is a Nash equilibrium and therefore rationalizable. Consider (A, C) leading to the payoffs (0, 2). C is a possible best response for Column if he thinks that Row is playing A. Why does Column believe this? Because he believes that Row believes that Column is playing B. Column justifies this belief by thinking that Row believes that Column believes that Row is playing C. The beliefs in this chain are all perfectly rational because each strategy of either player is a best response (or among the best responses) to some strategy of the rival player.

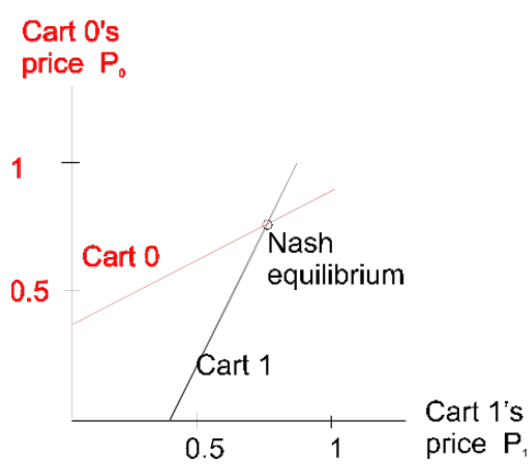
S5. No matter what beliefs Colin might hold about what Rowena is playing, South is never Colin's best response. Therefore, South is not a rationalizable strategy for Colin. Since Rowena recognizes this, and since Earth is Rowena's best response only against Colin's South, Rowena does not play Earth. Since North is Colin's best response only against Earth, Colin will not play North. Since Colin will never play North or South, Wind is never a best response for Rowena. The remaining strategies—Water and Fire for Rowena and East and West for Colin—are used in the two pure-strategy Nash equilibria, so they are certainly rationalizable.

S6. Using the third-round range of Q_A, we have that $Q_Z = 12 - Q_A/2$ must be at most $12 - 9/2 = 12 - 4.5 = 7.5$ (nothing new here) and at least $12 - 12.75/2 = 12 - 6.375 = 5.625$ (a narrowing of the range: after the third round, the lower bound was 4.5). Similarly, using the third-round range of Q_Z, we find that $Q_A = 15 - Q_Z/2$ must be at most $15 - 4.5/2 = 15 - 2.25 = 12.75$ (nothing new) and at least $15 - 7.5/2 = 15 - 3.75 = 11.25$ (a narrowing of the range: in the third round, the lower bound was 9). We see that in even rounds the lower bounds get tighter, and in odd rounds the upper bounds get tighter.

S7. (a) Cart 0 serves x customers and Cart 1 serves $(1 - x)$, where x is defined by the equation $p_0 + 0.5x^2 = p_1 + 0.5\,(1 - x)^2$. Expanding this equation yields $p_0 + 0.5x^2 = p_1 + 0.5 - x + 0.5x^2$, and solving for x yields $x = p_1 - p_0 + 0.5$. Thus Cart 0 serves $p_1 - p_0 + 0.5$ customers, and Cart 1 serves $1 - x$, or $p_0 - p_1 + 0.5$, customers.

(b) Profits for Cart 0 are $(p_1 - p_0 + 0.5)(p_0 - 0.25)$. Profits for Cart 1 are symmetric: $(p_0 - p_1 + 0.5)(p_1 - 0.25)$. Expanding the expression for Cart 0 profits yields $(p_1 - p_0 + 0.75)p_0 - (0.25p_1 + 0.125)$. Solving for the profit-maximizing value of p_0 by completing the square or differentiating with respect to p_0 yields $p_0 = 0.5p_1 + 0.375$. Cart 1's best-response rule is symmetric: $p_1 = 0.5p_0 + 0.375$.

(c) The graph is shown below. The Nash equilibrium prices are the values of p_0 and p_1 that solve simultaneously the two best-response rules found in part (b). Substituting Cart 1's best-response rule into that for Cart 0, we find $p_0 = 0.5(0.5p_0 + 0.375) + 0.375 = p_0 + 0.5625$. Solving for p_0 yields $p_0 = 0.75$ (75 cents); Cart 1's price is $p_1 = 0.75$ (75 cents) also.



S8. (a) South Korea's profit is

$$Y_{Korea} = q_{Korea} \times P - c_{Korea} \times q_{Korea} = q_{Korea}(180 - Q) - 30q_{Korea}$$

$$= q_{Korea}(180 - q_{Korea} - q_{Japan}) - 30q_{Korea}$$

$= -q_{Korea}^2 + (180 - 30)q_{Korea} - q_{Korea} \times q_{Japan}$.

