Solutions to Chapter 4 Exercises

SOLVED EXERCISES

If you find a blank space where an equation or figure may appear, please select that area for it to appear.

S1. (a) For Rowena, Up strictly dominates Down, so Down may be eliminated. For Colin, Right strictly dominates Left, so Left may be eliminated. These actions leave the pure-strategy Nash equilibrium (Up, Right).

(b) Down is dominant for Rowena and Left is dominant for Colin. Equilibrium: (Down, Left) with payoffs of (6, 5).

(c) There are no dominated strategies for Rowena. For Colin, Left dominates Middle and Right. Thus, these two strategies may be eliminated, leaving only Left. With only Left remaining, for Rowena, Straight dominates both Up and Down, so they are eliminated, making the pure-strategy Nash equilibrium (Straight, Left).

(d) Beginning with Rowena, Straight dominates Down, so Down is eliminated. Then for Colin, Middle dominates both Right and Left, so both are eliminated, leaving only Middle. Because Straight and Up both give a payoff of 1, neither may be eliminated for Rowena, so there are two pure-strategy Nash equilibria: (Up, Middle) and (Straight, Middle).

S2. (a)

(a) Zero-sum game (payoffs in all cells sum to 4).

(b) Non-zero-sum.

(c) Zero-sum game (payoffs in all cells sum to 6).

(d) Zero-sum game (payoffs in all cells sum to 7).

(b) In this solution, we use the fact that strategy X cannot possibly be superdominant to strategy Y for a player unless X (strictly) dominates Y. For this reason, one only needs to check for superdominance when the player has a (strictly) dominant strategy.

(a) As shown in the solution to Exercise S1, Up is dominant for Rowena and Right is dominant for Colin. Are these strategies superdominant as well? Rowena's lowest possible payoff when playing Up (3, if Colin plays Right) is higher than her highest possible payoff when playing Down (2, if Colin plays Left), so Up is superdominant for

Rowena. By contrast, Colin’s lowest possible payoff when playing Right (1, if Rowena plays Up) is lower than his highest possible payoff when playing Left (2, if Rowena plays Down), so Right is not superdominant for Colin.

(b) As shown in the solution to Exercise S1, Down is dominant for Rowena and Left is dominant for Colin. Are these strategies superdominant as well? Rowena’s lowest possible payoff when playing Down (4, if Colin plays Right) is higher than her highest possible payoff when playing Up (2, if Colin plays Left), so Down is superdominant for Rowena. Similarly, Colin’s lowest possible payoff when playing Left (4, if Rowena plays Up) is higher than his highest possible payoff when playing Right (2, if Rowena plays Down), so Left is superdominant for Colin.

(c) As shown in the solution to Exercise S1, Rowena does not have a dominant strategy, and Left is dominant for Colin. So, Rowena does not have a superdominant strategy, but what about Left for Colin? Colin’s lowest possible payoff when playing Left (4, if Rowena plays Straight) is higher than his highest possible payoff when playing Right (3, if Rowena plays Straight or Down) and equal to his highest possible payoff when playing Middle (4, if Rowena plays Up); so Left is superdominant to Right, but is not superdominant to Middle.

(d) As shown in the solution to Exercise S1, neither Rowena nor Colin has a (strictly) dominant strategy, so neither can possibly have a superdominant strategy. Note: Middle is weakly dominant for Colin, but because Colin is indifferent between Middle and Right when Rowena plays Down, Middle is not (strictly) dominant and hence cannot be superdominant.

S3. (a)

		Collette	
		Normal	Hoard
Rowena	Normal	1 , 1	–1 , 2
	Hoard	2 , –1	0 , 0

(b) Each player has a dominant strategy to Hoard; so, the unique Nash equilibrium is for both to Hoard.

(c) This game is a prisoners' dilemma: Each player has a dominant strategy (Hoard), but both are worse off in (Hoard, Hoard) than in (Normal, Normal).