Using the notation in the appendix yields A = 0, B = 150 $-q_{Japan}$, and C = 1, so South Korea's best response is

$q_{Korea} = B/(2C) = 150 - q_{Japan}/(2)$, or $q_{Korea} = 75 - 0.5q_{Japan}$.

This is South Korea's best-response function. You get the same answer by setting

$\partial Y_{Korea} / \partial q_{Korea} = -2q_{Korea} + 150 - q_{Japan} = 0$ and solving for q_{Korea}.

Since Japan has the same price and cost per ship as South Korea, Japan's profit is

$Y_{Japan} = -q_{Japan}^2 + (180 - 30)q_{Japan} - q_{Korea} \times q_{Japan}$.

Similarly, Japan's best-response function is

$q_{Japan} = 75 - 0.5q_{Korea}$.

(b) To find the solution for the equilibrium prices, substitute Japan's best-response function for q_{Japan} into South Korea's best-response function. This yields

$q_{Korea} = 75 - 0.5q_{Japan} = 75 - 0.5(75 - 0.5q_{Korea}) = 37.5 + 0.25q_{Korea}$

$\Rightarrow \quad q_{Korea} = 50$.

Therefore, $q_{Japan} = 75 - 0.5(50) = 50$.

The price of a VLCC is given by the expression P = 180 – Q, where $Q = q_{Korea} + q_{Japan} = 50 + 50 = 100$. Therefore, P = 180 – 100 = 80, or $80 million.

South Korea's profit is

$Y_{Korea} = -q_{Korea}^2 + (180 - c_{Korea})q_{Korea} - q_{Korea} \times q_{Japan}$

$= -(50)^2 + (180 - 30)(50) - (50)(50) = -2{,}500 + 7{,}500 - 2{,}500 = 2{,}500$.

Likewise, Japan's profit is

$Y_{Japan} = -q_{Japan}^2 + (180 - c_{Japan})q_{Japan} - q_{Korea} \times q_{Japan} = 2{,}500$.

Therefore, both countries make $2.5 billion in profits.

(c) South Korea's new best-response function is

$q_{Korea} = 90 - 0.5c_{Korea} - 0.5q_{Japan} = 90 - 0.5(20) - 0.5q_{Japan} = 80 - 0.5q_{Japan}$.

Japan's new best-response function is

$q_{Japan} = 90 – 0.5c_{Japan} – 0.5q_{Korea} = 90 – 0.5(40) – 0.5q_{Korea} = 70 – 0.5q_{Korea}$.

To find the solution for the new equilibrium prices, substitute Japan's new best-response function for q_{Japan} into South Korea's best-response function. This yields

$$q_{Korea} = 80 – 0.5q_{Japan} = 80 – 0.5(70 – 0.5q_{Korea}), \text{ or } q_{Korea} = 60.$$

Given this value for q_{Korea},

$$q_{Japan} = 70 – 0.5q_{Korea} = 70 – 0.5(50), \text{ or } q_{Japan} = 45.$$

South Korea's market share is 60 / (60 + 45) ≈ 57%, and Japan's market share is approximately 43%.

South Korea's profit is

$$\begin{aligned} Y_{Korea} &= –q_{Korea}^2 + (180 – c_{Korea})q_{Korea} – q_{Korea}q_{Japan} \\ &= – (60)^2 + (180 – 20)(60) – (60)(45) \\ &= –3{,}600 + 9{,}600 – 2{,}700 = 3{,}300, \text{ or \$3.3 billion.} \end{aligned}$$

Japan's profit is

$$\begin{aligned} Y_{Japan} &= –q_{Japan}^2 + (180 – c_{Japan})q_{Japan} – q_{Korea}q_{Japan} \\ &= – (45)^2 + (180 – 40)(45) – (60)(45) \\ &= –2{,}025 + 6{,}300 – 2{,}700 = 1{,}575, \text{ or \$1.575 billion.} \end{aligned}$$

S9. (a) South Korea's profit is

$Y_{Korea} = q_{Korea,} P – c_{Korea}q_{Korea} = q_{Korea}(180 – Q) – 30q_{Korea} = q_{Korea}(180 – q_{Korea} – q_{Japan} – q_{China}) – 30q_{Korea} = –q_{Korea}^2 + (180 – 30) \times q_{Korea} – q_{Korea} \times q_{Japan} – q_{Korea} \times q_{China}$.

Using the notation in the appendix, A = 0, B = 150 – q_{Japan} – q_{China}, and C = 1, so the solution is q_{Korea} = B/(2C) = (150 – q_{Japan} – q_{China})/2, or $q_{Korea} = 75 – 0.5q_{Japan} – 0.5q_{China}$. This is South Korea's best-response function. You get the same answer by setting $\partial Y_{Korea} / \partial q_{Korea} = –2q_{Korea} + 150 – q_{Japan} – q_{China} = 0$ and solving for q_{Korea}.

Since Japan and China face the same price (P = 180 – Q) and cost ($c_{Korea} = c_{Japan} = c_{China}$) as South Korea, the best-response functions for Japan and China are

$$q_{Japan} = 75 – 0.5q_{Korea} – 0.5q_{China}$$

and

$$q_{China} = 75 – 0.5q_{Korea} – 0.5q_{Japan}.$$

(b) To find the solution for the equilibrium prices, first substitute China's best-response function into Japan's best-response function:

$$q_{Japan} = 75 - 0.5q_{Korea} - 0.5(75 - 0.5q_{Korea} - 0.5q_{Japan})$$

$$\Rightarrow \quad q_{Japan} = 50 - q_{Korea}/3$$

Next, substitute Japan's best-response function into China's best-response function:

$$q_{China} = 75 - 0.5q_{Korea} - 0.5(75 - 0.5q_{Korea} - 0.5q_{China})$$

$$\Rightarrow \quad q_{China} = 50 - q_{Korea}/3$$

Then substitute these expressions for q_{Japan} and q_{China} into South Korea's best-response function:

$$q_{Korea} = 75 - 0.5q_{Japan} - 0.5q_{China} = 75 - 0.5(50 - q_{Korea}/3) - 0.5(50 - q_{Korea}/3) = 25 + q_{Korea}/3$$

$$\Rightarrow \quad q_{Korea} = 37.5$$

Japan and China have symmetric best response functions, so $q_{Japan} = 37.5$ and $q_{China} = 37.5$.

Each country produces 37.5, so each has a market share of 33.3%.

Since each country has the same market share, price, and cost, they will all earn the same profit. South Korea's profit is

$$Y_{Korea} = -q_{Korea}^2 + (180 - 30)q_{Korea} - q_{Korea} \times q_{Japan} - q_{Korea} \times q_{China}$$

$$= -(37.5)^2 + (150)(37.5) - (37.5)(37.5) - (37.5)(37.5) = 1{,}406.25, \text{ or}$$

$1,406.25 million.

(c) In the duopoly situation, each country produced 50 VLCCs per year at a price of $80 million each, for a profit of $2.5 billion per country. In the triopoly, each country produced 37.5 VLCCs per year at a price of $67.5 million each. The third producer increases the total output (supply), lowering the price per VLCC and therefore the profits made per country. There is a greater number of VLCCs available for purchase, but this lowers the price and therefore also the collective profits (roughly $4.2 billion versus $5 billion), which are now split among three countries instead of between two.

S10. (a) Since the joint profits of the partnership are split equally, Monica and Nancy each get a payoff of $0.5(4m + 4n + mn) = 2m + 2n + 0.5mn$, minus her own effort cost. Therefore Monica's payoff is $2m + 2n + 0.5mn - m^2$ and Nancy's payoff is $2m + 2n + 0.5mn - n^2$. If $m = n = 1$, each partner receives a payoff of $2(1) + 2(1) + 0.5(1)(1) - 1 = 3.5$, or $35,000.

(b) We find Nancy's best-response function by maximizing the expression $Y_n = 2m + 2n + 0.5mn - n^2$. Using the notation in the appendix, A = 2m, B = 2 + 0.5m, and C = 1, so Nancy's best-response function is

n = B/(2C) = 2 + 0.5m/(2) = 1 + 0.25m.

You get the same answer by setting $\partial Y_n / \partial n$ = -2n + 2 + 0.5m = 0 and solving for n.

When Monica puts in effort of m = 1, Nancy's best response is

n = 1 + 0.25m = 1 + 0.25(1) = 1.25.

(c) We know from part (b) that Nancy's best-response function is n = 1 + 0.25m. The game is symmetric; Monica's best-response function is m = 1 + 0.25n. To find the solution for the equilibrium effort levels, substitute Nancy's best-response function into Monica's best-response function:

m = 1 + 0.25(1 + 0.25m)

⇒ m = 4/3

Given this value of m, Nancy's effort will be

n = 1 + 0.25(4/3) = 4/3.

The Nash equilibrium to this game has both Monica and Nancy putting in an effort of 4/3.