S4. (a) (i) The minima for Rowena's strategies are 3 for Up and 1 for Down. The minima for Colin's strategies are 0 for Left and 1 for Right. (ii) Rowena wants to receive the maximum of the minima, so she chooses Up. Colin wants to receive the maximum of the minima, so he chooses Right. Again, the pure-strategy (minimax) Nash equilibrium is (Up, Right).

(b) Not a zero-sum game, so minimax solution is not possible.

(c) (i) The minima for Rowena's three strategies are 1 for Up, 2 for Straight, and 1 for Down. The minima for Colin's strategies are 4 for Left, 2 for Straight, and 1 for Right. (ii) Rowena wants the strategy that gets her the maximum of her minima, or 2, which she gets from playing Straight. Colin's maximum of the minima is 4, so he plays Left. This yields the pure-strategy (minimax) Nash equilibrium of (Straight, Left).

(d) (i) The minima for Rowena's strategies are 1 for Up, 1 for Straight, and 0 for Down. The minima for Colin's strategies are 1 for Left, 6 for Middle, and 4 for Right. (ii) Rowena wants to receive the maximum of the minima, so she chooses Up or Straight. Colin wants to receive the maximum of the minima, so he chooses Middle. Again, the two pure-strategy (minimax) Nash equilibria are (Up, Middle) and (Straight, Middle).

S5. (a) Rowena has no dominant strategy, but Right dominates Left for Colin. After eliminating Left for Colin, Up dominates Down for Rowena, so Down is eliminated, leaving the pure-strategy Nash equilibrium (Up, Right).

(b) Down and Right are weakly dominant for Rowena and Colin, respectively, leading to a Nash equilibrium at (Down, Right). Best-response analysis shows another Nash equilibrium at (Up, Left).

(c) Down is dominant for Rowena; Colin will then play Middle. The Nash equilibrium is (Down, Middle).

(d) There are no dominant or dominated strategies. Best-response analysis shows an equilibrium at (North, East) with payoffs of (7, 4). (The equilibrium is not in dominant strategies—another interesting point to convey to students.)

S6. (a) Neither Rowena nor Colin has a dominant strategy, because neither has one action that is the best response, regardless of the opponent's action.

(b) For Colin, East dominates South, so South may be eliminated. Then, for Rowena, Fire dominates Earth, so Earth may be eliminated. Doing so then allows East to dominate North for Colin, so

North may be eliminated. Finally, for Rowena, Water dominates Wind, so Wind may be eliminated. Iterative elimination of dominated strategies reduces the game to the following:

		Colin	
		East	West
Rowena	Water	2, 3	1, 1
	Fire	1, 1	2, 2

(c) The game is not dominance solvable, because a unique solution cannot be attained through iterated elimination of dominated strategies. See the game table in part (b) for the result of iterated elimination of dominated strategies.

(d) There are two pure-strategy Nash equilibria, which are (Water, East) and (Fire, West). (There is also a mixed-strategy Nash equilibrium, but that will be addressed in Chapter 7.)

S7. False. A dominant strategy yields the highest payoff available to the player against each of her opponent's strategies. Playing a dominant strategy does not guarantee that she will end up with the highest of all possible payoffs. In the Prisoners' Dilemma game, both players have dominant strategies, but neither gets the highest possible payoff in the equilibrium of the game.

S8. The game table is given below. Best-response analysis shows there are two pure-strategy Nash equilibria (underlined in the table): (Help, Not Help), with payoffs (2, 3) to (I, You); and (Not Help, Help) with payoffs (3, 2).

		You	
		Help	Not
I	Help	2, 2	<u>2</u>, <u>3</u>
	Not	<u>3</u>, <u>2</u>	0, 0

S9. (a) Best-response analysis shows that there are two pure-strategy Nash equilibria: (Lab, Lab) and (Theater, Theater).

(b) Chapter 4 describes numerous games with multiple equilibria, so we shall examine each. The game is not Chicken, because the pure-strategy Nash equilibria occur when the players choose the same strategy, whereas in Chicken, the pure-strategy Nash equilibria occur when the players choose different strategies. Pure coordination games, assurance games, and battle of the sexes have two pure-strategy Nash equilibria in which the players choose the same strategy. But due to the asymmetric payoffs, only one is a best response. In pure coordination games, the payoffs to both parties are identical, which is not the case in this game. In assurance games, although the payoffs are different, both parties clearly desire one pure-strategy Nash equilibrium over another. Therefore, the most similar multiple-equilibrium game is battle of the sexes, because the pure-strategy Nash equilibria occur when the parties use the same strategy, even though the parties desire different equilibria. For example, in this question, the science faculty clearly wants the lab more than the theater, and the humanities faculty wants the theater more than the lab, but both are better off choosing the same thing rather than disagreeing.

S10. (a) The Nash equilibria are (1, 1), (2, 2), and (3, 3). You could argue that (1, 1) is a focal point, because it's the only equilibrium giving payoffs of 10 to each, and it might be hard to coordinate on one of the other two equilibria that give payoffs of 15 to each.

(b) Expected (average) payoff from flipping a (single) coin to decide whether to play 2 or 3 is $0.25 \times 25 + 0.25 \times 25 + 0.5 \times 0 = 12.5$. The average payoff is therefore higher than would be achieved if (1, 1) were focal and each player got 10. The risk that the players might do different things is most important if they have only one opportunity to play, because then each would get zero 50% of the time. Such fears might make the (1, 1) equilibrium look more attractive.

S11. (a) If one player chooses Split, each player gets $5,000 (5K) if the other player also chooses Split, but if the other player chooses Steal, that player gets $10,000 (10K). If one player chooses Steal and the other also chooses Steal, each player gets $0. This gives us the following game table when players care only about how much money they win:

		Colin	
		Split	Steal
Rowena	Split	5K, 5K	0, 10K
	Steal	10K, 0	0, 0

(b) Weakly dominant. Choosing Steal always gives a player at least as much money as choosing Split. However, if the other player chooses Steal, then that player gets the same payoff (0), so Steal is not *strictly* dominant.

(c) There are three pure-strategy Nash equilibria: (Steal, Split); (Split, Steal); and (Steal, Steal).

(d) Adding a desire to avoid looking foolish causes players to *strictly* prefer to Steal when the other player Steals, causing the game to become a prisoners' dilemma. In particular, Steal becomes strictly dominant [changing the answer to part (b)] and (Steal, Steal) becomes the unique pure-strategy Nash equilibrium [changing the answer to part (c)].

(e) As a kind-hearted soul, Colin strictly prefers Split when Rowena chooses Steal. Thus, Colin does not have a dominant strategy [changing the answer to part (b)] and (Steal, Split) becomes the unique pure-strategy Nash equilibrium [changing the answer to part (c)].

S12. (a) The game tables follow:

Carlos	Yes	Bernardo	
		Yes	No
Arturo	Yes	2, 2, 2	2, 5, 2
	No	5, 2, 2	5, 5, 2

Carlos	No	Bernardo	
		Yes	No
Arturo	Yes	2, 2, 5	2, 5, 5
	No	5, 2, 5	0, 0, 0

(b) Best-response analysis shows that three pure-strategy Nash equilibria occur when two children say No and one child says Yes.

(c) A natural focal point is where Arturo and Bernardo write No and Carlos writes Yes, because Arturo and Bernardo did not break the lamp. They reason that if they both say No, then Carlos is forced to consider between saying Yes and receiving \$2 or saying No and receiving \$0. Thus, Carlos has an incentive to say Yes, and Arturo and Bernardo will receive a payoff of \$5.