S11. Because Xavier cannot rationally believe that Yvonne will charge a negative price, and because his best-response function is $P_x = 15 + 0.25P_y$, his own price P_x can never be below 15. A similar calculation done by Yvonne ensures that her price P_y cannot be less than 15 either. In the second round of this thinking, each sees through the other's first-round thinking, and therefore does not set a price less than 15 + 0.25(15) = 18.75.

However, this narrows the range of prices only from below the Nash equilibrium. To narrow from above, we need a starting point, namely an upper limit to the prices, such that no rational player would ever contemplate charging anything higher. If there were a price so high that you would sell nothing if you charged that, no matter the other's price, that would do. However, for the linear functions stipulated here, that is not the case. Consider Xavier. Might he charge P_x = 1,000? That would be his best response if he believed that Yvonne would charge $P_y = (P_x - 15)/0.25$ = 4 × 985 = 3,940, in which case he would sell Q_x = 44 – 2 × 1,000 + 3,940 = 1,984. And he might believe that Yvonne would charge P_y = 3,940 because he thinks she believes that he would charge P_x = (3,940 – 15)/0.25 = 15,700, in which case she would expect to sell Q_y = 44 – 2 × 3,940 + 15,700 = 7,864. And so on. Of course these are absurd prices for meals, but that is because the linear functions we stipulated are unlikely to be valid over such a large range. If it is not rational for anyone to charge a price higher than, say, 500, then because they know this, in the second round of thinking it is not rational for either to charge a price higher than $15 + 0.25 \times 500 = 140$. In the third round of thinking this upper limit drops to $15 + 0.25 \times 140 = 50$, and so on, eventually converging to the Nash equilibrium level of 20.

S12. (a) Because everyone has to choose a number less than or equal to 100, the average (X) cannot exceed 100, and half of the average ($X/2$) cannot exceed 50. Therefore choosing 79 will place Elsa closer to ($X/2$) than choosing 80 regardless of what the others choose. By definition of dominance, 80 is dominated. In fact the same argument shows that anything above 50 is (strictly) dominated.

(b) Suppose all of Elsa's classmates choose 40. If she chooses anything between 41 and 50 (remember that anything above 50 is dominated), the average X will be $\leq (50 + 49(40)) / 50 = 40.2$ and $(X/2)$ will be ≤ 20.1, so Elsa will be further away from (X/2) than her classmates. If Elsa chooses 40, she will tie with her classmates. Thus, we know that Elsa will select a number less than 40. Suppose Elsa chooses $n < 40$. The average of all numbers will be $X = [49(40) + n]/50 = (1{,}960 + n)/50$. To win, Elsa's number must be closer to X/2 than 40 which requires $40 - (X/2) > (X/2) - n$, or $40 + n > X = (1{,}960 + n)/50$. This expression simplifies to $2{,}000 + 50n > 1{,}960 + n$ which holds for all possible values of n. Therefore, Elsa can win by choosing any number (n) below 40; the range of winning numbers is from 0 to 39!

(c) Similarly, if Elsa knew that all of her classmates would submit the number 10, Elsa will do worse by choosing anything above 10 and tie with 10. If she chooses n between 0 and 9, then $X = [49(10) + n]/50 = (490 + n)/50$. Elsa will then win if $10 - (X/2) > (X/2) - n$, or $10 + n > X$. Then $10 + n > (490 + n)/50$ or $500 + 50n > 490 + n$ which again holds for all possible values of n. Therefore, Elsa's set of best responses in this situation is the same as in part (b) in the sense that any number less than what the rest of her classmates are submitting will win the game for her; her range of winning numbers is from 0 to 9.

(d) More generally, if all of her classmates choose $0 < m < 50$ and Elsa chooses $n < m$, $X = (49m+n)/50$, and we solve for $m - (X/2) > (X/2) - n$, or $m+n > X = (49m+n)/50$, or $50m+50n > 49m+n$, or $m+49n > 0$, which is true for all $n \geq 0$. We get equality if $m = n = 0$: if all others are choosing 0, Elsa cannot do any better than to choose 0. That is the Nash equilibrium. (Intuitively, there is an incentive to deviate from any number played by everyone else in the class if that number is positive. Zero, however, works as a symmetric Nash equilibrium strategy because it is its own best response: $0 = (1/2) * 0$).

(e) To find rationalizable strategies to this game, use iterated elimination of never-best responses. We know from part (a) that any number greater than 50 is never a best response. On the basis of this fact, we determine that $(1/2) \times 50 = 25$ is also never a best response. You continue this iterated elimination until you are left with the only rationalizable strategy in the game, which is choosing zero.