S13. There are three ticket buyers, and each ticket buyer can do three things: not purchase a ticket (represented as $0), purchase a $15 ticket, and purchase a $30 ticket. To represent this game, we need three three-by-three payoff tables, where each table represents the strategies of the first two players and one strategy of the third. In the payoff tables below, the first number represents Larry's payoff, the second number represents Curly's payoff, and the third number represents Moe's payoff. In all cases, payoffs are calculated by subtracting the cost of a purchased ticket and adding the value of any prize won; all payoffs are in dollars, with the dollar signs omitted to save space. Best responses are underlined in the following tables and cells that are Nash equilibria are shaded.

Moe	$0	Curly		
		$0	$15	$30
Larry	$0	0, 0, 0	0, 15, 0	0, 0, 0
	$15	15, 0, 0	0, 0, 0	−15, 0, 0
	$30	0, 0, 0	0, −15, 0	−15, −15, 0

Moe	$15	Curly		
		$0	$15	$30
Larry	$0	0, 0, 15	0, 0, 0	0, 0, −15
	$15	0, 0, 0	−5, −5, −5	−15, 0, −15
	$30	0, 0, −15	0, −15, −15	−15, −15, −15

Moe	$30	Curly		
		$0	$15	$30
Larry	$0	0, 0, 0	0, −15, 0	0, −15, −15
	$15	−15, 0, 0	−15, −15, 0	−15, −15, −15
	$30	−15, 0, −15	−15, −15, −15	−20, −20, −20

Best-response analysis shows that there are no pure-strategy Nash equilibria when any player spends $30 to purchase a ticket. There are six pure-strategy Nash equilibria. Three occur when two purchasers spend nothing and the other spends $15. The other three occur when two players spend $15 and the third spends nothing.

S14. (a) The game may be described as a zero-sum game, but for clarity, we have included both payments in the game table:

		Bruce	
		1	2
Anne	1	1, 0	0, 1
	2	0, 1	1, 0

(b) Best-response analysis shows that no combination of actions is a pure-strategy Nash equilibrium.

S15. (a) With only two men, two brunettes, and one blonde, the payoffs are as follows:

		Young Man 2	
		Approach	Approach
Young Man 1	Approach	0, 0	10, 5
	Approach	5, 10	5, 5

There are two Nash equilibria in which one young man approaches the blonde and one the brunette: (Approach blonde, Approach brunette) and (Approach brunette, Approach blonde) with payoffs of (10, 5) and (5, 10).

(b) For three young men with three brunettes and one blonde, the payoffs are as follows:

Young Man 3

Approach blonde

		Young Man 2	
		Approach blonde	Approach brunette
Young Man 1	Approach blonde	0, 0, 0	0, 5, 0
	Approach brunette	5, 0, 0	5, 5, 10

Approach brunette

		Young Man 2	
		Approach blonde	Approach brunette
Young Man 1	Approach blonde	0, 0, 5	10, 5, 5
	Approach brunette	5, 10, 5	5, 5, 5

This time, there are three Nash equilibria. Each has the same characteristics: one young man approaches the blonde, and the other two approach brunettes. The young man approaching the blonde gets a payoff of 10; the other two get payoffs of 5.

(c) With four young men, four brunettes, and one blonde, there will be four Nash equilibria. In each equilibrium, one of the young men approaches the blonde (and gets a payoff of 10), and the other three approach brunettes (and get payoffs of 5 each).

(d) For n young men, with n brunettes and 1 blonde, there will be n Nash equilibria. Let k be the number of other men approaching the blonde. If you are one of the young men and $k = 0$, you get a payoff of 10 from approaching the blonde and a payoff of 5 from approaching a brunette. For any other k, you get 0 from approaching the blonde and 5 from approaching a brunette. Therefore, if any one of the n young men approaches the blonde and the rest approach brunettes, everyone's choice is optimal, given the choices of the others. All such strategy configurations are Nash equilibria. In each equilibrium, one young man approaches the blonde (for a payoff of 10), and the rest each approach a brunette (for a payoff of 5). The outcome in which all the men approach brunettes cannot be a Nash equilibrium. It yields payoffs of 5 to each young man, but each could have gotten 10 if he had chosen to approach the blonde instead.